Matricial methods for the systematic analysis of fiber optic bidirectional birefringent networks

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ABSTRACT

A great variety of methods for network analysis have been developed both in the frequency domain and in the time domain. The analysis methods in the frequency domain are applicable both to coherent sources (linearity in the electric field) as in incoherent sources (linearity in intensity) and they also make the analysis of the most complex structures possible. In this communication several matricial methods in the frequency domain, such as the scattering matrix in conjunction with the graph theory, the Yeh 4x4 matrices, and the transfer matrices are introduced and compared. They permit the exact and systematic analysis of the birefringent bi-directional concatenated photonic two-port and four-port networks. This analysis technique has been applied to the design and simulation of the polarisation-based in-line wavelength filters made by directional couplers and hi-bi optical fibres, such as all-fibre Fabry-Perot resonators, and Lyot-Ohman, Solc-fan, and Solc-folded all-fiber filters. The use of these matricial analysis methods gives the precise optical spectral response.

Keywords: Photonic networks, Fiber optics, Matricial methods, Polarisation, Birefringent filter, Fabry-Perot filter.

1. INTRODUCTION

With the arrival of the optical fibres and the integrated optic technology, the bulk optics devices have been progressively substituted by the optical fibre. In these photonic networks (PN) the state of polarisation (SOP) of the optical radiation must be taking into account, since the optical devices are anisotropics and sensitive to the SOP. This implies the using of matricial methods for modelling the referred photonics networks (PN), and in this particular case the transfer function is a transfer matrix. This means that it is necessary to define and develop a methodology for the systematic analysis of the PN taking as starting points the methods of analysis, which are broadly used in the field of microwaves, but taking into account the complexity caused by the differences that exist between both zones of the electromagnetic spectrum. Since the end of last century matricial methods have been developed for the analysis of devices and photonics networks, such as the Stokes and Mueller calculus and the Jones, Yeh and transference matrices 4×4 ¹⁻⁴. These methods, in general, only consider two port devices and optical networks. But, there is not a method for the systematic analysis of the concatenated N port networks, being the lack of this method the main purpose of this communication. The scattering matrix (SM) with the graph theory (GT) makes it possible to obtain the transfer function in reflection and transmission in all kinds of PN straightforward. However, this method presents an inconvenience, which is the impossibility of multiplying the SMs of the complete network. In the case of networks with few elements, it is possible to calculate the total matrix solving the graph that results from the individual graphs. Nevertheless, when the number of the simple networks, that form the composed network, is high, the complexity of the resulting graph for one scattering parameter may be very complicated. On the other hand, the matricial formalism based on 4×4 matrices, which is valid for two-port networks without feedback, presents the advantage that the complete matrix of several concatenated devices can be obtained multiplying their matrices in a certain order. This communication begins with an extension of the Jones calculus (JC) to the bi-directional photonic networks. So, considering the basis of the JC, a matricial formalism has been developed that takes into account the two directions of propagation. According to this formalism, the optical elements are represented by means of 4×4 matrices that include reflective and transmissive parameters. Section III introduces the use of the SM for characterising optical components. The fundamental concepts of the optical SM are defined. Their parameters are 2×2 matrices. Graph theory is introduced in conjunction with the SM as a complement for being applied to the topological description of devices and photonic networks. Section IV presents a systematic methodology in the frequency domain, for the analysis of the concatenated bi-directional photonic networks, which has the advantages of the matricial methods described. Finally in section V the above mentioned systematic matricial method is applied to the analysis and design of all-fiber birefringent filters.

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2. THE 4x4 M MATRICES AS A JONES CALCULUS EXTENSION FOR THE ANALYSIS OF BI-DIRECTIONAL BIREFRINGENT OPTICAL NETWORK

As it is commonly accepted, the evolution of the SOP in photonic devices or networks can be analysed by using JCM. However, it only considers the propagation in one direction, avoiding the partial reflections which let a coupling between counterpropagating waves. Fig 1 shows a bi-directional optical network composed by several optical elements. If the four complex vectors (E_0 , E'_0) and (E'_L , E_L) are known, the fields in both sides of the structure are completely determined.

