STUDY OF THE PERMANENT DEFORMATION OF BINDERS AND ASPHALT MIXTURES USING RHEOLOGICAL MODELS OF FRACTIONAL VISCOELASTICITY

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ABSTRACT

The accumulation of load on asphalt pavement as a result of increased vehicle traffic generates problems in the asphalt layer due to permanent deformation. For correct design, it is essential to carry out a rheological characterization of the aggregate-binder materials that make up the asphalt mix. This article shows the analysis of permanent deformation based on the rheological behaviour of asphalt mixtures and binders. Experimental tests based on creep and recovery phenomena allow the study of permanent deformations using theoretical models of fractional viscoelasticity. The rheological characterization allows us to detail the elasticity of the aggregate, $\xi_2$, and the elastic-viscous properties of the different binders used, $\xi_1$ and $\eta$. The results obtained show that it is possible to predict the deformations of the recovery phenomenon in asphalt mixtures from the rheological values (aggregate-binder) obtained in the creep process. Besides, the properties of the asphalt binder ($\xi_1$ and $\eta$) correlate with the recovery phenomenon of the MSCR test for conventional and modified materials. The methodology proposed allows a better understanding of the states of permanent deformation to improve the design of binders and asphalt mixtures.

Keywords: Permanent deformation; rheological properties; MSCR; viscoelastic; creep-recovery; asphalt binder

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1. INTRODUCTION

Asphalt mixtures have for decades been the most commonly used composite material for the construction of flexible pavements [1]. The use of mineral aggregate and asphalt binder as the skeleton and agglomerate of the asphalt mix has made it possible to ensure the quality and comfort of the road structure for controlled periods. However, the increase in vehicle traffic and tremendous climatic variations have caused permanent damage to the asphalt layers along the path of the wheels, resulting in traffic accidents due to the poor quality and comfort of driving [2].

The asphalt binder is a petroleum-based product that has little elastic capacity when subjected to different stresses, generating plastic deformations due to its anelastic nature [3]. Therefore, in recent decades, the rheological study of this material has generated great scientific discussions and comparisons on its mechanical behaviour in the asphalt mix. The rheological analysis generates various methodologies that establish parameters for different static and dynamic stresses. The methodology proposed by Superpave for tests on asphalt binders using a dynamic shear rheometer (DSR) makes it possible to establish a limit with the parameter \( G^*/sen(\delta) \) to predict the failure of permanent deformations in asphalt mixture [4–6]. However, the technological advance of new binder-modifying materials and asphalt mixtures, such as oils, waxes, polymers, among other materials [7–10], do not generate an adequate representation of permanent deformation since it is known that Superpave parameters are focused on linear viscoelastic deformation [11]. In this case, the pavement will be designed to be able to withstand the stresses and strains of the traffic, which are not reflected in the deformations caused by rutting, since these occur in a non-linear viscoelastic range. For this reason, the Federal Highway Administration (FHWA) has conducted studies based on the phenomenon of multiple stress creep recovery (MSCR) to understand the ability of the asphalt binder to recover some initial deformation [12], generating a complementary test for the rheological analysis of asphalt binders. The studies based on analysis of these phenomena mention that the MSCR test characterizes the properties of the asphalt binder in a different way than the DSR-test [12] due to the different levels of loading and natures of the tests [13]. In this sense, the MSCR is a test that is more related to the rutting performance of the asphalt mix, reaching greater detail on the behaviour in base and polymer binders [14] and evaluating the modification efficiency rate of additives and modifiers in asphalt binders [15].

Creep and recovery phenomena generate with static cycles in controlled time intervals [16]. The creep phenomenon is the deformation that occurs in the process of loading the material, which generates elastic-viscous strains that depend on different factors such as the test temperatures, load magnitude or nature of the load [17,18]. Subsequently, the recovery phenomenon details the capacity of the material to redeem the deformations obtained in the creep process and to avoid the complete plastic deformation of the binder or asphalt mix [19]. The behaviour exhibited by the asphalt mix and binder are complemented by experimental analysis with numerical models that allow understanding of permanent deformations, generating various methodologies that address the phenomena of creep and recovery.

