Simulation of the behavior of bidders in first-price sealed-bid auctions on communication networks

P. GARCIA-DIAZ (1), K. HACKBARTH (2), A. PORTILLA-FIGUERAS (1), S. SALCEDO-SANZ (1)
Signal Theory and Communications Department (1)
University of Alcalá
Escuela Politécnica, Campus universitario. 28871 Alcalá de Henares, Madrid
SPAIN
Communication Engineering Department (2)
University of Cantabria
ETSII y de Telecomunicación, Santander, Cantabria
SPAIN

Abstract: - Auction mechanisms have arisen as very efficient methods for scarce resource allocations. For this reason there is a current research topic based on the application of these economics mechanisms to the design of communication networks, mainly in the provision of QoS to new multimedia services. The main research works are oriented to the assignment and pricing rules, that is, the network resource assigned to the user (the bandwidth on a link, or the route of a traffic demand) and the amount of money it is going to cost to him respectively. However, as far as we know, there is a little research focused in the analysis of the behaviour of the user. Note that an auction is in fact a competition where the user will behave selfishly in order to obtain the greater profit. In this paper we present the first results of a simulator which analyzes the behaviour of the user in an auction over a communication link. They obtain the revenue of the auctioneer that is the network provider, and the resource allocation under different schemes of user behaviour, from a very conservative character up to most risky behaviours.

Key-Words: - Network capacity, congestion, auction, bidders’ behavior, and simulator.

1 Introduction

Internet network is composed by several interconnected autonomous systems (AS). Each one can be described as a specific network administrated by an independent entity. In our globalized world, a user works and communicates not only with users in his native network, but also with users in other networks. On the other hand, there is a lot of new multimedia services, as VoIP (voice over IP) or “Triple Play” services (telephone+Internet+TV) [1] which are demanding a guaranteed quality of service (QoS). QoS can be measured mainly in terms of bandwidth and transmission delay. The bandwidth is associated to the capacity of the transmission links and the delay depends critically on the route assigned to the demand of the user. Note that a very large route implies the traffic has to go through a large number of network elements and links and therefore the delay increases.

The increasing bandwidth demand of the users causes a high probability of network congestion. In those situations, the network may not fulfill the QoS constrains of real time services. A feasible way to solve this problem may consist of prioritizing the traffic in the network.

We could think of a system where the users prioritize their own traffic themselves. Obviously this pattern would not work, because users (PC’s, autonomous systems or humans) will behave selfishly. Hence they will always establish the highest priority for their traffic because they would consider their own demands as the most important ones. This would take us again to a network congestion situation. A more effective strategy for resources assignment and control congestion may be based on traffic priority according to pricing.

Pricing mechanisms have been broadly applied in communication networks [2]. Specifically, economic mechanisms, like auctions, have revealed as appropriate methods for assigning efficiently network resources to the users [3]. This assignment not only consists of bandwidth allocation but also routing inside a single provider (intra-domain) and/or through different autonomous systems (inter-domain) [4]. It is straight forward that these three problems have an increasing complexity as it is shown in Figure 1.
An auction is composed of several parts: the auctioneer (network provider), the bidders (service providers and final users), the award auctioned (total transmission capacity through the network) and the assignment and pricing rules. A key factor is the bidders’ behavior because an auction is, in fact, a competition, where the users behave selfishly trying to obtain their maximal profit [5]. Our work consists of a study of the users’ behavior during the competition for transmissions resources in a communication network through auctioning.

When the award auctioned is a single item, there is a unique winner of the process. But in our case, the award consists of a transmission capacity \( Q \), which maybe divided in multiple units. Due to existence of network congestion, not all demands can be served and an allocation rule is needed. The allocation rule determines how to split and assign the resource, i.e., which bidders will access to the network and which capacity \( q_i \) will use each one. Due to the limited transmission capacity of the network, resources assignment must fulfill the condition of the equation (1.1), where \( M \) users demand a portion of \( Q \) capacity. \( a_i \) is the assigned capacity to the user \( i \) by the auctioneer. \( x_i \) represents a binary value that is “one” when the user receives some capacity (is a winner of the auction) and “zero” in other case. Note that \( a_i \) maybe does not correspond to the demanded capacity \( q_i \). Nevertheless \( a_i \leq q_i \),

\[
\sum_{i=1}^{M} x_i a_i \leq Q \tag{1.1}
\]

In auction mechanisms in economy environments with multiple goods, it is not always guaranteed that all winners pay the same price per unit. Pricing rule fixes the price \( c_i \) to pay by each bidder \( i \) which receives a portion of the award.