Fig.1:Diagram of a bidirectional optical structure of length L made of different linear optical elements

If the method of defining Jones vector transformations in a simple way of propagation is used, a linear operator can be defined, relating the output and the input fields in a bidirectional PN:

$$\begin{bmatrix} E'_L \\ E_L \end{bmatrix} = M \cdot \begin{bmatrix} E_0 \\ E'_0 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \cdot \begin{bmatrix} E_0 \\ E'_0 \end{bmatrix} \quad (1)$$

The M linear operator is a 4×4 complex matrix, and M_{11} y M_{22} are 2×2 transmission matrices, and M_{12} y M_{21} are 2×2 reflection matrices, which produce the coupling between counterpropagating waves. In general the M_{ij} matrices can not be considered as JM. Only in the case of free reflection devices, where $M_{12} = M_{21} = 0$ (zero matrix), M_{11} and M_{22} are JM. The M matrix of n concatenated optical devices defined by the matrices $M_i (i=1, n)$, is described by a matricial product, and it is represented by $M = M_n M_{n-1} \dots M_2 M_1$. This formalism allows to model optical devices with isotropic and anisotropic losses, both with and without reflections. In the case of free reflection optical elements the M matrix can be expressed by means of the Pauli matrices, which are well known in quantum mechanics².

3. EXTENSION OF THE SCATTERING PARAMETERS THEORY TO THE OPTICAL CONTEXT

The use of the S matrix for characterising the optical components is a very helpful theoretical tool for the global analysis of the PNs. The S matrix in optics is different that the S matrix in microwaves due to the fact that the single-mode optical devices can have two modes of polarization which are given by a Jones vector. In this way the scattering parameters become 2×2 matrices and the S matrix is a supermatrix whose elements are 2×2 matrices instead of simple complex numbers. This is the difference between the optical S matrix and the S matrix used in microwaves. If we consider Fig. 2, in which a number

Fig.2: Diagram of a N-port generical optical component

N of Jones vectors $E_1, E_2, \dots, E_N$ enter a N -port component, E_k being the Jones vector entering the k port. $\tilde{E}$ and $\tilde{E}'$ are the complete Jones vectors at the input and at the output of the component, respectively. And in the case of one linear and time independent component these supervectors can be related by mean of the S matrix: $\tilde{E}' = S \cdot \tilde{E}$. The S linear operator is a N

$\times N$ complex matrix which can be expressed in terms of 2×2 complex matrices⁴. Each combination input-output (i,k) is characterized by a two-range S_{ki} matrix. For a N-port component, all the possible input-output combinations are N^2 , so N^2 different matrices will be needed to characterize it completely. These matrices are elements of the S matrix. The definition of this S matrix for a linear optical component can be extended for any optical network or structure made of different linear optical elements⁴.

3.1. Graph theory for optical networks

The analysis of the network has two parts: the obtention of the network equations and their solution. In these equations, the unknown parameters are the guided fields in the network nodes, and the coefficients are the scattering parameters of the different components of the network. The obtention of the network equations directly from the physical scheme is usually difficult and produces mistakes. It is much easier and safer to obtain these equations from a scheme in which the relationships between unknown variables are shown in it. The graph is the properest one for this objective, as it gives a clear and flexible definition of the topology of the network. This topology regards only the interconnection way of the elements, and not the characteristics of each different element. According to this description, each component is represented by a graph, and the complete network graph is made joining the different individual graphs in the same way the different components are connected. The values of each branch are simply the Scattering parameters of every component. Once the network graph is determined, a systematic reduction process is made. The graph reduction rules in photonic networks are different from the electric networks, as in a photonic one the different branches represent Jones matrices. In conventional graph theory, some rules have been developed to reduce complex graphs, which can be resolved analytically⁴⁻⁶. These rules must be extended to the situation of non-commutative transmission parameters (Jones matrices) in order to allow their application to the analysis of photonic networks. In photonic networks with polarisation maintaining devices or incoherent sources, it is possible to apply the well-know Mason Rule for the resolution of electric graphs⁵⁻⁸.

4. METHODOLOGY FOR THE SYSTEMATIC ANALYSIS OF THE CONCATENATED BI DIRECTIONAL PHOTONIC NETWORKS

Once the two basic components of photonic networks have been characterized, it is necessary to find a fast and easy method to describe both simple and complex networks, these ones being the result of concatenating different simple structures. In this section, a method of matricial analysis in frequency domain is defined, based in the relation between the S and M matrices, specially good for the complex structures analysis. Different ways of concatenation of simple structures are defined to obtain complex structures, as a function of the existence or not of reflections in the network ports.