In the literature, several models allow the establishment of rheological values for elastic-viscous deformations. The fractional Burgers model proposed by Oeser requires four parameters to
describe the progressive deformation of an asphalt binder [20]. This model allows the correct
adoption of the deformations of a viscoelastic material from its elastic to a viscous state, but it
was not a solution when submitted to the analysis of composite materials such as an asphalt mix
(aggregate-binder). The use of the aggregate-binder matrix as a single material does not allow
the elastic-viscous transitions of the asphalt binder to be detailed independently. Therefore, one
of the disadvantages of rheological characterization by these mechanical models is that it is hard
to control the variability or uncertainty of the experimental tests, resulting from the complex
mechanical and environmental variations in the laboratories [21]. In this regard, some research
claims that no single model commonly used in the literature is ideal for predicting asphalt mix
deformation, as they all have several disadvantages, such as predicting creep and recovery
deformation with a single model [20].

From the above, there is a need to establish a model centred on the materials that make up the
asphalt mix separately, respecting the elastic rheology of the mineral aggregate and the
viscoelasticity of the asphalt binder. To this end, a rheological model has been proposed in
previous studies that establishes a set of springs and fractional dampers, representing a particle of
mineral aggregate within an asphalt binder [20]. The mathematical detail of this study is
achieved with the use of fractional differential equations, which generate operators that
symbolize the rheological transition from the elastic to the viscous state of the binder [22]. In this
way, the classification of the rheological properties of the asphalt mix is detailed based on the
aggregate-binder interface jointly and independently. The proposed model establishes the typical
instantaneous elastic jump of viscoelastic materials. The first spring represents the critical point
of linear viscoelasticity (Fig. 1, phase 1). Then, the change of curvature gives rise to the non-
linear viscoelasticity (Fig. 1, phase 2) and corresponds to the complete system (aggregate-
binder); the material is still in the stage of no plastic damage, acting at the same time as the
elasticity of the aggregate. The last state of the creep phenomenon, or the plastic damage stage
(Fig. 1, stage 3), is determined by a fractional damper, which indicates the final deformation
from which the mixture will not be able to recover. The recovery phenomenon begins with an
initial instantaneous strain defined by a fit factor from Maxwell’s model. Subsequently, the
model allows for detailing the recovery curve with the same rheological properties obtained from
the creep phenomenon to determine the projection of the permanent deformation curves.

The objective of this study is to optimize the asphalt mix design methodology for states of
permanent deformation based on a rheological correlation of the properties of the mix and the
asphalt binder. The methodology proposed allows us to distinguish the rheological properties of
the aggregate-binder materials. The equations were obtained using fractional order derivatives
and were implemented in computer codes using MATLAB© based on nonlinear viscoelasticity.

2. MATERIALS AND METHODS

2.1. Asphalt binder and filler

The asphalt binders used are conventional and modified. The physical properties are
summarised in Table 1, with essential characteristics and specifications. The mineral aggregate
used is ophthalmic. This material is used in rolling layers due to its anti-slip qualities,
guaranteeing the necessary surface texture for a period.
The above set of materials was used to make a semi-dense asphalt mix type AC16S. Marshall specimens were manufactured at a temperature of 135°C. The examples were compacted at 75 strokes per side. A series of samples made with only asphalt binders were then produced at a temperature of 135°C to test in the dynamic shear rheometer (DSR).

![Figure 1](image)

**Fig. 1.** Representative diagram of the phenomena of creep and recovery.

**Table 1**

Properties of the asphalt binders

<table>
<thead>
<tr>
<th>Test</th>
<th>Units</th>
<th>Standard</th>
<th>B50/70</th>
<th>B70/100</th>
<th>PMB45/80-65</th>
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<tr>
<td>Penetration at 25°C</td>
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<td>EN 1426</td>
<td>57.0</td>
<td>70.0</td>
<td>49.5</td>
</tr>
<tr>
<td>Softening point R&amp;B</td>
<td>°C</td>
<td>EN1427</td>
<td>51.6</td>
<td>48.5</td>
<td>72.3</td>
</tr>
<tr>
<td>Fraass breaking point</td>
<td>°C</td>
<td>EN 12593</td>
<td>-13.0</td>
<td>-10.0</td>
<td>-13.0</td>
</tr>
<tr>
<td>Density at 25°C</td>
<td>G cm⁻³</td>
<td>EN 15326</td>
<td>1.035</td>
<td>1.020</td>
<td>1.030</td>
</tr>
</tbody>
</table>

