Observation of the Baryonic Flavor-Changing Neutral Current Decay $\Lambda^0_b \to \Lambda \mu^+ \mu^-$


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Rare decays of hadrons containing bottom quarks through the process $b \to s \mu^+ \mu^-$, where $b$ is a bottom quark and $s$ is a strange quark, occur in the standard model (SM) with $O(10^{-6})$ branching ratios [1,2]. The $b$ and $s$ quarks carry the same charge but different flavor, so this process is a flavor-changing neutral current (FCNC) decay. FCNC decays are suppressed at tree level in the SM, and must occur through higher order, and more suppressed, loop amplitudes. Their suppressed nature and clean experimental signature, along with reliable theoretical predictions for their rates [1,3,4], make them excellent search channels for new physics. With multibody final states, these decays offer sensitivity to new physics in a number of kinematic distributions in addition to the total branching ratio. In this Letter, we report measurements of the total branching ratios of FCNC decays, as well as their differential branching ratios as a function of $q^2 = M_{\mu\mu}^2$, where $M_{\mu\mu}$ is the dimuon invariant mass. Exclusive decays of $B \to K^{(*)} \mu^+ \mu^-$ have been observed by BABAR [5], Belle [6], and CDF [7]. The CDF experiment also recently reported the observation of $B_0^+ \to \phi(1020) \mu^+ \mu^-$ [7].

We report the first observation of the baryonic flavor-changing neutral current decay $\Lambda_b^0 \to \Lambda \mu^+ \mu^-$ with 24 signal events and a statistical significance of 5.8 Gaussian standard deviations. This measurement uses a $p\bar{p}$ collisions data sample corresponding to $6.8 \, \text{fb}^{-1}$ at $\sqrt{s} = 1.96 \, \text{TeV}$ collected by the CDF II detector at the Tevatron collider. The total and differential branching ratios for $\Lambda_b^0 \to \Lambda \mu^+ \mu^-$ are measured. We find $\mathcal{B}(\Lambda_b^0 \to \Lambda \mu^+ \mu^-) = [1.73 \pm 0.42(\text{stat}) \pm (\text{syst})] \times 10^{-6}$. We also report the first measurement of the differential branching ratio of $B_b^0 \to \phi \mu^+ \mu^-$, using 49 signal events. In addition, we report branching ratios for $B^+ \to K^+ \mu^+ \mu^-$, $B_0^+ \to K^0 \mu^+ \mu^-$, and $B \to K^+(892) \mu^+ \mu^-$ decays.

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No significant departure from the SM has been found thus far. In addition, the study of the baryonic $b \rightarrow s \mu^+ \mu^-$ decays is very important, since the baryonic FCNC decays are sensitive to the helicity structure of effective Hamiltonian, which is lost in the hadronization of the mesonic decays [8]. Although the theoretical calculations of the exclusive baryonic $b \rightarrow s \mu^+ \mu^-$ decays have large uncertainties compared to the mesonic decays due to additional degrees of freedom in the baryon bound states, the measurements of the total and the differential branching ratios can help the improvement of the theoretical treatments. One can also compare the measurements of the mesonic $b \rightarrow s \mu^+ \mu^-$ decays with the baryonic decays, which follow the common quark transition. Measurements of both mesonic and baryonic FCNC decays therefore provide additional tests of the SM and its extensions.

One can also compare the measurements of the mesonic $b \rightarrow s \mu^+ \mu^-$ decays with the baryonic decays, which follow the common quark transition. Measurements of both mesonic and baryonic FCNC decays therefore provide additional tests of the SM and its extensions. However, no $b$ baryon FCNC decay has been observed and there are few experimental constraints on their decay rates. The $\Lambda_b^0 \rightarrow \Lambda \mu^+ \mu^-$ decay is considered promising in this respect [8–11] and experimentally accessible since the branching ratio is predicted as $(4.0 \pm 1.2) \times 10^{-6}$ [10].

