New approach using a bare fiber optic cantilever beam as a low-frequency acceleration measuring element

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Abstract. A theoretical study is carried out in which a bare fiber optic cantilever beam, with no concentrated seismic mass, is proved to be suitable for measuring low-frequency vibrations accurately. Its behavior was experimentally verified in the laboratory and can be approximated by a very simple transfer function that leads to straightforward signal processing. Some practical configurations, with accuracies as good as 0.5%, are given that depend on the operating frequency range. The simplicity of the architecture is highlighted. © 1996 Society of Photo-Optical Instrumentation Engineers.

Subject terms: fiber optic sensor; fiber optic cantilever beam; low-frequency acceleration; mechanical vibration; smart structures.

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1 Introduction

Traditionally, low-frequency accelerometers have been based on the seismic mass effect, that is, on a mass fastened to a housing by means of an elastic element, whose movement related to the housing registers the acceleration experienced by the system. It is a very simple system, with only one degree of freedom (the displacement of the seismic mass), and its response is well known.¹

Recently, some accelerometers with optical fiber technology have been manufactured, with differing philosophies. In one of them, the mobile mass interposes into the beam of light transmitted between two fibers, and the modulation of that light reveals the position of the seismic mass.² Other authors use the mobile element to reflect the light emitted by a fiber into the same fiber or in another one.^{3,4} A different way of detecting the movement of the seismic mass is by allowing it to produce microbendings in the fiber, which are detected thereafter.⁵

In other methods, the stiffness of the fiber substitutes for the spring-damper system, keeping the concentrated seismic mass at the end of the fiber beam.^{6,7} These methods establish an equivalence between the mechanical system and the fiber system through the stiffness constant of the optical fiber. The equation of movement and the response continue to be those of the single degree of freedom system.

Other authors used a fiber beam to detect pressure waves,⁸ but until now a complete theoretical study of the movement of an optical fiber beam has not been presented.

Until now, the necessity of measuring vibrations in large structures, such as those in civil engineering or in electrical energy generation and transformation plants, has not been met satisfactorily from the technical-economic point of view. Thus, it is necessary to fabricate low-cost accelerometers (made of inexpensive materials and of simple electronics, which do not require the use of a digital processor),

in an easy way (without the need of putting a concentrated mass at the end). Furthermore, the use of optical fiber devices in adverse industrial environments has other well known advantages.

In this paper, the feasibility of using an optical fiber cantilever without a mass at the free end as the key element of a low-frequency acceleration detector is demonstrated. On the one hand, this enormously simplifies the fabrication process, due to the absence of the need to put a mass of reduced dimensions and perfect symmetry on the end of an optical fiber without degrading it, and additionally, contributes to the accuracy of the device, since the dependency on factors such as the value of the mass, its exact location on the beam, its degree of symmetry and the effect of its fastening on the fiber is avoided.

2 Philosophy Description

With the objective of obtaining an optical fiber based accelerometer for low frequencies, with ease of construction, that improves the features of reflective accelerometers,⁹ and that competes by its reduced cost with the interferometric accelerometers¹⁰ and the laser vibrometers,¹¹ a very simple architecture is proposed, composed of an optical fiber beam fixed in a housing in touch with the vibratory element whose acceleration is to be measured. One end of the fiber is embedded in the housing and the other is free. A sketch is shown in Fig. 1. The relative displacement of the free end of the beam with respect to the housing $y(t)-u(t)$ will be detected with the aid of other fibers and will provide information about the acceleration to be measured.

The construction facility is patent, because it is a simple fiber beam embedded in a housing, and, depending on the material which it is made of, the device will be able to operate in electromagnetically adverse environments and will admit wide temperature variations (a change of 200 °C

Fig. 1 Proposed architecture.

in the system temperature produces a change of 0.02% in the detected acceleration).

3 Theoretical Analysis

In this section, the vibrational behavior of the cantilever beam is theoretically analyzed and a new approach to its application in an acceleration measuring device is given. First, its general behavior is shown, which is the sum of infinite modal contributions, and the possibility of using a first mode approximation for this architecture is discarded. Next, the equation that governs the movement of the beam within a device submitted to an external acceleration is developed and its capacity to measure low-frequency accelerations is proved. And finally the practical dimensions of various fiber beams are given, depending on the operational frequency range and on the required degree of accuracy.