2.2. Creep and recovery of asphalt mixtures

The creep and uniaxial static recovery tests were carried out using the universal testing machine (UTM) for asphalt mixtures at different temperatures and load frequencies. Permanent deformation of the material was determined at a predetermined constant load within a set period of 10 minutes [21]. The tests were carried out for a Heaviside-type load with a load sweep of 3 and 6 kN (with a scale of 1kN) at a temperature of 20°C.

2.3. Multiple stress creep recovery (MSCR) for asphalt binders

The multiple stress creep and recovery (MSCR) test was performed on the dynamic shear rheometer (DSR) for conventional and modified binders. The deformation of the samples was
determined for a temperature of 20°C, developing 10 continuous load-discharge cycles. The load was determined for deformation of 0.1% in 1s, and the recovery was prolonged for a time of 39 s.

2.4. Rheological model for asphalt mixtures

The mechanical model allows the rheological characterization of aggregate-bonded materials. It obtains the properties of the mineral aggregate and of the different types of asphalt binders by means of the equations that describe the phenomena of creep and recovery. The representation of the asphalt mixes was based on an aggregate particle wrapped in a continuous film of asphalt binder—that is, an elastic element surrounded by an elastic-viscous assembly [20]. For each magnitude of static stress applied, Eq. (1) and Eq. (2) describe the creep and recovery phenomena for asphalt mixes, respectively.

\[
F(t) = \frac{\sigma_0}{\Gamma(\alpha)} \left( \frac{t}{\tau} \right)^{\alpha - 1} e^{-\frac{t}{\tau}}
\]

\[
R(t) = \frac{\sigma_0}{\Gamma(\beta)} \left( \frac{t}{\tau} \right)^{\beta - 1} e^{-\frac{t}{\tau}}
\]

where \( F(t) \) is the creep deformation, \( R(t) \) is the recovery deformation, \( M = \xi_2 + \xi_1 \) is the elastic property of the asphalt mix, \( \xi_2 \) is the elasticity of aggregate, \( \xi_1 \) and \( \eta \) are the elastic and viscous constants of the asphalt binder, \( \sigma_0 \) is the initial stress, \( \Gamma(.) \) is the Gamma function, \( \alpha \) and \( \beta \) are the fractional exponents, \( \varepsilon_M^0(\theta) \) is the adjustment factor of Maxwell’s model, \( \varepsilon_M^0(\theta) \) is the adjustment factor of the parallel system, and \( t \) is the time.

3. RESULTS AND DISCUSSION

3.1. Rheological model for asphalt binders

To develop a rheological correlation using elastic-viscous models for the creep and recovery phenomena of binders and asphalt mixtures, a mechanical model was established (Fig. 2) based on a rheological modification of the Burgers model [16,23,24]. The proposed model for asphalt binders was achieved by extracting the mechanical representation of the aggregate \( \xi_2 \) from the model proposed for asphalt mixtures [20]. The resulting model differs from the one proposed by Burgers and other models because it has only two rheological constants (\( \xi_1 \) and \( \eta \)), representing the elasticity and viscosity of the asphalt binder. In this way, all deformation states of the asphalt binder can be explained, reducing the number of rheological variables involved in curve settings. The use of fractional exponents (\( \alpha \) and \( \beta \)) allows the elastic-viscous transition of the binders to established, since they are not considered Newtonian fluids at a temperature of 20°C [3]. The differential equation (Eq. 3) that represents the model is determined by the sums of the elasticities of the fractional Maxwell and modified Kelvin-Voigt model (see Appendix A) [25–27].