The data sample used in the measurements reported in this Letter corresponds to an integrated luminosity of $20.80 \pm 0.12 \text{ fb}^{-1}$ from $p \bar{p}$ collisions at a center-of-mass energy of $\sqrt{s} = 1.96 \text{ TeV}$ collected with the CDF II detector between March 2002 and June 2010. The $\Lambda_b^0 \rightarrow \Lambda \mu^+ \mu^-$ decay is reconstructed and measurements are made of the total branching ratio and the differential branching ratio as a function of $q^2$. Besides the updated branching ratios of $B^0_s \rightarrow \phi \mu^+ \mu^-$, $B^+ \rightarrow K^+ \mu^+ \mu^-$, and $B^0 \rightarrow K^*(892)^0 \mu^+ \mu^-$, we report the branching ratios of $B^0 \rightarrow K^0 \mu^+ \mu^-$ and $B^+ \rightarrow K^*(892)^+ \mu^+ \mu^-$, which are measured for the first time in hadron collisions. We also report the first measurement of the differential branching ratio as a function of $q^2$ of $B^0_s \rightarrow \phi \mu^+ \mu^-$. To cancel the dominant systematic uncertainties, decay rates for each rare channel $H_b \rightarrow h \mu^+ \mu^-$ are measured relative to the corresponding resonant channel $H_b \rightarrow J/\psi h$ with $J/\psi \rightarrow \mu^+ \mu^-$, used as a normalization, where $H_b$ represents the $b$ hadron and $h$ stands for $\Lambda$, $\phi$, $K^+$, $K^0$, $K^-$, $K^{*+}$, and $K^{*-}$. Charge conjugation is implied throughout the Letter.

The reconstruction of the exclusive $b$ hadron events starts with a dimuon sample selected by the online trigger system [12] of the CDF II detector [13]. The trigger system utilizes information from muon detectors and the central outer tracker [14]. Muon chambers CMU and CMX [15] cover $|\eta| < 0.6$ and $0.6 < |\eta| < 1.0$, respectively, [16]. The CMU muon chamber covers $|\eta| < 0.6$ and is located behind the CMU and an additional steel absorber. The dimuon trigger requires a pair of oppositely charged particles with a momentum transverse to the beam line $p_T \geq 1.5 \text{ GeV}/c$, which are matched to track segments in the CMU or CMX chambers. At least one of the muon tracks is required to have a CMU track segment. The trigger also requires that the dimuon pair satisfies either $L_{xy} > 100 \mu\text{m}$, where the transverse decay length $L_{xy}$ is the flight distance between the dimuon vertex and the event primary vertex [17], or $p_T \geq 3.0 \text{ GeV}/c$ and matched segments in both CMU and CMP chambers for one of the muon candidates.

Offline event selection starts with the triggered dimuon pairs. Each offline track is required to satisfy more stringent requirements on the number of hits used to reconstruct the track. The dimuon selection requirements used in the trigger are repeated with the higher quality offline tracks. The decay length and invariant mass of each dimuon pair are calculated after a vertex fit using the muon tracks. Dimuon pairs are classified according to their invariant mass $M_{\mu\mu}$. Dimuons from FCNC $b$ hadron decays are required to be inconsistent with decaying from $J/\psi (\psi(2S))$ mesons by requiring $q^2$ values outside the window of $8.68(12.86) < q^2 < 10.09(14.18) \text{ GeV}^2/c^2$ [7]. The $J/\psi \psi(2S)$ candidates are required to have $M_{\mu\mu}$ of 50 MeV/$c^2$ of the known $J/\psi$ mass [18].