One of the main objectives in the analysis of fiber optic structures is the obtention of the transfer functions in transmission and reflection, which characterize them. The problem in this characterization is that it is not possible to multiply the individual scattering matrices of two concatenated networks to find the full network matrix. In networks with few elements the total matrix can be found resolving the graphs result of joining every individual graph. However, when networks are more complex or the number of simple networks is high, the final graph for each scattering parameter can be very difficult to solve. In the following, the obtention of the transfer functions of the photonic networks by means of the scattering parameters, and the matricial transformations in order to find a full description of the method for different kinds of concatenation are presented.

4.1. Obtention of the transfer functions of transmission and reflection with the scattering parameters

We consider a photonic network of two bidirectional ports, see Fig. 3, in which two independent input and output waves characterized by Jones vectors E_1 and E_2 , and E'_1 y E'_2 , respectively. In a generic way, each vector is made of two orthogonal polarization modes s and p, and is represented by:

$$\begin{array}{c}
 E_1 \longrightarrow \boxed{S, M, N} \xrightarrow{2} E'_2 \longrightarrow \\
 \longleftarrow E'_1 \longleftarrow \boxed{S, M, N} \longleftarrow E_2 \\
 E'_i = \begin{bmatrix} E'_{is} \\ E'_{ip} \end{bmatrix} \quad i = 1, 2 \qquad E_i = \begin{bmatrix} E_{is} \\ E_{ip} \end{bmatrix} \quad i = 1, 2
 \end{array} \quad (2)$$

Fig.3: Two-port bidirectional photonic network

If the network is made of linear components, there will be a linear operator which relates the input and output vectors. This linear operator may be the M matrix, the S matrix, or the Yeh matrix (N)³. Anyway, this network will be completely characterized through 8 transfer functions in reflection and 8 in transmission⁴. As the elements of a scattering matrix contains directly and in an explicit way the transfer functions of the networks, it is possible to obtain them, by means of the S matrix of the photonic network.

4.2. Concatenation of bi-directional photonic networks

Two-port and four-port simple structures can be concatenated to form a more complex structures. It is possible to obtain in a fast and systematic way the scattering matrices of the compound structures if S matrices in every simple structure are known. The concatenation of simple structures can be made in different ways according to their number of ports. Two cases of concatenation are going to be studied: a) two gates networks, and b) four gates networks.

a) Two-port networks

Fig.4 shows a diagram of a concatenated structure of two gates both in transmission and reflexion:

Fig.4: Composed structure of two-port networks both in transmission and reflection

To analyze this kind of structures in an easy and systematic way is necessary to apply the transformation matrices S to every simple network. The complete matrix of the composed structure is given by:

$$M = M_{n-1n} M_{n-2n} \cdots M_{12} \quad (3)$$

Once the total M matrix is obtained, obtaining the S matrix of the concatenated network is achieved applying the equations of inverse transformation. In this way, it is possible to know in an immediate way the transfer functions both in transmission and reflection of the composed structure. So the steps to analyze in a exact way a cascaded network will be as follow: 1. Obtention of the simple network graph. 2. Obtention of the S matrix of the simple network from the graph resolution. 3. Transformation of S matrix in M matrix. 4. Obtention of the complete M matrix multiplying the individual M matrices. 5. Inverse transformation of the complete M matrix in complete S matrix.

b) Four port Networks

Fig.5 shows a diagram of a composed structure both in transmission and reflection of four-port networks. If the signal is introduced only at one of the input ports and leaves through one of the output ports the analysis can be reduced to a two-port network one.

Fig.5: Composed structure of four-port network both in transmission and reflection

5. APPLICATION TO THE STUDY AND DESIGN OF FILTERS WITH BIREFRINGENT OPTICAL FIBERS

Next, the design of three types of optical filters based on the interference of their polarization modes, using birefringent optical fibers is described. In all the situations where the 4×4 matrix method is used, normal incidence is assumed, using high linear birefringence PANDA optical fiber (with linear birefringence $\Delta n = n_p - n_s = 10^{-4}$)⁹, filters are designed for the 3th window, 1550 nm, and the polarizers are LAMIPOL class, and only the transmittance are represented. The design of the filter for wavelength λ_m , with a wavelength width $\Delta\lambda_{1/2}$, depends on three parameters: the length of the fibers pieces, their

birefringence, and the number of elements in the device. Both groups of parameters are related by specific expressions for each kind of filter, obtained from their reflectances and transmittances.