\[
\frac{\partial^\alpha \varepsilon}{\partial t^\alpha} + \frac{\partial \varepsilon}{\partial t} + \mathbf{C} \varepsilon = \mathbf{F}
\]
where $\epsilon$ is the deformation, $\sigma$ is the stress, and $D_t^\alpha$, $D_t^\beta$, and $D_t^\gamma$ are defined as fractional derivatives with respect to time $t$ and the fractional exponents $\alpha$, $\beta$, and $\gamma$. It is important to emphasize that the fractional exponents must have the same ranges, between 0 and 1, to satisfy classical rheological equations. Applying the Laplace transform \[22,28\] a la Eq. (3) and assuming that the initial conditions are zero, the following stress-strain equation is obtained:

$$
\dot{\epsilon}(s)\left[s^{\alpha+\beta} + \frac{\xi_1}{\eta} s^\alpha \right] = \delta(s) \left[\frac{1}{\xi_1} s^{\gamma+\beta} + \frac{1}{\eta} s^\beta + \frac{1}{\eta} s^\alpha + \frac{1}{\eta} s^\gamma + \frac{\xi_1}{\eta^2} \right] \tag{4}
$$

The recovery phenomenon begins when the initial stress applied to the material in the creep test is released [29]. To describe this process, it is necessary to use the concept of initial stress ($\sigma_0 = 0$), resulting in the setting function for asphalt binders (Eq. 5, Appendix B):

$$
R_2(t) = \epsilon_\infty' (0) + \epsilon_0' (0) \sum_{k=0}^{\infty} \left( -\frac{\xi_1}{\eta} t^\beta \right)^k \frac{1}{\Gamma(1 + \beta k)} + \epsilon_\infty (0) \tag{5}
$$

where $R_2(t)$ is the recovery strain for asphalt binders, $\epsilon_\infty' (0)$ the fit factor from Maxwell’s model, and $\epsilon_0' (0)$ the fit factor from the parallel system. The model has a relaxation time [30] that depends exclusively on the properties defined by the asphalt binder ($\eta$, $\xi$), unlike Eq. (1) and Eq. (2), which have a relaxation time determined by the variables $M$ and $\eta$, as is typical of asphalt mixing. The proposed model is adjusted to the recovery phenomenon of the MSCR test to verify whether the asphalt binder rheological variables are equivalent to the adjustment obtained in the asphalt mixes.

### 3.2 Creep and recovery analysis for asphalt mixtures

The deformation that occurs when asphalt mix specimens are subjected to different magnitudes of loading depends on the type of binder used and its strength. In this respect, asphalt mixes will be directly influenced by the states of non-recoverable deformation due to the softening point of each asphalt binder (Table 1). The conventional asphalt binders generate less thermal inertia, resulting in higher permanent strains. The asphalt mix with B70/100 (Fig. 3b) achieves the most significant deformation over the entire range of loads tested. The 6 kN load...
creates a maximum creep deformation of 0.6 mm. However, this result is due to a higher activation energy, $E_f$, compared to the remaining binders [3]. This growth generates a more significant kinetic movement of the internal particles of the B70/100, causing the softening of the material with higher speed. Secondly, the mixture manufactured with B50/70 (Figure 3a) achieves lower deformation than B70/100 for 6 kN loads due to a higher softening point of the binder. Finally, asphalt mixes with PMB 45/80-65 (Figure 3c) are shown to achieve lower deformation for 6 kN compared to mixes with conventional binders. This effect is due to the incorporation of the rubber elastomer polymers in the modified binder. These change the asphalt binder to reduce the heat generated by the loads, delaying the softening and plastification of the mixture.

However, when adjusting the experimental data from creep Eq. (1) [20], the results obtained (table 2) show that the instantaneous jump is directly proportional to the load magnitude, since its determination is related to the quotient between the deformation and the force magnitude. The mixtures with B70/100 acquire higher values of $\xi_1$ compared to those containing B50/70. This is because the modulus of elasticity is inversely proportional to the elastic jump. Therefore, the mixture with B70/100 has a shorter linear viscoelastic deformation time. The asphalt mixture with PMB45/80-65 increases the elastic value to a maximum of $\xi_1 = 7.238$ MPa, demonstrating that the rubber polymer generates higher recoverable non-linear development.