The $\Lambda_b^0 \rightarrow \Lambda \mu^+ \mu^-$ candidates are selected by combining the dimuon pairs with $\Lambda$ baryons reconstructed from decays $\Lambda \rightarrow p\pi^-$. The $p\pi^-$ pairs are required to have invariant mass consistent with the known $\Lambda$ mass [18], $p_T \geq 1.0 \text{ GeV}/c$, and a vertex displaced from the dimuon vertex. The transverse momentum of the $\Lambda_b^0$ candidate is required to be greater than 4.0 GeV/$c$. Candidates with an invariant mass calculated from two or three daughter particles compatible with $J/\psi$, $\psi(2S)$, $D^0$, $D^*$, $D^{**}$, or $\Lambda$, masses are rejected to remove backgrounds from these charm-hadron decays [7]. The $B^0_s \rightarrow \phi \mu^+ \mu^-$ candidates are reconstructed from dimuons together with a pair of oppositely-charge kaons consistent with a $\phi$ decay with a selection similar to that of $\Lambda_b^0 \rightarrow \Lambda \mu^+ \mu^-$. The $B^0_s \rightarrow K^{0\ast+} \mu^+ \mu^-$ candidates, where $K^{0\ast+}$ is one of $\{K^+; K^0; K^0\ast; K^{*0}; K^{*+}\}$, are formed from a dimuon combined with up to three charged tracks. The $K^0_s$ meson is reconstructed in its $\pi^+\pi^-$ final state by requiring the dipion mass to be consistent with the known $K^0_s$ mass [18]. Details about the reconstruction of the decays of $K^{0\ast0} \rightarrow K^+\pi^-$ and $\phi \rightarrow K^+K^-$ can be found in Ref. [7]. Cross feed between $\Lambda_b^0 \rightarrow \Lambda \mu^+ \mu^-$ and $B^0 \rightarrow K^0_s \mu^+ \mu^-$ is suppressed by evaluating the momentum imbalance of $\Lambda$ and $K^0_s$ daughters [19]. We utilize the correlation between invariant mass and the asymmetry $\alpha \equiv (q^-_L - q^+_L)/(q^+_L + q^-_L)$, where $q^+_L$ ($q^-_L$) is the longitudinal momentum of the positive (negative) decay product relative to the direction of the $\Lambda$ or $K^0_s$. We reject candidates that satisfy $-0.26 < -1.9M(K^0_s) + |\alpha| < -0.15$ for $K^0_s \mu^+ \mu^-$ and $4.73 < 3.6M(\Lambda) + |\alpha| < 4.78$ for $\Lambda \mu^+ \mu^-$. We remove 76 (90)% of the cross feed while the signal loss is 11 (7)% for $\Lambda \mu^+ \mu^- (K^0_s \mu^+ \mu^-)$. A residual cross-feed contamination of 0.1% (0.6%) to the $\Lambda \mu^+ \mu^- (K^0_s \mu^+ \mu^-)$ signal is considered as a systematic uncertainty. To further optimize the event selection, an artificial neural network (NN) classifier is trained using simulated signal events and
The signal yield of the $\Lambda_b^0 \to \Lambda \mu^+ \mu^-$ candidates is obtained by an unbinned maximum likelihood fit to the $\Lambda_b^0$ invariant mass distribution with the signal probability density function (PDF) parametrized by Gaussian distributions using simulated signals and the background PDF modeled by a linear function. We fix the $\Lambda_b^0$ mass width for the rare decay while it is floated for the normalization channel. Different mass width between data and the simulated signal is corrected by measured mass width ratio of the normalization channel between data and the simulated signal. The signal region is defined within $\pm 40$ MeV/c$^2$ from the world average $\Lambda_b^0$ mass [7]. The statistical significance is obtained through a likelihood-ratio test between the signal plus background and background-only hypotheses interpreted assuming it distributed as a $\chi^2$ variable. The invariant mass distribution of the $\Lambda_b^0 \to \Lambda \mu^+ \mu^-$ candidates is shown in Fig. 1. In the signal region, we observe 24 $\pm$ 5 events from $\Lambda_b^0 \to \Lambda \mu^+ \mu^-$ decays while the total number of the signal candidates is 34. The statistical significance of the signal $s$ corresponds to 5.8 Gaussian standard deviations. The signal yields of $B^\pm \to \phi \mu^+ \mu^-$ and other FCNC $B$ meson decays are obtained by a similar procedure as that of $\Lambda_b^0 \to \Lambda \mu^+ \mu^-$. Each channel uses independent NN weight and PDF. The fit range for $B^+$ and $B^0$ decays is from 5.18 to 5.70 GeV/c$^2$ to avoid the region of 5.0–5.18 GeV/c$^2$, which is dominated by the feed-down background from multibody decays of $b$ hadrons. While the contribution from charmless $H_b$ decays is negligible due to the muon identification, we estimate a 1% cross talk between $B^0 \to K^0 \mu^+ \mu^-$ and $B^0 \to \phi \mu^+ \mu^-$ using simulation, and correct for it. Invariant mass distributions of $B^0 \to \phi \mu^+ \mu^-$ and other FCNC $B$ meson decays are shown in Fig. 1 and signal yields are listed in Table I.

