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A bilevel mathematical programming model to optimize the design of cycle paths

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Abstract

In this article, we present a methodology to simultaneously modelize car, bus, and bicycle transport modes, considering the interactions among the three modes through the modelling of the modal split and the network assignment of the different travels of each mode. Later, this model was utilized to optimize the design of cycling paths network to achieve an efficient and sustainable transport system. The proposed methodology has two levels. In the lower level there is a transport network, over which cars users, bus passengers and bicycles users could be simulated at the same time Applied to this is a combined model (modal split-assignment model) with its inputs come from a global matrix (car, bus and bicycle trips). The Multinomial Logit model for modal split and network assignment models will follow an iterative process, to provide the final matrices and service variables for each mode of transport. Finally, in the upper level, an optimization model has been developed, based on bilevel mathematical programming. The objective is to optimize the design of cycling paths, determining which typology of bike lane will be the optimal for each street. For this specific model, we considered only three typologies (segregated, non-segregated, and no bike lane).
The optimization criteria utilized aims to maximize the number of cycling users. These have been applied to the real scenario of the city of Santander (Spain).

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Keywords: Transport planning; cycle paths; optimization; sustainable mobility; mathematical programming;

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1. Introduction

During the 1990s, science started to propose alternatives to existing mobility models. Society at the time had doubts about the validity of the existing model, the pollution management and the loss of public space. Society needs to address these subjects through urban planning and environmental policies. Attempts are being made to recuperate part of these lost public spaces, the most obvious being the pedestrianization of historic town centers and increasing investment in public transport. The increasing presence of the bicycle in urban areas has meant that; people friendly spaces are being created to allow circulation either on foot or by bicycle, safe zones are being created with fewer or no vehicles and as a result, a changing mentality is beginning to spread among the population. The current challenges cities face are to plan a mobility system aimed at the citizen and to plan the city on a more local scale, where the concept of nearness has greater relevance (Pozueta, Daudén, & Schettino, 2009).

Although the bicycle is not the only solution to traffic and environmental problems in urban areas, it does constitute a response that can easily be inserted into any urban renewal legislation and policy at a relatively low economic outlay. The individual evolution of different towns and cities has led to great diversity in urban morphologies over the years and the cyclist has always looked for the shortest available route to optimize their trip that, depending on the particular urban morphology of the town in question, will be longer or shorter.

Urban architecture is a key element in the promotion of cycling, and the type of infrastructure (the cycle lane) has a great influence on speed, physical exertion and safety, which are all important factors when considering whether to travel by bicycle. A positive correlation between the number of bicycle journeys and the density of the bicycle lanes has been observed in a study of the largest cities in the United States of America (Dill y Carr, 2003). Cyclists make variable choices, they adjust their routes in order to use the infrastructure prepared specifically for their use (Howard and Burns, 2001).

Nevertheless, cycling infrastructure does come at a cost, which depends on the characteristics of the dedicated cycle lane: segregated or non-segregated, width, type of surface, etc. This infrastructure is also limited by the space available for it in urban areas.

Modeling the mobility of an urban or intercity area is a frequent topic in international transportation literature; sometimes considering just private transport users (the mode that historically has been studied the most), and other times including different motorized transport modes, both private and public (buses).

However, it is difficult to find a model that simultaneously considers private and public transport and the bicycle mode, taking into account the interactions between the three modes through the modeling of the modal split, and the assignment of the different trips to the network.

There is a small amount of international literature on the subject of the simultaneous assignment of private, public and cyclist traffic and the reflection upon user's behavior when choosing between these three available modes of transport.

Therefore, the aim of this research is to propose a cycling network optimization model by establishing the type of infrastructure required on each link, which will maximize number of cyclists and is subject to budgetary constraints. The methodology is applied to the city of Santander, which allows a sensitivity analysis to be performed as well as an evaluation of the cost increases resulting from improvements made to the network to provide a quality sustainable transport system. The resulting network is capable of reducing the conflict between motorized traffic and bicycles.

This article firstly presents a bi-level mathematical programming model, which is used to optimize the location of public bicycle docking stations. Its lower level is a modal split and assignment model capable of jointly simulating private and public transport and bicycle modes, considering the interactions between them (Romero, 2012). Secondly, the models developed are applied to the real case of Santander city, determining the optimum location of cycle paths and its typology. The article ends with a section that enumerates the most important conclusions reached.