After the first elastic jump, a recoverable non-linear elastic-viscous deformation starts, which is developed by the parallel system of the mentioned model of mixtures. The elastic jump that comes from this deformation establishes a value that adds up the elasticities of the asphalt binder and the aggregate. The curvature obtained experimentally is demonstrated by the Mittag-Leffler infinite series, symbolized by the fractional exponent. The value of this second elastic jump $M$ establishes by the difference of elasticities that the aggregate of the mix has a Young’s constant of approximately $\xi_2 = 636.4$ MPa as the standard average. This result explains why the aggregate is not deformable in a mixture, delivering volumetric and load dissipation properties capable of increasing the second elastic jump and delaying the viscosity. The fractional exponent $\beta$ remains approximately constant for conventional mixtures for loads of 3, 4, and 5 kN. Therefore, the magnitude of the load does not affect the transient behaviour of the recoverable deformation before the constant slope is achieved (fractional exponent $\alpha$). Mixtures with PMB45/80-65 have an elastic-viscous development that reaches higher values of $\beta$ compared to conventional ones.

The last phase of creep is characterized by a straight line with a constant slope, generating non-linear and non-recoverable viscoelasticity. Each asphalt mix adopts a certain final deformation determined by the constants $\eta$ and $\alpha$, defined by the last shock absorber of the model. The value of the parameter $\alpha$ decreases as the load applied increases. The mixtures with B70/100 have higher slopes, causing more significant non-recoverable deformations. The asphalt mixtures with B50/70 and PMB45/80-65 reach similar values of $\alpha$, having lower viscous components compared to B70/100 binder. Thus, the final strain of the asphalt mixes is mainly influenced by the rheological properties of the asphalt binder. It becomes indispensable to obtain the rheological variables $\xi_1$ and $\eta$ for each load magnitude to determine the final slope of the creep phenomenon, which determines the degree of ultimate plasticity of the recovery phenomenon.
The recovery phenomenon that occurs when the load is removed demonstrates the degree of plasticity caused in the creep process that leads to permanent deformation of the asphalt mix. When the strain is elastic, its recovery is complete, i.e., the mixing cylinder returns to its initial physical state. Of the tests carried out at 20°C, none shows this condition, since the overload and the duration generate plastic deformations. Conventional mixtures recover less from strain at the end of each cycle compared to asphalt mixtures with a modified binder (Fig. 3). Mixing with B70/100 binder produces a loss of recovery from 6kN of 0.82%, obtaining a permanent deformation superior to mixing with B50/70, which ends with 0.65% for the same load conditions. This corresponds to the presented analysis on the last creep phase, since the adjustments determined higher values of $\alpha$ for the mixture with B70/100. Samples with PMB 45/80-65 managed to recover from higher deformations due to a more top-elastic component (lower values from $\alpha$) [3], generating a final permanent deformation of 0.54% for 6kN.

The recovery phenomenon showed through curve fits (Eq. 2) that the rheological properties of the aggregate and the asphalt binder remain the same as those obtained for the creep phenomenon. The constant $\beta$ is of equal rheological value for both events, demonstrating the capacity of this model to predict the recovery phenomenon in the asphalt mixture. The curve adjustment established that PMB45/80-65 is the binder that recovers the most from deformation, since it has more significant development of the exponent $\beta$. The instantaneous elastic jump of the recovery $\epsilon_0^\alpha$ increases with the magnitude of the load—the mixtures with B70/100 reach a higher elastic jump of recovery, reaching a maximum value for 6 kN of 0.4831 mm. After the elastic jump, a time-dependent recovery begins to develop, which is mathematically related to the multiplication of the infinite Mittag-Leffler series and an adjustment factor $\epsilon_{M0}$. The results indicate that the adjustment value is directly proportional to the load, and the infinite series maintains the values of the creep phenomenon.