3.1 General Behavior of the Beam

The behavior of an isotropic beam such as the one in question can be modeled mathematically by the following differential equation¹²:

$$EI_z \frac{\partial^4 y(x,t)}{\partial x^4} + \rho \frac{\partial^2 y(x,t)}{\partial t^2} = 0 \quad (1)$$

with the appropriate boundary conditions; E is Young's modulus of the fiber, I_z is its transverse moment of inertia, ρ is the linear mass density, and $y(x,t)$ is the transverse displacement of the x section of the beam at the instant t .

The solution for the free vibration, which is the sum of an infinite number of oscillation modes, and the characteristic equation are¹²

$$\Phi_i(x) = C_i \left\{ \sin(k_i x) - \sinh(k_i x) - \left[\frac{\sin(k_i L) + \sinh(k_i L)}{\cos(k_i L) + \cosh(k_i L)} \right] \times [\cos(k_i x) - \cosh(k_i x)] \right\} \quad (2a)$$

$$\cos(k_i L) \cosh(k_i L) + 1 = 0 \quad (2b)$$

where L is the length of the fiber beam, and $k_i L$ are the roots of Eq. (2b); k_i are, therefore, the inverse lengths that see the characteristic equation for a given beam length. The natural vibration frequencies of the cantilever beam are

Fig. 2 Shape of the first three vibration modes of the beam.

$$f_i = \frac{1}{2\pi} (k_i L)^2 \left(\frac{EI_z}{\rho L^4} \right)^{1/2} \quad (3)$$

The shape of the first three modes is shown in the Fig. 2 for a normalized beam length. The amplitudes of deflection of the modes are represented in arbitrary units. To obtain a concrete amplitude, the system must be driven by a force function, and the weight of each modal component calculated. It can be seen that the greater transverse displacement takes place in the free end of the beam, which is the key point for the philosophy proposed, since the acceleration to which the set is submitted is obtained from its deflection.

At this stage, the uncertainty of the influence of the different modes on the low-frequency vibrational movement of the beam arises, and whether the first mode approximation will be valid, such as that made by other authors whose beams have a mass at the free end. To solve it, a modal analysis was carried out by means of the finite element simulator ANSYS, version 5.0 A (Swanson Analysis Systems, Inc.). The relative contributions of the first three modes to the movement of the beam free end were obtained for two configurations of the system, one without a concentrated mass and another with a mass of 50 mg at the fiber end. In both, the fiber beam was 25 mm long and had a diameter of 125 μm . The results are given in Table 1.

The conclusion of this first study is that for a fiber beam with no mass in its free end, the contribution of the upper modes is not negligible. The first mode approximates quite well the low-frequency vibration of the beam with the concentrated mass, but this is no longer true for the bare beam.

Table 1 Comparison between the weights of the second and third modes relative to the first mode, for the same cantilever beam with and without a mass concentrated at its free end.

	Relative Weight of 2nd Mode	Relative Weight of 3rd Mode
Beam with a 50-mg mass at the end	0.01%	0.0005%
Beam without mass at the end	2.5%	0.3%

which is the proposed configuration. The whole set of modes contributes to the transverse displacement of the free end in the frequency zone of interest, and therefore a complete study of the vibration of the cantilever beam must be carried out if one wants to apply it to the construction of an acceleration detector

3.2 Application as Accelerometer

Once the invalidity of the first mode approximation for this architecture is demonstrated, the transfer function of the cantilever beam as an accelerometer is developed. That function takes into account all the vibration modes of the system as a whole.

For the system of Fig. 1 to be capable of measuring accelerations, it must detect a sinusoidal movement applied to the housing

$$u(t) = U \sin(\omega t). \quad (4)$$

Equation (1) must be solved again, with the appropriate boundary conditions for the solution $y(x, t)$. The embedded end of the beam follows the sinusoidal movement of the housing, without tilt, and the bending moment and the shearing force at the free end are both zero

$$y(x, t)|_{x=0} = U \sin(\omega t)$$

$$\left. \frac{dy(x, t)}{dx} \right|_{x=0} = 0$$

$$\left. \frac{d^2 y(x, t)}{dx^2} \right|_{x=L} = 0 \quad (5)$$

$$\left. \frac{d^3 y(x, t)}{dx^3} \right|_{x=L} = 0.$$

The resulting stationary state solution is a sinusoidal function with the same frequency as the force function ω and has a spatial part in which all the modes influence implicitly in terms of such a driving frequency