2. Methodology

The outlined problem's structure perfectly satisfies the requirements of a bi-level mathematical programming model (Bard, 1998).

Bi-level programming constitutes one of the most important areas of overall system optimization. Currently there are countless problems associated with practical applications that take advantage of their own structural hierarchy to outline and solve formulations through bi-level programming. At the upper level, the number of bike users are maximized, while at the lower level characterizes the behavior of the users of the transport system.

A more comprehensive understanding can be found by simultaneously looking at two points of view: on one hand, the logical extension of the mathematical programming and, on the other hand, the generalization of a problem specific to game theory, such as a Stackelberg game (Stackelberg, 1952). In a Stackelberg competition there is a special player known as the leader, who knows how the other players will react to his strategy. The other players are known as followers. The leader can choose his strategy from a certain group, independently of the strategies of his followers, but each follower can only choose a strategy from a set defined by the choice made by the leader. A follower's strategy depends on the leader's strategy, and his usefulness depends as much on the strategies of the other followers, as on that of the leader.

Various problems in the field of transport planning can be formulated using a Stackelberg equilibrium problem, because their hierarchical structure is suited to reflecting the decision making process. The system operators (Leaders) plan or design the transport system, keeping in mind the behavior of the users (Followers) in response to their decisions about management policy or investment. Some applications described in the literature, which have been modeled using bi-level programming, are presented below:

- Application to network design, where this type of model is defined to use the traffic assignment problem, TAP, at the lower level. For this type of linear bi-level programming, there are applications in Ben-Ayed et al. (1992) and dell'Olio et al. (2006); network design applications bearing the effect of congestion on the network, as in Marcotte (1986); various algorithms and heuristic implementations such as those in Marcotte and Marquis (1992) and non-linear bi-level programming, as in Suh and Kim (1992).
- Another common application is the problem of estimating demand, as in Florian and Chem (1991) and in Kim (2001), where bi-level programming is presented to estimate the O-D matrix with traffic counts for some links. These models use traffic volume data, including more economic information, as opposed to the expensive home survey.

In our case, the leader is the public bicycle network planner/manager; and the followers are car, bus and bicycle users, who modify their behavior (route and mode choice) based on the characteristics of each mode of transport. In

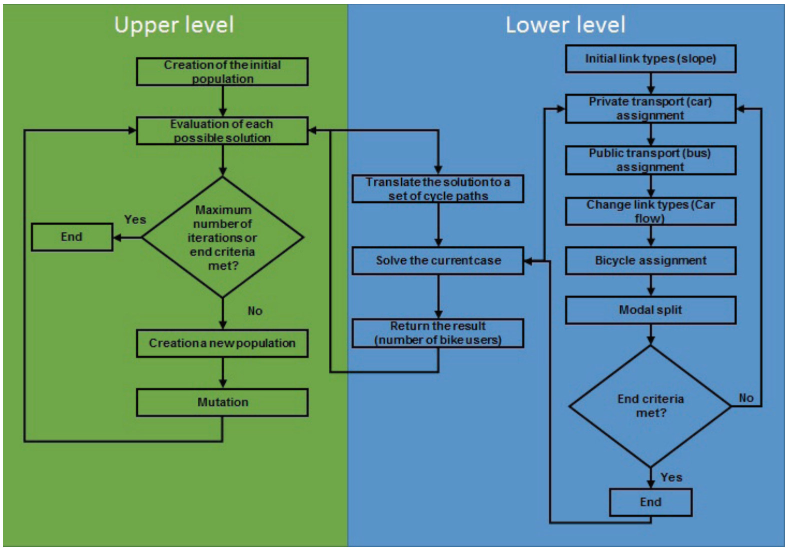

Figure 1. Flowchart of the bi-level optimization problem.

the upper level, an optimization algorithm will search for the distribution of cycle paths that maximizes the number of bicycle users. The lower level firstly readies the network where cars and bicycles will be simultaneously simulated, and then resolves the iterative modal split and network assignment model.

This optimization model is applied with the Python library called Pyevolve (Perone 2009), which has some methods for solving a genetic algorithm. This library has been modified due to be able to solve the particular problem of determining the optimum cycle path and its typology. The flowchart (Figure 1) shows the process of the model, explained below with all the peculiarities that it has.