The parameter $\alpha$ is not considered for the recovery phenomenon, since its physical interpretation is based on the non-recoverable deformation. Therefore, an asphalt mix that obtains more significant development of the parameter $\beta$ will demonstrate less plastic damage for permanent deformation failures. When varying the magnitude of the load, conventional mixes tend to keep the parameter $\beta$ constant, since the physical transformation of the asphalt binder is fast, creating more work from the fractional exponent $\alpha$. The mixtures with the modified binder increase the value of $\beta$'s product of the slow transformation of the asphalt binder, demonstrating lower values of non-recoverable deformations. The rheological properties $\xi_1$, $\xi_2$, and $\eta$ obtained in the load-discharge cycles are kept constant throughout the process of the flow and recovery cycles for each binder and aggregate.

### 3.3. Correlation of creep and recovery analyses for asphalt mixes and binders

Creep and recovery analyses for conventional and modified binders were carried out using a dynamic cutting rheometer to correlate the data provided by the asphalt mix simulation. The MSCR test generates deformations in asphalt binder samples utilizing multiple cycles of angular torsional stress. The correlation of the rheological properties of binders and asphalt mixtures was carried out for percentage deformations at a temperature of 20°C, and only for recovery phenomena.
In Fig. 4a, the multiple deformations of the three binders mentioned for the MSCR test are illustrated. It can be seen that the B70/100 binder is the one that obtains the highest accumulated strain over the 10 cycles, ending with deformation of 1.72%, which ratifies the behaviour that occurred in the asphalt mixtures. B50/70 binder adopts a lower plastic deformation compared to B70/100, reaching a maximum of 0.87% in accumulated strain. PMB45/80-65 binder keeps its strain in a lower range due to the aggregate polymers; its maximum deformation does not exceed 0.60%.

**Fig 3.** Creep-recovery test and asphalt mixture B50/70 (a), B70/100 (b) and PMB45/80-65 (c). Comparison between experimental data (solid line) and prediction (dashed line) at different load, sizes shown
The curve fitting analysis of the MSCR test was considered with the model intended for asphalt binders (Eq. 5). No fitting was carried out in the case of creep deformation, as the curves obtained in the rheometer show no apparent difference in the recoverable and non-recoverable states for 1 s, this being due to the viscous component of the asphalt binders. The simulation results for asphalt binders are summarized in Table 3. The simulation values relate to the number of cycles depending on the type of asphalt binder, with a higher number of cycles resulting in a lower softening point (Table 1). It can be seen that the rheological properties of the asphalt binders, $\xi_1$ and $\eta$, remain the same as for the previous calculations in asphalt mixtures ($R^2 > 0.85$). This means that the model proposed for asphalt mixes and binders allows the data in the corresponding tests to be correlated with a good fit.

Fig. 4b shows the recovery of the binder B50/70 by correlating the number of cycles with the load size. The results show that the ninth period induced by the rheometer achieved the same recovery deformation as applying a 6 kN load to an asphalt mix. Adjustment shows that the fractional operator $\beta$ remains stable from the third to the eighth cycle and then increases its value to generate a slower rate of change, causing more significant plastic deformation. The number of cycles required to match the rheological values obtained in mixtures and binders is lower for the B70/100 binder compared to the B50/70 binder (Fig. 4c). This is due to increased plastic deformation and a percentage increase in the non-recoverable deformation stage. For 6kN, the average strain is 0.1%, which is achieved in the fifth cycle of the MSCR test.

The percentage deformation of asphalt mixtures containing PMB45/80-65 binders (Fig. 4d) does not cause the same abrupt changes as with conventional asphalt binders. The ratio between the tests is achieved with subsequent cycles, as the polymer-based asphalt binder generates a lower plastic deformation. The range of deformation is lower than for conventional binders, and the adjustment is achieved with a strain of approximately 0.8%, using up to the 10th cycle of the MSCR test. The use of polymers in binders generates a better effect for permanent deformations compared to the use of conventional asphalt binders. This is due to the polymers’ ability to delay.