It is straightforward that bidders wish to pay as less as possible. On communications, the valuations of the bidders are private, the bidders and auctioneer do not know the valuation of others bidders. Each user \( i \) value the auctioned capacity \( q_i \) in a different way. The value is represented as a valuation function \( \theta_i \), Figure 2 shows three types of valuation functions. Functions as case (A) corresponds to non-elastic demands, where the valuation does not change with the amount of capacity requested, whereas cases (B) and (C) belong to situations of elastic demands. In (B) the valuation function has a constant gradient, more capacity is more valued in a proportional way. Case (C) has a variable gradient valuation/capacity.

The auctioneer can assign the demanded capacity \( q_i \) to the user \( i \), or a value \( a_i < q_i \).

Let \( \theta_i(q_i) \) the valuation function of the capacity \( q_i \) of user \( i \) as \( \theta_i(q_i) \). The user always bids a price \( p_i \leq \theta_i(q_i) \) because of he wants to buy as cheap as possible. We denote the utility function (or user’s benefit) as the subtraction of the valuation of an assigned resource \( a_i \) and the cost \( c_i \) that the user pay for it. See the equation (1.2) where \( U_i \) means the utility function of user \( i \).

\[
U_i = \theta_i(a_i) - c_i \tag{1.2}
\]

As said before, in an auctioning process the bidders behave selfishly and untruthfully, bidding fewer amounts than they value the resource.

On the other hand, the network provider wishes to maximize his revenue. The auctioneer will split the total resource \( Q \) in a manner that he gets the maximum profit, see equation (1.3), where again \( x_i \) is equals to “one” if the user \( i \) receives some capacity and “zero” in other case.
The two main disadvantages are “winner’s curse” and collusions. “Winner’s curse” is produced when the bidder pays more than what the award is worth.\(^1\)

### 2.2 Dutch auction

A Dutch auction starts with an exceedingly high price for the item. The price is decreasing progressively until a buyer shows his interest to acquire it. When there are multiple units of an item, the bidders get in units with at lower prices successively until the units run out. The allocation and pricing rules of Dutch auctions are different to the English ones. During an English auction, a high interested buyer has more response time to buy, in the sense that if other bidder offers a higher price, the first one has the opportunity to pay more and get the item. In a Dutch auction, the very interested buyer must ahead of the rest of bidders. The first one which accepts the price, anyone more can get out the award. In this way, the auctioneer wins more usually in Dutch auctions than in English ones.

### 2.3 First-price sealed-bid auction

In a sealed bid auction, no others bidders know the bid price of anyone. The process has two stages: at first, the bidders inform to the auctioneer their bid prices and at second step the winners are announced. The term “first-price” specifies the allocation rule mentioned in the introduction. In the second stage, the bid prices are sorted from high to low. For single item, the winner is who bid the highest price, for multiple units, they are selected from this sorted list until the units are exhausted. Reynolds [2] refers to this characteristic as “discriminatory” because not all winners pay the same per unit. The bidder strategy will be to put near the winning bid prices interval but inside the lowest part, so he would pay less than the others.

### 2.4 Vickrey auction

Under a Vickrey auction, the price paid for each user, is the loss of declared welfare he imposes to others users due to his presence in the auction. This mechanism satisfies the following conditions:

- Incentive compatibility: for each user, bidding truthfully is a dominant strategy. They present offers similar to their real valuation functions.

\(^1\) Collusions consist of the agreement among bidders (called “ring” in economy world), where they agree on a highest amount, and anybody bid more than this fixed amount.
• Individual rationality: each truthful bidder obtains a non-negative utility as equation (1.2).