$$y(x, t) = \frac{U}{2} \left\{ \frac{\sin(kL) \cosh(kL) + \cos(kL) \sinh(kL)}{1 - \cos(kL) \cosh(kL)} \times [\sin(kx) - \sinh(kx)] + \frac{1 + \cos(kL) \cosh(kL) - \sin(kL) \sinh(kL)}{1 + \cos(kL) \cosh(kL)} \times [\cos(kx) - \cosh(kx)] + 2 \cosh(kx) \right\} \sin(\omega t) \quad (6a)$$

with

$$k = \left(\frac{\rho}{EI} \right)^{1/4} \sqrt{\omega}. \quad (6b)$$

Note that, on the one hand, the solution given by Eq. (6a) fulfills Eq. (1), and, on the other, for the natural frequencies of the beam ω_1 , the resulting k 's are the same that are seen in Eq. (2b).

It is the amplitude deflection of the beam free end relative to the housing displacement that the proposed device is able to detect, that is,

$$z(t) = y(L, t) - u(t) = \left[\frac{\cos(kL) + \cosh(kL)}{1 + \cos(kL) \cosh(kL)} - 1 \right] \times U \sin(\omega t). \quad (7)$$

Taking into account Eqs. (6b) and (3) and the first root of Eq. (2b), the transfer function of the system is

$$f(r) = \frac{\cos(1.8751\sqrt{r}) + \cosh(1.8751\sqrt{r})}{1 + \cos(1.8751\sqrt{r}) \cosh(1.8751\sqrt{r})} - 1 \quad (8a)$$

where r is the vibration frequency relative to the first resonant frequency of the beam:

$$r = \omega/\omega_1. \quad (8b)$$

Equation (8a) gives the whole frequency response of the system, that is, it includes the contributions of all the modes. This function, together with the one that considers only the fundamental mode, is shown in Fig. 3(a) and an amplification of it in the zone of the lower frequencies, the zone of application as an accelerometer of this system, is given in Fig. 3(b). The great difference between the functions can be observed, which reaffirms the fact that all the vibration modes have an effect on the actual behavior of the beam, even at the lower frequencies, and consequently it is not possible to make a first order approximation in this case.

On the other hand, an ideal acceleration measuring system is the one that starting from a vibratory movement of the type in Eq. (4) gives a response

$$\ddot{u}(t) = -\omega^2 U \sin(\omega t) \quad (9)$$

so that the amplitude of its frequency function is

$$a(r) = \omega^2 = r^2 \omega_1^2. \quad (10)$$

The proposed system, whose transfer function is given by Eq. (8a), is intended to be able to substitute for the ideal accelerometer, governed by Eq. (10), and therefore could be applied in measuring low-frequency accelerations reliably. A study of both functions was made and shows a great similarity between them, conveniently scaled, that is, one is proportional to the other, with a minimum error that depends on the given frequency range. Fig. 4(a) shows such functions in a frequency range from zero to 0.2 times the system's main resonance. And the relative error that is made by substituting the first function for the second one is given in Fig. 4(b). The maximum error is 2% within the mentioned range.

The result arrived at is that the function that governs the deflection of the cantilever beam end is practically propor-

Fig. 3 Actual behavior of the cantilever beam end and the component due to the fundamental mode: (a) up to a frequency 20 times greater than the fundamental and (b) in the low-frequency range.

tional to the one that defines the behavior of the ideal accelerometer within the frequency range of interest, that is, that the system based on optical fiber is perfectly valid as a base element for measuring low-frequency accelerations from zero to a frequency that depends on the dimensions of the beam. The error that is made on substituting the ideal accelerometer by the proposed architecture is very small. Furthermore, the ability to interchange them by merely multiplying by a constant enormously simplifies the subsequent processing of the signal given by the detector.

It has also been found that such relative error becomes smaller as the operation frequency range of the device decreases. Table 2 shows the ranges where this topology can measure accelerations for some maximum permitted error percentages.

3.3 Actual Geometric Configurations

Since the operation range of the device depends on the fundamental frequency of the beam, and this, in turn, according to Eq. (3), depends on the length and the outer diameter on the optical fiber beam used, some geometric configurations for the cantilever and their operation frequency ranges are given. Table 3 shows the frequencies up to which it is possible to measure vibrations for various 125 μm diameter telecommunication standard fiber lengths for three maximum percentage errors allowed.