### Table 2
Creep-recovery of asphalt mixtures for a temperature of 20°C

<table>
<thead>
<tr>
<th>Type of binder</th>
<th>Load (kN)</th>
<th>$\xi_1$ (MPa)</th>
<th>$\xi_2$ (MPa)</th>
<th>$\eta$ (MPa* s)</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\varepsilon^{0.5}_{\infty}$ (mm)</th>
<th>$\varepsilon^{0.5}_{M}$ (mm)</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B 50/70</td>
<td>3</td>
<td>3.2700</td>
<td>639.7</td>
<td>7.5070</td>
<td>0.2656</td>
<td>0.1106</td>
<td>0.2441</td>
<td>7.1420</td>
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<td>4</td>
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<td>635.0</td>
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<td></td>
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<td>5.9000</td>
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<td>0.3634</td>
<td>16.6300</td>
<td>0.99</td>
</tr>
</tbody>
</table>
the viscoelastic-to-viscous transformation process of the binder [3]. From the above, it is established that the development of creep-recovery cycles in asphalt mixtures could be limited and studied with the predictions of the rheological behaviour in asphalt binders.

Table 3
Recovery of asphalt binder at a temperature of 20°C

<table>
<thead>
<tr>
<th>Type of binder</th>
<th>No. of cycles</th>
<th>( \tilde{\sigma}_r ) (MPa)</th>
<th>( \eta ) (MPa*ls)</th>
<th>( \beta )</th>
<th>( \varepsilon_r^{(0)} ) (mm)</th>
<th>( \varepsilon_h^{(0)} ) (mm)</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>B 50/70</td>
<td>5</td>
<td>3.2700</td>
<td>7.5070</td>
<td>0.1106</td>
<td>0.2441</td>
<td>7.1420</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>4.3600</td>
<td>5.1990</td>
<td>0.1200</td>
<td>0.2995</td>
<td>12.0900</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>3.8710</td>
<td>4.6300</td>
<td>0.1276</td>
<td>0.3790</td>
<td>13.5900</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>5.9000</td>
<td>3.7870</td>
<td>0.1288</td>
<td>0.3915</td>
<td>18.4500</td>
<td>0.91</td>
</tr>
<tr>
<td>B 70/100</td>
<td>4</td>
<td>4.7470</td>
<td>3.6700</td>
<td>0.1200</td>
<td>0.3533</td>
<td>13.6000</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>6.6700</td>
<td>3.7380</td>
<td>0.1200</td>
<td>0.3866</td>
<td>15.3200</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>6.6670</td>
<td>3.6750</td>
<td>0.1525</td>
<td>0.4478</td>
<td>15.9100</td>
<td>0.89</td>
</tr>
<tr>
<td>PMB 45/80-65</td>
<td>7</td>
<td>4.1180</td>
<td>5.9900</td>
<td>0.0900</td>
<td>0.1784</td>
<td>14.0000</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>4.9980</td>
<td>5.9980</td>
<td>0.1203</td>
<td>0.2243</td>
<td>14.4600</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>7.2380</td>
<td>4.4240</td>
<td>0.3190</td>
<td>0.3270</td>
<td>14.4600</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>7.2000</td>
<td>4.2400</td>
<td>0.3955</td>
<td>0.3634</td>
<td>16.6300</td>
<td>0.89</td>
</tr>
</tbody>
</table>

**Fig. 4.** MSCR tests for binders (a). Comparison between experimental data (solid line) and predictions (dashed line) at different loads and number of cycles—sizes shown for B50/70 (b),
4. CONCLUSIONS

Based on the results obtained in this research, the following conclusions are drawn:

The model allows the adjustment of experimental data for permanent deformations. It provides rheological values for the asphalt binder ($\zeta_1$ and $\eta$) and the aggregate used ($\zeta_2$). This model reduces the number of rheological constants compared to other classical models, allowing the fractional exponents to describe the physical significance of transition between the elastic and viscous state of the material.

The proposed methodology allows the correlation through mechanical models of the permanent deformations that occur in asphalt mixtures and binders based on the uniaxial compression test and MSCR, respectively, generating details of the creep and recovery phenomena for the prediction of permanent deformations in both mixtures and binders.