	Design equations	Transmittances	Structure
a)	$\lambda_m = \frac{\Delta n \cdot t}{m}$ $\Delta\lambda_{1/2} = \frac{0.886 \cdot \lambda_m^2}{2^N \cdot \Delta n \cdot t}$
b)	$\lambda_m = \frac{2 \cdot \Delta n \cdot t}{(2m + 1)}$ $\Delta\lambda_{1/2} = \frac{0.8 \cdot \lambda_m^2}{N \cdot t \cdot \Delta n}$
c)	$\lambda_m = \frac{\Delta n \cdot t}{m}$ $\Delta\lambda_{1/2} = \frac{0.8 \cdot \lambda_m^2}{N \cdot t \cdot \Delta n}$

Fig.6: Equations, transmittances and structure of a) a Lyot filter b) a Solc folded filter c) a Solc fan filter, made of optical fibers.

In Fig.6 m is an integer number. For the 3th window, and $\Delta\lambda_{1/2}=1\text{nm}$., the number of fiber pieces needed is $N=5$ (Lyot - Ohman) or $N=25$ (Folded, Solc Fan), and the maximum length of the fiber piece is $t=0,775 \text{ m}$.

In Lyot - Ohman filters , PANDA fiber pieces are used, whose length has a geometric progression, rotated 45° each other., separated by polarizers. Solc-Folded are made with an even number of fiber pieces of the same length, rotated alternatively an angle and situated between two crossed polarizers. Solc Fan filters are implemented with high birefringence optical fibers of the same length, rotated an angle, function of the position, and the same total number of pieces, which are situated between two paralel polarizers.

5.1. Filter based on the concatenation of the concatenation of three all fiber Fabry-Perot structures

The filters to be analysed use hi-bi optical fibres and polarisation dependent fiber optic couplers^{9,10}. Fig.7 shows a filter based on the concatenation of three all-fiber Fabry-Perot resonator with two pieces of length L between them. The filter is made of identical couplers with coupling constant K_x and K_y an losses γ_x and γ_y . The reflectors rings are composed of single-mode optical fibres with low-birefringence with a length of L_0 and a phase delay $\beta L_0 = \pi/2$. The cavity is composed by a hi-bi fibre with $\Delta n=10^{-4}$, and a length L , and are designed at 1550nm .

Fig.7: Network made of concatenation of three structures

Fig.8: a) Transmittance T_{ss} for a length $L=0.93$ meters with a) $K_x=K_y=0.8$ b) $K_x=0.8$ and $K_y=0.3$

The most outstanding consequence by the concatenation of the three structures is a significative improvement of $BW_{1/2}$. However, four additional bandpass appear next to the central wavelength of the bandpass (1550 nm) (see Fig.8a). The additional bandpass, for certain applications are not suitable and it is possible to eliminate them by using polarization-dependent couplers with $K_x > 0.5$ and $K_y < 0.5$. Fig.8b shows the transmittance and T_{ss} for $K_x=0.8$, $K_y=0.3$ and a fiber length $L=0.93$ m. The $BW_{1/2}$ decreases 0.75 to 0.53 nm in relation to a filter with two Fabry-Perot resonators.

6. CONCLUSIONS

A powerful method for the characterization of bidirectional cascaded photonic networks based on the S-parameter theory for microwave circuits, the graph theory for electrical circuits and the 4×4 M matrices, has been presented. The rules for the graph reduction, including the Mason's rule, which simplifies the analysis of polarisation maintaining photonic networks and the networks with non-coherent sources, have been commented. This matricial method is based on the algebraic transformation of the S matrix into the 4×4 M matrix, and presents several advantages: It allows to characterise, independently of the devices, concatenated structures. There is no need to solve a great number of equations. This formalism is very easy to implement with a computer program, and it makes possible to obtain automatically the transfer functions of the photonic networks. The method has been applied to the design of wavelength polarisation-based optical fibre filters composed by the concatenation of simple all-fiber Fabry-Perot birefringent resonators. The dependence on the separation and width of the filter passbands with respect to the parameters of the coupler, the length and birefringence of the fibre in the cavity has been studied and analysed. This matricial method has been applied to the calculus of the spectral response of the Lyot-Ohman, Solc folded, and Solc fan optical filters, based on optical fibers with high linear birefringence, and allows the obtention of their exact expressions, which makes possible the simulation and later, the design of them.

7. ACKNOWLEDGEMENT

This work has been partially funded by the Spanish C.I.C.Y.T. Commission through the project TIC'98-0397-C03-02.