- **Initializer:** the first population is a list of possible solutions that are lists of zeros and ones. Every feasible solution represents a possible distribution of bike lanes in the network, and each solution is composed of as many elements as candidate links to be bike lanes are being considered. Each element indicates whether its correspondent candidate link has been selected or not. The final solution will depend on the initialization criteria considered. We considered 3 different ones:
    - No initialization criterion: the program randomly selects as many possible solutions of one link as the population size considered.
    - Highest traffic: the program randomly selects as many possible solutions of one link as the population considered; where a link has more possibilities to be selected if the traffic of this link is one of the highest flows. This criterion tries to design cycle paths next to the main streets of the city and some of these streets are the highways, as expected.
    - Pondered bike and car flow: the program randomly selects as many possible solutions of one link as the population considered; where the possibilities of a link to be selected are proportional to a combination of the bike and car flow, giving more weight to the bike flow rather than the car traffic. This other criterion leads to more realistic solutions.
- **Evaluation of each solution**, which consists of the following steps:
    - Translate the solution into a list of the link where a bike lane will be placed in the network.
    - Solve the current case:
        - A Multinomial Logit model for modal split and network assignment models will follow an iterative process (MSA) to provide the final matrices and service variables for each mode of transport.
        - Each link's maximum bicycle speed will depend on that link's slope and car traffic volume (only if both traffics go in the same infrastructure). The first attribute is constant, but the second one varies in each iteration.
    - Return the result: the evaluator function sets, as the value of the solution's raw score, the number of bicycle users.

  For the evaluation of each case, we used the commercial software called VISUM. In order to make VISUM work as we wanted (calculating the bicycle speed depending on the slope and the car flow), we were forced to intervene via com-ports as Figure 2 shows.
- **End criterion meet:** the program will stop if a previously set number of iterations is reached, or if a solution's population is uniform.
- **Creation of the next population:** in each iteration a new population is created, with the following criteria:
    - The fittest individual is always one of the new elements (elitism). We can activate or not this criterion.
    - The rest of the population is chosen semi randomly. Each element of the previous population has a chance to be elected proportional to its fitness score, which depends on its raw score through a linear scaling function, which indicates the ratio between the fittest individual's fitness score and that population's mean fitness score.
    - Once all elements have been selected, they increase their lengths until they achieve the maximum length limited by the budgetary constraint. The growth of every element follow two sequential steps:
        - In the first step, the program decides from which vertex the corridor will grow.
        - In the second step, also the program choses from all the available links (all the links that starts in the objective node, excluding all the links that also are in the corridor) which of them will be incorporated in the new corridor (feasible solution).
- **Mutation:** to reach the final solution before, a mutation has been implemented. This mutation increments the length of the corridor as the same way that was commented before.

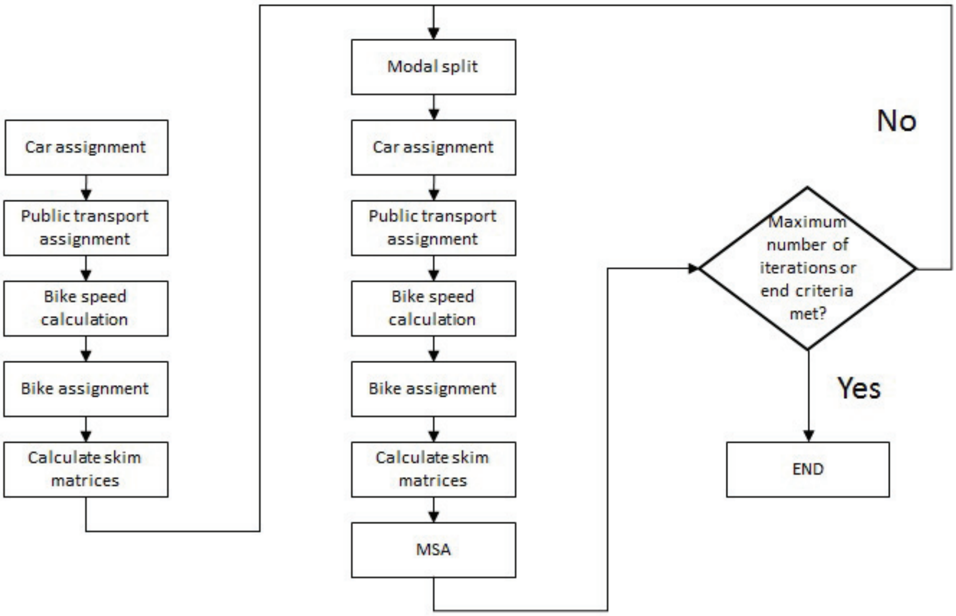

Figure 2. Procedure sequence of the VISUM model (lower level).