Fig. 4 Comparison between the ideal accelerometer function and the scaled function of the optical fiber based system: (a) the two curves and (b) their relative error.

According to what has been developed in this section, the behavior of a beam like the one in Fig. 1 can not be approximated by a first order system with a single vibration mode. Its actual behavior has been found, and what is more important, it has been proved that such behavior resembles the ideal accelerometer to a large extent. As an example, such as Table 3 indicates, a 20 mm long, 125 μm diameter fiber beam, operating as shown, is capable of measuring accelerations up to 25 Hz with a maximum error of 0.5%.

4 Experimental Results

With the objective of validating the theoretical deductions obtained, some experimental work was done in the laboratory that, together with results, is presented next. First, the fundamental resonance frequency of the beam was measured. Next, it was proved that the optical fiber damping factor is so small that not considering it in the theoretical calculations is valid. And, finally, a structural reliability

Table 2 Relative frequency up to which this system can measure accelerations in terms of the error tolerated.

Error Allowed (%)	0.5	1	2	3	5
Max. Frequency (r)	0.099	0.140	0.196	0.239	0.307

Table 3 Upper frequency limit of operation as an accelerometer for various beam lengths in terms of the tolerated maximum error.

	0.5%	2%	5%
$L = 10$ mm	99 Hz	196 Hz	307 Hz
$L = 15$ mm	44 Hz	87 Hz	136 Hz
$L = 20$ mm	25 Hz	49 Hz	77 Hz
$L = 25$ mm	16 Hz	31 Hz	49 Hz

study was accomplished by subjecting several beams to large mechanical efforts, with the purpose of proving their long term vibration performance.

4.1 Checking the Value of the Fundamental Resonance Frequency

The value of f_1 is essential in the development accomplished, not only because it determines the validity range of Eq. (8a) in measuring accelerations for a given accuracy, but also because it is basic in the calculation of the absolute value of the detected acceleration, according to Eq. (10). That is why the first experimental check refers to the fundamental resonance frequency of the fiber beam. The mentioned value, which is given theoretically by Eq. (3) in terms of the geometric dimensions and the elasticity properties of the optical fiber, was measured in the laboratory by means of the setup shown in Fig. 5.

The beam was subjected to a vibration of a constant amplitude and varying frequency. It was a set composed of a Tiravib model 5100 shaker and associated power amplifier that produced the vibration, and an SRS model SR850 digital signal processing (DSP) lock-in amplifier accomplished the frequency sweep. By means of a reference accelerometer the frequency response of the shaker was obtained, and using a position detector, the response of the shaker-system under test was recorded. Starting from these two signals, the lock-in amplifier extracted the frequency response of the optical fiber based system.

Fig. 5 Experimental setup for testing the frequency response of the fiber beam. **Fig. 6** Frequency response of the fiber beam around the principal resonance.

Fig. 6 shows the response of an 18.9 mm long, 125 μm diameter beam. The fundamental resonance peak is centered in 280 Hz, while Eq. (3) gives a value of 279.95 Hz for f_1 .

4.2 Measurement of the Optical Fiber Damping Factor

In the theoretical calculation carried out, the damping in the system was not taken into account, because the structural damping of the optical fiber is very low, such that it does not alter the vibrational behavior of the system. It was proved experimentally in the laboratory, by means of the frequential curve in Fig. 6.

The damping factor of the beam ζ is a function of the ratio between the width of the fundamental resonance peak at half height and the frequency of the peak, such as¹³

$$\zeta = \frac{\Delta f}{2f_1} \quad (11)$$

A value of 0.0045 was obtained for the mentioned damping factor, a value which is actually very small, as the narrowness of the beam resonance peak indicates.

4.3 Mechanical Reliability Test of the Fiber Beam

Last, once the validity of using this optical fiber beam as the base element in an acceleration transducer is demonstrated, it is necessary to ensure its vibration reliability in long term operation, to demonstrate its viability.

With that objective, an accelerated aging experiment was performed on several bare fibers with similar geometric dimensions—outer diameter of 125 μm and about 20 mm long. They were loaded with a mass at the free end and were forced to vibrate in resonance with a great amplitude for more than 1000 h. During this period, they experienced much harder mechanical conditions than those they might encounter in normal use within an acceleration detector, which will not have the concentrated mass and will vibrate outside the resonance. In Fig. 7 a picture of the vibration movement of one of these beams is shown, where the considerable amplitude of deflection reached can be appreciated.