4 Implementation of simulator

In this section we present the implementation of a simulator to auction specific network resources. It analyzes the users’ behavior in competition. As said before, we select the first-price sealed-bid auction. Sealed auctions are nearer to communication applications than English and Dutch ones. The computational complexity of a Vickrey auction is NP ("Non-deterministic Polynomial time") whereas the complexity of a first-price sealed-bid auction is only polynomial time. We select first-price auctioning as an approximation of second-price one.

The system is composed by the following parts: the network provider (the auctioneer), the transmission capacity of a single link ($Q$, the item to sell by multiple portions $a_i$), and the potential users (PC’s, autonomous systems or humans, who are the $M$ bidders). The link under analysis connects two any nodes $N_1$ and $N_2$ of a network. Figure 3 represents the described system. We considerer the link is under congestion, so the network provider must prioritize the traffic through auctioning. The assigned capacity by an auction must keep the equation (1.1) and also is advisable to produce the maximum revenue to the network provider.

Consider a vector $q = (q_1, q_2...q_M)$ which represents the capacity bided by M users. Obviously they have different valuations of the demands so they are willing to pay different prices, named as $p = (p_1, p_2...p_M)$. Remember that bidders really want to pay as least as possible as long as it gets some capacity.

We define that a user $i$ has an offer range as show in Figure 4. He values the $q_i$ capacity as $P_{Max,i}$, which is the valuation function $\theta_i(q_i)$, whereas the user offers a value $p_i < P_{Max,i}$. $P_{Min}$ represents a minimum of the valuation of the good.

The result of each auction is a set of winners who share the available capacity. Let us write the vector $a = (a_1, a_2...a_M)$ where $a_i$ means the assigned capacity to the bidder $i$. The vector $a$ participate in equation (1.1). The cost of the assigned capacity is written as a vector $c = (c_1, c_2...c_M)$, that characterizes the price that users pay for the transmission service.

Due to an auction is really a competition, the users’ behavior has a lot of influence on the results of the process, chiefly with successive auctions. We consider systems where bidders remember the results of the last auction. That means bidders fix their offers according to the capacity assigned in the last auction.

The basic behave consists of increasing the price for next auction if they didn’t get any capacity, and decreasing the offered price if they got before some capacity. A first classification of bidders according to their behavior distinguishes three kinds of users: risk-seeking (sometimes called risk-loving), risk-averse (or risk-hating) and neutral-risk users:

- A risk-seeking bidder has high variation of pricing between two consecutive auctions, his behavior is aggressive in sense of he changes sharply prices.
- Risk-averse bidders change slightly the price of the last auction, they have a previsible behavior.
- Neutral-risk bidders have a middle behavior among risk-seeking and risk-averse one.

Of course it is possible to expand this first classification finding new groups inside each large type. For example, risk-averse users can be more or less conservative. A conservative bidder never decreases his offered price, he only increases it if he didn’t get any capacity before. We establish the width of the decision window of these kinds of behaviors.

- A risk-seeking user with a value “zero” of $x_i$ selects always a random bid between the previous $p_i$ and his valuation of the goods ($P_{Max}$). See Figure 5(A) dotted area. The range from $p_i$ and $P_{Max}$ is named decision window. A risk-seeking bidder who won the previous auction fix his
following offer as a random price between \( p_i \) and the minimum \( P_{Min} \) as we can see in Figure 5(A) scratched area. In this case the decision window is the gap between \( p_i \) and \( P_{Min} \).

- A risk-neutral bidder behavior is showed in Figure 5(B), where the decision window is the 50% of the risk-seeking one. If the user won the last auction then he selects a random price between \( p_i \) and \( (P_{Max}) \).

- Figure 6 represents behavior of risk-averse bidders. We assign a width of 20% for the decision windows. If the user has a value of \( x_i \) equals 1, the behavior of conservative and non-conservative users is the same (Figure 6 dotted areas). They increase the offer a random percentage from 0% to 20%. If the user has a value of \( x_i=0 \) then only the non-conservative bidders decrease their following offers if they won the last auction, see Figure 6(A) scratched area whereas the conservative ones keep their previous price \( P_0 \), as shows Figure 6(B).