3. Application

The methodology presented has been applied to a real case: the city of Santander, Spain. It is a medium-sized city, with approximately 180,000 inhabitants and the population density is 5055 people/km^2, located on the north coast of the Iberian Peninsula. The city lies on a hilly peninsula particularly rough along its southern side; it is characterized by a lineal structure, with a highly developed commercial town center and various residential areas on the outskirts, with varying population density. Most of the workforce is employed in the service sector (76%), followed by industry (12.9%), and construction (9.8%). The motorization rate is 462 veh/1000 inhabitants. 50 % of the trips in the city are walking, 42 % by car, 7 % by bus, 0.7 % by bicycle and 0.3 % by train.

For developing this study, we based on previous works of the Transport Systems Research Group where a conventional 4- stage model for Santander was created and calibrated.

When applying the methodology described in the previous section, to achieve an optimum performance for the optimization algorithm, its different methods take into account the singularities of the problem, reducing the number of cases to study, thus accomplishing a faster convergence. In addition, for this application, different sub-models has been implemented into the bi-level optimization model.

3.1. Sub-Models

In order to reduce the number of possible links, we introduced the constraint that only links which slope is less than 6%. As in Romero et al. (2012) demonstrated, if the slope is bigger than 6%, bicycle users dismount and continue their journey on foot pushing the bicycle, which makes no sense that money is invested on a bike path that will not be used.

In addition, we considered that a bike line has to be continuous; it does not make sense that the final solution has bike lines distributed randomly along the city. It is not comfortable that the type of bike path changes along a corridor. Therefore, we designed the model to calculate what the optimal corridor is. Two of the sub-models considers the influence of the slope and the influence of the car flow in the bicycle speed. These interactions have been studied in

Romero et al. (2012). The first sub-model introduces the relation between the slope and the free flow speed of the bicycle users (Equations: 1 to 4). This model is calculated only once at the beginning of the model.

$$\left[\left(V_{ij}\right)_{Bk}\right]_0 = 27.296 \cdot e^{0.1072 \cdot p_{ij}} \qquad if \qquad p_{ij} \leq -0.92 \tag{1}$$

$$\left[\left(V_{ij}\right)_{Bk}\right]_0 = 20.832 \cdot e^{-0.188 \cdot p_{ij}} \qquad if \qquad -0.92 < p_{ij} \leq 6 \tag{2}$$

$$\left[\left(V_{ij}\right)_{Bk}\right]_0 = 3 \qquad if \qquad 6 < p_{ij} \leq 10 \tag{3}$$

$$\left[\left(V_{ij}\right)_{Bk}\right]_0 = 0 \qquad if \qquad 10 < p_{ij} \tag{4}$$

The second sub-model brings in the real bicycle speed in function of the traffic flow of a street. This sub-model suffers an iterative process as it appears previously. Each time that the program calls VISUM to resolve the multimodal split and assignment, we use this sub-model to calculate the new speed.

$$\frac{\left[\left(V_{ij}\right)_{Bk}\right]_0}{\left[\left(V_{ij}\right)_{Bk}\right]_i} = 1 + \alpha \bullet \left\{ \frac{\left[\left(F_{ij}\right)_{Veq}\right]_i}{C_{ij}} \right\}^{\beta} \tag{5}$$

Where:

$[(V_{ij})_{Bk}]_0$	Free flow speed for the bicycle mode in the link ij.
p_{ij}	Slope of the link ij.
C_{ij}	Capacity of the link ij.
$[(V_{ij})_{Bk}]_i$	New speed for the bicycle mode in the link ij, affected by the motorized flow.
$[(F_{Veq})_{Bk}]_i$	Motorized flow in equivalent vehicles in the link ij.
α	Constant that takes the value of 2.1.
β	Constant that takes the value of 7.8.

3.2. Budgetary constraint

If we do not consider a limitation in the bike line length, it is logical that the optimal solution is to allocate a cycle path in all the links of the network. Therefore, we needed to consider this constraint.

We had two different options to include this limitation. The first one is to limit the length and the second one is to limit the total budget. With the second option, indirectly we limit the length, if we consider the cost of build for each different type of bike path (Liñan et al., 2013). Moreover, it makes the work easy for the decision makers.