Fig. 7 Large amplitude resonant movement of fiber.

As a result, none of the beams was broken, in spite of the extreme stresses experienced. Furthermore, these beams were observed with a microscope Olympus AHMT3 Research Photomicrographic Microscope System, which has Nomarski facilities available, to look for possible faults or microcracking, and no visual defect was detected. Furthermore, no flaws were detected on introducing laser light in the fiber cladding in search of flaws or imperfections that dispersed the light.

Considering that in the foreseen utilization of this architecture the fiber beam will be massless and will experience small amplitude displacements, the structural reliability of the beams is high enough to ensure viability within the scheme proposed for measuring accelerations.

5 Summary and Conclusions

A theoretical analysis of the vibrational behavior of a fiber optic cantilever beam was developed with special attention to the low-frequency range. The first mode approximation was discarded, because it was found that all the modes influence the vibration appreciably, even at the lowest frequencies. A new approach to the use of the fiber beam in an acceleration detector element was carried out, showing the similitude of the vibrational behavior of the beam with the ideal accelerometer. Errors of less than 1% were found, and they become lower as the frequency range decreases. Moreover, the approximation of the characteristic function of the ideal accelerometer by the transfer function of the cantilever simplifies the consequent signal processing enormously, so it can be applied to the construction of a low-cost accelerometer.

Theoretical and experimental results obtained in the laboratory have shown to be in substantial agreement. The experimentally measured values of the first resonant frequency, which is the determinant factor of the device as an acceleration measuring element, validated the values calculated in theory. The damping factor of the optical fiber was so small that it had no effect on the vibration of the beam. Several frequency ranges of operation of the cantilever in an acceleration transducer were given in terms of the dimensions of the fiber beam and the maximum error allowed.

Experimental results for the performance of the optical fiber cantilever, obtained in more stringent conditions than

normal anticipated operation, guarantee its reliability as a part of a device subjected to vibrations.

The main significance of the study developed is its utility in the building of a fiber optic based accelerometer, with great simplicity of construction, very low cost and applicability to harsh industrial environments.

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References

1. J. P. Den Hartog, *Mechanical Vibrations*, Dover Publications, Inc., New York (1985).
2. W. B. Spillman, Jr., "Multimode fiber hydrophone based on a schlieren technique," *Appl. Opt.* **20**(3), 465-471 (1981).
3. G. Conforti, M. Brenci, A. Mencaglia, and A.G. Mignani, "Fiber optic vibration sensor for remote monitoring in high power electric machines," *Appl. Opt.* **28**(23), 5158-5161 (1989).
4. S. Inaba, K. Akaishi, T. Mori, and K. Hane, "Analysis of the resonance characteristics of a cantilever vibrated photothermally in a liquid," *J. Appl. Phys.* **73**(6), 2654-2658 (1993).
5. F. T. S. Yu, J. Zhang, K. Pan, D. Zhao, and P. B. Rutkin, "Fiber vibration sensor that uses the speckle contrast ratio," *Opt. Eng.* **34**(1), 236-239 (1995).
6. G. A. Rines, "Fiber-optic accelerometer with hydrophone applications," *Appl. Opt.* **20**(19), 3453-3459 (1981).
7. P. A. A. Laura, J. L. Pombo, and E. A. Susemihl, "A note on the vibrations of a clamped-free beam with a mass at the free end," *J. Sound Vib.* **37**(2), 161-168 (1974).
8. W. B. Spillman, Jr. and R. L. Gravel, "Moving fiber optic hydrophone," *Opt. Lett.* **5**(1), 30-31 (1980).
9. Takaoka Electric, Ltd., "Fiber Accelerometer FA2010," Technical specifications, Tokyo (1992).
10. CSO, "HC250 Micro-Laser Interferometer," Technical specifications, Grenoble, France (1995).
11. TSI Inc., "Laser Vibrometer Model 1942," Technical specifications, St. Paul, MN (1991).
12. L. Meirovitch, *Elements of Vibration Analysis*, McGraw-Hill Book Company, Inc., New York (1975).
13. J.T. Broch, *Mechanical Vibration and Shock Measurements*, Bruel & Kjaer Publications, Naerum Denmark (1980).

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