The branching ratios of $\Lambda_b^0 \to \Lambda \mu^+ \mu^-$, $B^\pm \to \phi \mu^+ \mu^-$, and $B \to K^{(*)} \mu^+ \mu^-$ are calculated by comparing their signal event yield to that of the normalization decay modes $\Lambda_b^0 \to J/\psi \Lambda$, $B_s^0 \to J/\psi \phi$, and $B \to J/\psi K^{(*)}$, where $J/\psi \to \mu^+ \mu^-$, after the reconstruction efficiency correction:

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\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Mode & $N_{\mu\mu}$ & $s(\sigma)$ & $N_{J/\psi}$
\hline
$\Lambda_b^0 \to \Lambda \mu^+ \mu^-$ & 24 & 5.8 & 1740 $\pm$ 50

$B_s^0 \to \phi \mu^+ \mu^-$ & 49 & 7.0 & 4560 $\pm$ 80

$B^+ \to K^+ \mu^+ \mu^-$ & 234 & 19 & 72200 $\pm$ 300

$B^0 \to K^0 \mu^+ \mu^-$ & 164 & 15 & 28300 $\pm$ 200

$B^0 \to K^0 \mu^+ \mu^-$ & 28 & 9 & 9470 $\pm$ 90

$B^+ \to K^+ \mu^+ \mu^-$ & 20 & 6 & 4560 $\pm$ 80
\hline
\end{tabular}
\caption{Summary of observed yields, the statistical significance $s$, and the relative efficiency $e_{rel}$.}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Mode & $\mathcal{B}(10^{-3})$ & $\mathcal{B}(10^{-6})$
\hline
$\Lambda_b^0 \to \Lambda \mu^+ \mu^-$ & 2.45 $\pm$ 0.59 & 1.73 $\pm$ 0.42