Table 1. Characterization of the types of bike lanes considered (Liñán et al., 2013)

Type of bike lane	Recommended width	Applied width	Required budget (€/m)
No bike line	3 m.	-	-
Non segregated bike lane on sidewalk	2.5 m.	2.5	200
Segregated bike lane on asphalt	2.5 – 2.8 m.	3	250

In this study, an only one typology of cycle path was considered in due to reduce the computational time. In the case that more than one typology are taken into account, the model works the same, with the only distinction that the program has to be executed as many times as the number of typologies is. After this, a comparison between the different solutions has to be developed, in order to know what the better solution is.

3.3. Other considerations

For this application, the consideration of use only the second of the three different initialization criteria has been taken in order to simplify the process, but the model is calibrated for working properly in all the cases.

3.4. Results

A study to find out a good population size for Santander was performed. To carry this study out, several population sizes were tried to solve the same problem. The population size means how many solutions are tested in each iteration. For example, if a population size of 100 has been considered, this means that in every iteration 100 of different corridors have been evaluated. After being evaluated, all corridors have a score that depends on the optimization criterion, which, in this case, is the maximization of the number of bicycle users.

The figure below shows a comparison of the different population size, comparing the solutions they return, and how much time they spend to reach them. The calculation was made with a Intel® Core™2 Duo Processor E8400 (3 GHz and 4 GB RAM).

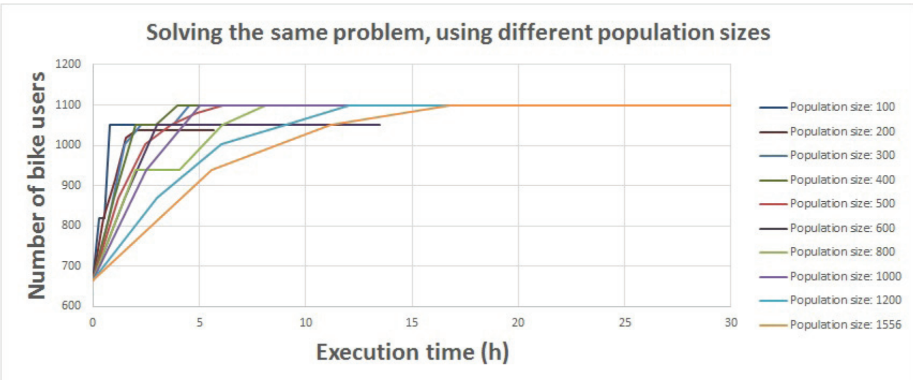

Figure 3. Solving the same problem, using different population sizes

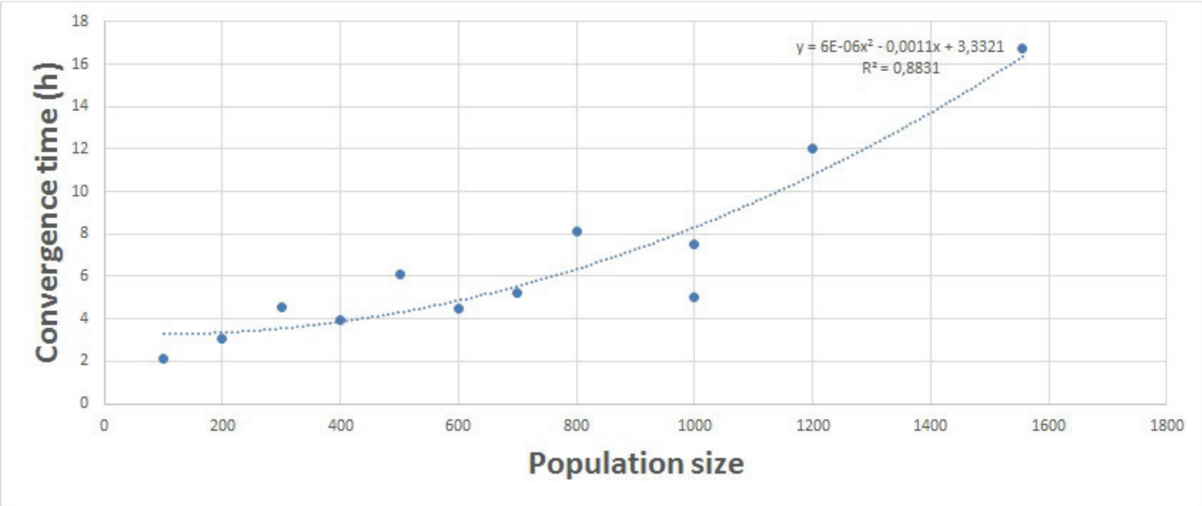

Figure 4. Relation between the population size and the convergence time

This graph shows the relation between the final solution, in terms of the maximum number of bicycle users, and the execution time for different population sizes. For this study, a population size of 100, 200, 300, 400, 500, 600, 800, 1000, 1200 and 1556 were used. The population size of 1556 was utilized because is the maximum number of possible links in the city of Santander, the rest of them have a slope bigger than 6%, and as was commented before, this is the limit when a cyclist gets off the bicycle.