5 Results
A large set of experiments have been performed but for the sake of simplicity we are going to present only three considering the user classification mentioned above. We assume that winner always obtain the demanded capacity \( q_i \) it implies \( a_i=q_i \). Next we define the three different simulation scenarios explained in this section. They are represented in Figure 7:

- Scenario 1: all bidders are risk-neutral.
- Scenario 2: 50% of users have risk-seeking behavior and 50% risk-averse (non-conservative) one.
- Scenario 3: distribution of population as 20% risk-seeking bidders; 30% of risk-neutral users; 25% conservative and 25% non-conservative risk-averse users.

![Figure 7 Distribution of the population of the three different scenarios according the risk characteristic of users.](image)

The size of population is fixed to 40, as it may be the number of employers of a small-medium company that demands transmission facilities to a service provider. The auctioneer, that is, the service provider, auctions his capacity every 10 minutes during the 24 hours of days. We have selected this time slot because this is the average duration of an ftp session. This frequency means 148 auctions per day. Therefore each simulation contains 150 auctions as approximation of this scenario.

First we compare the total revenue of the service provider for the three scenarios. Figure 8 shows the accumulated revenue for each distribution. Note that scenario 3 contributes with a higher income to the auctioneer than the others, whereas scenarios 1 and 2 produce similar revenue.
Concerning scenario 2, Figure 9 shows the accumulated revenue produced by each set of users (risk-seeking and non-conservative ones) and the percentage of assigned capacity of each set of users represented by the striped and shadowed areas. After some transitory time, risk-averse bidders reach more capacity than the risk-loving ones. Note further that the risk-seeking group wins less capacity, paying more, than non-conservative group whose decision window is tighter (remember Figure 6).

Simulations with population from scenario 3 have some important conclusions obtained from Figure 10 and Figure 11. The first one is that the system reaches stabilization after a transitory period. After 50 auctions the capacity is allocated in the same average proportions: 40.5% of the goods are assigned to the conservative users; 26.5% are allocated to neutral bidders; 19.5% are assigned to risk-seeking and 13.5% of the facilities are located to the non-conservative users. This percentage is represented on the column F in Figure 10. The column A on the same figure represents the distribution of the total population.

Finally, Figure 12 maps the allocation of capacity to the users on the three scenarios. The assignment is represented by a simple black dot in the user column. Rows represent consecutive auctions. Note that in the third scenario the conservative strategy is the best the conservative users obtain capacity in the majority of the auctions. This means that they get profit from the aggressive strategy of the risky users.

6 Future research
This paper shows the first results in the development of a simulator of the user behavior in auctions for
communication network resources. We have considered a single link with capacity constraints and different user characters. The bids of the users change with time depending on the past auctions and on their own natures. Under the defined conditions we can observe that the results tend to stabilize. The maximum revenue of the auctioneer is obtained in the real scenario due to the strong competition of the users. Note that the risk seeking users pay the highest individual amount although they obtain less capacity than the other groups. This situation is produced by the kind of auction.

Regarding future work lines, the first research work consists of improving the simulator with new, and more realistic, behavior patterns, based mainly in Gaussian distributions. The assignment and pricing rules will be also improved changing the first price sealed mechanism by a Vickrey auction which satisfies the Nash criteria. Finally we will compare the results of the simulator with analytical studies derived from the application of modern heuristics to this kind of economic mechanisms.

7 Acknowledgments
The research explained in this paper is being financed by the Network of Excellence EuroNGI: Design and Engineering of the Next Generation Internet, of the VI European Framework, contract Nº IST 50-7613 and the project of the University of Alcala: “Desarrollo de métodos y modelos algorítmicos para la optimización multicapa de redes dorsales basadas en GMPLS”, UAH PI 2005/079.

References:
Figure 12 Representation of the allocation to the users in the successive auctions in the considered scenarios.