$B_s^0 \to \phi \mu^+ \mu^-$ & 1.13 $\pm$ 0.19 & 1.47 $\pm$ 0.24

$B^+ \to K^+ \mu^+ \mu^-$ & 0.46 $\pm$ 0.04 & 0.46 $\pm$ 0.04

$B^0 \to K^0 \mu^+ \mu^-$ & 0.77 $\pm$ 0.08 & 1.02 $\pm$ 0.10

$B^0 \to K^0 \mu^+ \mu^-$ & 0.37 $\pm$ 0.12 & 0.32 $\pm$ 0.10

$B^+ \to K^{*+} \mu^+ \mu^-$ & 0.67 $\pm$ 0.22 & 0.95 $\pm$ 0.32

$B \to K^{*+} \mu^+ \mu^-$ & $\cdots$ & $\cdots$

$B \to K^* \mu^+ \mu^-$ & $\cdots$ & 1.01 $\pm$ 0.10
\hline
\end{tabular}
\caption{Measured branching ratios of rare modes. First (second) uncertainty is statistical (systematic). The last two values are for the isospin average.}
\end{table}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1}
\caption{(color online). Invariant mass of (a) $\Lambda_b^0 \to \Lambda \mu^+ \mu^-$, (b) $B_s^0 \to \phi \mu^+ \mu^-$, (c) $B^+ \to K^+ \mu^+ \mu^-$, (d) $B^0 \to K^0 \mu^+ \mu^-$, (e) $B^0 \to K^0 \mu^+ \mu^-$, (f) $B^+ \to K^{*+} \mu^+ \mu^-$, with fit results overlaid. The histograms are the data. Solid, dashed-dotted, and dotted curves show the total fit, the signal PDF, and the background PDF, respectively.}
\end{figure}
where $N_{h\mu^+\mu^-}$ is the $h\mu^+\mu^-$ yield, $N_{J/\psi h}$ is the $J/\psi h$ yield for the normalization channel, and $e_{rel} = e_{h\mu^+\mu^-}/e_{J/\psi h}$ is the relative reconstruction efficiency determined from the simulation. The calculated relative and absolute branching ratios are listed in Table II. The absolute branching ratios are obtained using world averages of the $J/\psi h$ decay rates [18]. The branching ratios of $B^0 \to K^0\mu^+\mu^-$ and $B^+ \to K^{*+}\mu^+\mu^-$ are measured for the first time in hadron collisions.

The dominant sources of systematic uncertainty are the scale-factor reweighting of the simulated signal (the trigger efficiency near the threshold) which ranges from 0.5% to 4.0% (0.8% to 7.2%), depending on the channel. We estimate the former uncertainty from the comparison of the relative efficiencies with and without reweighting and the latter uncertainty from the different $p_T$ requirements for each trigger. In the $\Lambda_b^0 \to \Lambda\mu^+\mu^-$ case we consider an additional uncertainty of 6.6% due to the unknown $\Lambda_b^0 \to J/\psi\Lambda$ polarization.

For the absolute branching ratio measurements we assign the uncertainties on the world average $B(H_b \to J/\psi h)$ [18] or the most recent measurement [20]. Contributions from other sources (e.g., background PDF shape or the decay model of the simulated event) are minor (0.3%–3.4%). The combined branching ratio is calculated by assuming isospin symmetry and using the $B^+$ and $B^0$ total widths [18]. These numbers are consistent with our previous results [7], $B$-factory measurements [5,6], and theoretical expectations [9,10].

We also measure differential branching ratios with respect to $q^2$. We divide the signal region into six bins in $q^2$. We fit the signal yield in each $q^2$ bin. In each fit, we fix the mean of the $H_b$ mass and the background slope to the value from the global fit, so that only the signal fraction is allowed to vary in the fit. Figure 2 shows the differential branching ratios for $\Lambda_b^0 \to \Lambda\mu^+\mu^-$, $B^+_s \to \phi\mu^+\mu^-$, and $B \to K^{0}\mu^+\mu^-$. For illustration, we superimpose the SM expectations, which are based on the formula in Ref. [1], with the form factors in Ref. [21], except for the case of $\Lambda_b^0 \to \Lambda\mu^+\mu^-$ decays which follows Ref. [10]. The cusp at $q^2 \sim 7$ GeV$^2$/c$^2$ is due to a change in parameter approximations. Tables III and IV summarize the differential branching ratio measurements. The two bottom rows in each table show the results for the semi-inclusive bins, which are included with ranges covering theoretically well-controlled regions.

In summary, we have updated our previous analysis of the flavor-changing neutral current decays $b \to s\mu^+\mu^-$ using data corresponding to an integrated luminosity of 6.8 fb$^{-1}$ and adding new decay channels. We report the first observation of $\Lambda_b^0 \to \Lambda\mu^+\mu^-$ and measure the total and differential branching ratios of this decay with respect to $q^2$. We also measure the total and differential branching ratios of $B \to K^{(*)}\mu^+\mu^-$ and $B^+_s \to \phi\mu^+\mu^-$, with respect to $q^2$. All measurements are consistent and competitive.