With all of these probes, the population size that achieves the maximum number of bicycle users in the less computational time is 400. With a population size over 300, the algorithm reaches the optimal solution in most cases. When it is around 1000 the model gets the optimal in less time than it was expected, this is because the population size is big enough to find the optimal solution in two iterations, but studying this graph, we reach the conclusion that whether the population size is similar to the number of possible links, most of the solutions are the same, because in the initial population some of the links are chosen more than once. This means that the number of different solutions tested do not increase as the same form as the population size does. Therefore, the benefits of increasing the population size are not as big as we might imagine, and the disadvantages are the expected. Otherwise with a small population size like 100 or 200, it is very possible that the model does not reach the global optimum, it is easy that the model gets stuck in a local optimum.

The solution reached by the bi-level optimization program is shown in Figure 5. This solution is the optimal solution achieved by the algorithm in all the cases that the algorithm got the maximum score (1196 bicycle users). As it appeared in the previous chapter, the streets in Santander have steep slopes in the edge north – south, as it is shown in Figure 6. Therefore it was expected that the solution proposed by the model was a cycle path along one of the streets parallel to the sea (east –west edge). That is because these streets are the only ones that are flat enough for having a bike lane.

Figure 5. Solution achieved by the bi-level optimization program

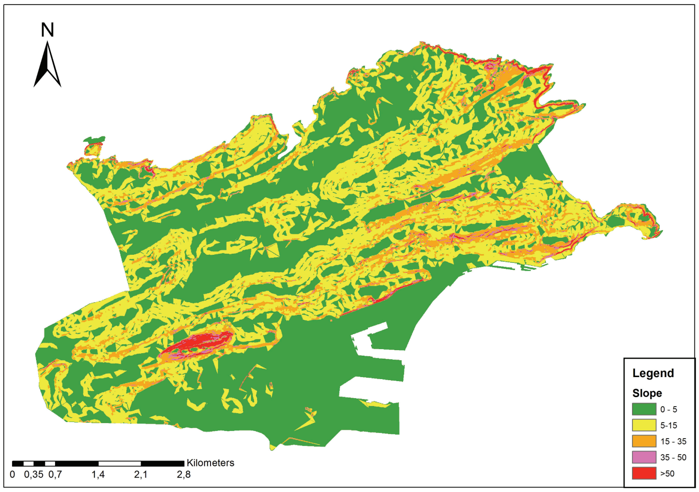

Figure 6. Slope map of the city of Santander (Own elaboration source)

The cycle path proposed by the bilevel model is placed along the Constitution Avenue, this street is one of the two main entrances of the city. The solution reached contains one main line and some branches that connects the peripheral zone in the north of the city with road called Los Castros Avenue, which is characterized for being there the main part of the campus of the University of Cantabria.

Moreover the proposed model in previous calculations considered other cycle paths that are not the best solution, but they are good ones. Currently, one of these bike lanes is already built, but it was not take into account because it was not operative when the calculations were done.

4. Conclusions

A bi-level mathematical programming model that optimizes the location of cycle paths, with an optimization algorithm, has been presented. Its lower level is a modal split and assignment model, capable of reflecting the interactions between car and bicycle modes. The model has been developed, tested, and applied to a real case.

The location of the cycle paths is important to encourage bicycle use. Bicycle users have a predisposition of travel along a cycle path and they adapt their routes in order to circulate on a separate infrastructure. The length of the cycle path network makes the bicycle mode more competitive against other modes.

It has been verified that the time-reducing strategies applied to the optimization algorithm methods are valid because the program returns similar solutions, with great computing time savings.

The model has been able to replicate the behavior of the public bicycle system in the real case of the City of Santander.

Through planning the optimal placement of bike lines, using our bi-level programming model, it can encourage a great number of people to switch greatly increase the number of people who are encouraged to switch to utilizing bicycles.

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