The behaviour of the conventional mixtures under the different loads used generated higher permanent deformations with the B70/100 binder, since it has a lower softening point than B50/70 and makes strains in the non-recoverable range of the creep phenomenon, causing an increase in the fractional parameter $\alpha$.

The mixtures with the PMB45/80-65 binder obtained the least plastic deformation due to their rheology’s generating more significant strain in the recoverable range (increase in the parameter $\beta$). This is due to its ability to decrease softening due to the delayed kinetic movement of the binder particles produced by the integrated polymers.

Further work is planned to compare the methodology outlined in this document with experimental tests of fatigue failures and permanent deformation of mastic asphalt under dynamic loads.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the institutional support provided by the Department of R+D of the University Austral of Chile (DID UACCh) and to the Santander Bank Iberoamerican Scholarship Program, which made it possible to carry out this research. The authors would also like to thank the GITECO and GCS research groups from the University of Cantabria (Spain) for their support.

Appendix A

The total system deformation shown in Figure 2 is given by

$$\text{(A.1)}$$
The fractional differential equations are given by

\begin{equation} \tag{A.2} \end{equation}

and

\begin{equation} \tag{A.3} \end{equation}

Taking the fractional derivative $\alpha + \beta$ of Eq. (A.1) to time,

\begin{equation} \tag{A.4} \end{equation}

Now, taking the fractional-order derivative $\beta$ from Eq. (A.2) and the order $\alpha$ from Eq. (A.3), we get

\begin{equation} \tag{A.5} \end{equation}

and

\begin{equation} \tag{A.6} \end{equation}

Substituting Eqs. (A.5) and (A.6) in Eq. (A.4), we get

\begin{equation} \tag{A.7} \end{equation}

To express Eq. (A.7) as a function of total deformation, we take the fractional-order derivative $\alpha$ of Eq. (A.1) to time:

\begin{equation} \tag{A.8} \end{equation}

Solving $D_t^{\alpha} \epsilon_2$ in Eq. (A.8) and using Eq. (A.2) gives the following equation:

\begin{equation} \tag{A.9} \end{equation}

**Appendix B**

The total recovery function for asphalt binders is obtained by eliminating the concept of initial load $\sigma_0$ of Eq. (4); we get
Note that the fractional \( \alpha \) and \( \beta \) can reach maximum values of 1, then \( m-1 = 0 \). Therefore, the sums in Eq. (B.1) are cancelled, and only the initial conditions of deformation remain. Using first the Maxwell model and later the kelvin-Voigt model, we obtain

\[
(B.1)
\]

**REFERENCES**


Fig. 1. Representative diagram of the phenomena of creep and recovery. 

- **Phase 1**: Elastic jump
- **Phase 2**: Viscoelastic, non-linear (recoverable)
- **Phase 3**: Maxwell model
- **Phase 4**: Elastic jump
- **Phase 5**: Viscoelastic, non-linear (recoverable)
Fig. 2. Schematic diagram of the proposed model of asphalt binde

Click here to download Figure: FIG2.pdf
Fig 3. Creep-recovery test and asphalt mixture B50/70 (a)

Click here to download Figure: FIG3a
Fig 3b. Creep-recovery test and asphalt mixture B70/100

Click here to download Figure: FIG3b.pdf
Fig 3c. Creep-recovery test and asphalt mixture PMB45/60-65
Fig. 4. MSCR tests for binders
Click here to download Figure: FIG4a.pdf
Fig. 4B. Comparison between experimental data (solid line) and p
Click here to download Figure: FIG4b.pdf

![Graph showing strain over time for different cycles and load levels.]

- 5° Ciclo: 3 kN
- 6° Ciclo: 4 kN
- 8° Ciclo: 5 kN
- 9° Ciclo: 6 kN
Fig. 4c. Comparison between experimental data (solid line) and predicted results (dashed line) for different load levels: 3 kN, 4 kN, and 5 kN. The plots show the strain percentage over time for three different load cycles: 3° Ciclo, 4° Ciclo, and 5° Ciclo.
Fig. 4d. Comparison between experimental data (solid line) and predicted trends. Click here to download Figure: FIG4d.pdf