### Table III: Differential branching ratios of $\Lambda_b^0 \to \Lambda\mu^+\mu^-$, $B^+ \to K^{+}\mu^+\mu^-$, $B^0 \to K^{0}\mu^+\mu^-$, combined $B \to K\mu^+\mu^-$, in units of $10^{-7}$. The $q^2_{\text{max}}$ is 20.30 (23.00) GeV$^2$/c$^2$ for $\Lambda_b^0 \to \Lambda\mu^+\mu^-$ ($K^{+}\mu^+\mu^-$). The first (second) uncertainty is statistical (systematic).

<table>
<thead>
<tr>
<th>$q^2$ (GeV$^2$/c$^2$)</th>
<th>$\Lambda_b^0 \to \Lambda\mu^+\mu^-$</th>
<th>$B^+ \to K^{+}\mu^+\mu^-$</th>
<th>$B^0 \to K^{0}\mu^+\mu^-$</th>
<th>$B \to K\mu^+\mu^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.00, 2.00]</td>
<td>0.15 ± 0.01 ± 0.05</td>
<td>0.36 ± 0.11 ± 0.03</td>
<td>0.312 ± 0.372 ± 0.024</td>
<td>0.33 ± 0.10 ± 0.02</td>
</tr>
<tr>
<td>[2.00, 4.30]</td>
<td>1.84 ± 1.66 ± 0.59</td>
<td>0.80 ± 0.15 ± 0.05</td>
<td>0.929 ± 0.485 ± 0.070</td>
<td>0.77 ± 0.14 ± 0.05</td>
</tr>
<tr>
<td>[4.30, 8.68]</td>
<td>−0.20 ± 1.64 ± 0.08</td>
<td>1.18 ± 0.19 ± 0.09</td>
<td>0.663 ± 0.510 ± 0.052</td>
<td>1.05 ± 0.17 ± 0.07</td>
</tr>
<tr>
<td>[10.09, 12.86]</td>
<td>2.97 ± 1.47 ± 0.95</td>
<td>0.68 ± 0.12 ± 0.05</td>
<td>−0.030 ± 0.223 ± 0.005</td>
<td>0.48 ± 0.10 ± 0.03</td>
</tr>
<tr>
<td>[14.18, 16.00]</td>
<td>0.96 ± 0.73 ± 0.31</td>
<td>0.53 ± 0.10 ± 0.03</td>
<td>0.726 ± 0.257 ± 0.055</td>
<td>0.52 ± 0.09 ± 0.03</td>
</tr>
<tr>
<td>[16.00, $q^2_{\text{max}}$]</td>
<td>6.97 ± 1.88 ± 2.23</td>
<td>0.48 ± 0.11 ± 0.03</td>
<td>0.214 ± 0.182 ± 0.016</td>
<td>0.38 ± 0.09 ± 0.02</td>
</tr>
<tr>
<td>[0.00, 4.30]</td>
<td>2.65 ± 2.52 ± 0.85</td>
<td>1.13 ± 0.19 ± 0.08</td>
<td>1.268 ± 0.622 ± 0.096</td>
<td>1.07 ± 0.17 ± 0.07</td>
</tr>
<tr>
<td>[1.00, 6.00]</td>
<td>1.27 ± 2.08 ± 0.41</td>
<td>1.41 ± 0.20 ± 0.10</td>
<td>0.980 ± 0.614 ± 0.076</td>
<td>1.29 ± 0.18 ± 0.08</td>
</tr>
</tbody>
</table>
TABLE IV. Differential branching ratios of \( B_s^0 \rightarrow \phi \mu^+ \mu^- \), \( B^0 \rightarrow K^0 \mu^+ \mu^- \), \( B^+ \rightarrow K^+ \mu^+ \mu^- \), and combined \( B \rightarrow K^* \mu^+ \mu^- \), in units of \( 10^{-7} \). The \( q^2 \) is 18.90 (19.30 \( \text{GeV}^2/c^2 \)) for \( \phi \mu^+ \mu^- (K^* \mu^+ \mu^-) \). The first (second) uncertainty is statistical (systematic).

<table>
<thead>
<tr>
<th>( q^2 (\text{GeV}^2/c^2) )</th>
<th>( B_s^0 \rightarrow \phi \mu^+ \mu^- )</th>
<th>( B^0 \rightarrow K^0 \mu^+ \mu^- )</th>
<th>( B^+ \rightarrow K^+ \mu^+ \mu^- )</th>
<th>( B \rightarrow K^* \mu^+ \mu^- )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.00, 2.00]</td>
<td>2.78 ± 0.95 ± 0.89</td>
<td>1.80 ± 0.36 ± 0.11</td>
<td>1.30 ± 0.98 ± 0.14</td>
<td>1.73 ± 0.33 ± 0.10</td>
</tr>
<tr>
<td>[2.00, 4.30]</td>
<td>0.58 ± 0.55 ± 0.19</td>
<td>0.84 ± 0.28 ± 0.06</td>
<td>0.71 ± 1.00 ± 0.15</td>
<td>0.82 ± 0.26 ± 0.06</td>
</tr>
<tr>
<td>[4.30, 8.68]</td>
<td>1.34 ± 0.83 ± 0.43</td>
<td>1.73 ± 0.43 ± 0.15</td>
<td>1.71 ± 1.58 ± 0.49</td>
<td>1.72 ± 0.41 ± 0.14</td>
</tr>
<tr>
<td>[10.09, 12.86]</td>
<td>2.98 ± 0.95 ± 0.95</td>
<td>1.77 ± 0.36 ± 0.12</td>
<td>1.97 ± 0.99 ± 0.22</td>
<td>1.77 ± 0.34 ± 0.11</td>
</tr>
<tr>
<td>[14.18, 16.00]</td>
<td>1.86 ± 0.66 ± 0.59</td>
<td>1.34 ± 0.26 ± 0.08</td>
<td>0.52 ± 0.61 ± 0.09</td>
<td>1.21 ± 0.24 ± 0.07</td>
</tr>
<tr>
<td>[16.00, ( q^2_{\text{max}} )]</td>
<td>2.32 ± 0.76 ± 0.74</td>
<td>0.97 ± 0.26 ± 0.07</td>
<td>1.57 ± 0.96 ± 0.17</td>
<td>0.88 ± 0.22 ± 0.05</td>
</tr>
<tr>
<td>[0.00, 4.30]</td>
<td>3.30 ± 1.09 ± 1.05</td>
<td>2.60 ± 0.45 ± 0.17</td>
<td>2.01 ± 1.39 ± 0.27</td>
<td>2.53 ± 0.43 ± 0.15</td>
</tr>
<tr>
<td>[1.00, 6.00]</td>
<td>1.14 ± 0.79 ± 0.36</td>
<td>1.42 ± 0.41 ± 0.12</td>
<td>2.57 ± 1.61 ± 0.40</td>
<td>1.48 ± 0.39 ± 0.12</td>
</tr>
</tbody>
</table>

with other results, and the differential measurements of \( B_s^0 \rightarrow \phi \mu^+ \mu^- \) and \( \Lambda^0_b \rightarrow \Lambda \mu^+ \mu^- \) are the first such measurements. At present there is no evidence of discrepancy from the SM prediction.

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\[ q^2 = 18.90 (19.30) \text{GeV}^2/c^2 \]

\[ B_s^0 \rightarrow \phi \mu^+ \mu^- \]

\[ B^0 \rightarrow K^0 \mu^+ \mu^- \]

\[ B^+ \rightarrow K^+ \mu^+ \mu^- \]

\[ B \rightarrow K^* \mu^+ \mu^- \]

\[ \Delta q^2 \]
We use a cylindrical coordinate system in which $\theta$ is the polar angle with respect to the proton beam line and pseudorapidity $\eta = -\ln(\tan \theta/2)$. 

We refer to references [7] through [21] for more details.