FROM DISCRETE TO CONTINUOUS-TIME TRANSITION MATRICES IN INTRA-DISTRIBUTION DYNAMICS ANALYSIS: AN APPLICATION TO PER CAPITA WEALTH IN EUROPE

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Abstract. Previous studies focusing on the intra-distribution dynamics analysis have usually computed, in a Markov chain framework, discrete-time transition matrices. Such approach, however, can involve some limitations, especially when using stock variables. In order to illustrate the importance of the time-scale issue when estimating transition matrices, this paper applies both discrete and continuous-time approaches to a set of cross-national European data on per capita wealth for the period 2000-2010. The results reveal, on the one hand, that the continuous-time estimation provides a most accurate estimation of transition probabilities and, on the other, that the differences between both approaches are especially remarkable in the long-term equilibrium distribution.

Keywords: Intra-distribution dynamics, transition probabilities, transition intensities, continuous-time estimation, ergodic distribution.

JEL classification numbers: C13, C21, D31

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I. INTRODUCTION

In the last decades, major advances have been made in the proposal of alternative methods to analyze the dynamics of a distribution, the foremost, undoubtedly, being the intra-distribution dynamics approach. This approach examines directly how the whole distribution changes over time, its main advantage being that it allows us to see changes in the relative positions of the elements that comprise the distribution, and therefore to analyze its internal dynamics.

In this framework, discrete-state, discrete-time stochastic processes (the so-called transition matrix approach) have been extensively used in a variety of areas of inquiry. In the last decades, some advances were made regarding the discrete-state nature of these processes. As noted by Quah (1997) and Bulli (2001), discretization of the state-space may distort dynamics and even remove the Markovian property (Bickenbach and Bode, 2003).[1] To resolve this drawback, Quah (1997) proposed the use of a Markov transition function (or stochastic kernel), which is essentially a continuous-state, discrete-time Markov process; this, indeed, has been the main and more promising way forward in such a research context (Magrini, 2004).[2] In any case, the estimation of discrete-state stochastic processes for examining distribution dynamics keeps being quite widespread in the literature and, in fact, is employed in this paper. As indicated by

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[1] Loss of the Markovian property would mean that the state that the process arrives in a forward step would depend not only on its immediate predecessor, but also on others before that.

[2] In addition, Johnson (2000) extended the analysis by providing a way to estimate the long-run equilibrium (or ergodic density). Recently, Fotopoulos (2006, 2008) has also suggested a non-parametric quantile method based on the stochastic kernel.

Maza et al. (2010), its use as a complementary approach to the estimation of continuous-state processes is highly recommended. While not denying drawbacks coming from the use of a discrete state-space approach, it is the only methodology that really allows us to, by formalizing the process of successive transitions between income states and through the definition of some scalar measure, quantify the mobility degree within a distribution. Following this vein, a discrete-space Markov process is employed in this paper.

On a different note, and with respect to the discrete-time nature of the stochastic processes, the choice between continuous or discrete-time modelling in dynamic analysis is another moot question (see Gandolfo, 1997, for a thorough discussion on this issue). Empirical studies on intra-distribution dynamics have commonly relied on simple estimation procedures, in which the discrete-time assumption plays a key role.[3] This assumption has been often made on the basis that the availability of continuous data is not the general rule in economics. Note, however, that the distinction between flow and stock variables should be borne in mind for selecting the appropriate time-scale for such stochastic processes. More specifically, whereas a flow variable (as GDP per capita), defined for a given period, cannot be modeled by a continuous-time process except in the limiting case where the interval of observation is going to zero, a stock variable (for example, wealth and human capital) should be modeled by continuous-time stochastic processes as it is measured at one point in time. This distinction is far from being trivial, as discrete and continuous-time estimations may differ significantly.

Accordingly, this paper tries to contribute to the existing literature on intra-distribution dynamics by illustrating, both from a theoretical and an empirical

[3] Another common assumption is stationarity. This is relaxed in the paper by Hierro and Maza (2009).

perspective, the differences between discrete-time and continuous-time estimation procedures. For a set of data on wealth in Europe, the paper shows that, without much more computational effort, the continuous-time formulation provides a finer estimation of transition probabilities over any time horizon and, therefore, a more realistic framework than the discrete-time one

More specifically, a set of national data on per capita wealth for forty European countries during the period 2000-2010 is used. We opt for using the European case as a sort of laboratory because the study of national/regional disparities in the oldest continent has become a heated topic in the last two to three decades, where the interest on this topic has been fostered by concerns about the ongoing process of economic integration (see, e.g, papers by Quah, 1996, 1997; López-Bazo et al., 1999; Magrini, 1999; Fingleton, 2003; López-Bazo, 2003; Le Gallo, 2004; Ezcurra et al., 2005; Tortosa et al., 2005; Ertur et al., 2006; Fotopoulos, 2006; Geppert and Stephan, 2008; Desli, 2009; Petrakos and Artelaris, 2009; Archibugi and Philippetti, 2011; Cavenaile and Dubois, 2011); furthermore, among these studies those that apply the distribution dynamics approach have resorted to the estimation of discrete-time rather than continuous-time transition matrices (see, e.g., Le Gallo, 2004; Le Gallo and Chasco, 2008; Sakamoto and Islam, 2008; and Bosker, 2009).[4] Additionally and due to the characteristics of our estimation procedure, it is convenient to emphasize that our variable of analysis is, instead of the traditional per capita income (productivity), the

[4] A contribution in this field has been made by Rummel (2009) by considering a hidden Markov chain approach. Unlike Rummel, in this paper we assume that wealth states are observable like is common practice in the literature. In fact, we choose the classification of states traditionally used in European studies to gain interpretation by discerning between poor, middle-poor, middle, middle-rich and rich countries.

average wealth per inhabitant. The reason for this choice is twofold: firstly, as already mentioned, that stock variables are more suitable for the estimation of continuous-time models than flow variables; secondly, that wealth is one of the main determinants of well-being and, therefore, an appropriate variable when it comes to evaluating the dynamics of cross-country disparities.

The remainder of this paper is divided into two parts. The first one, Section II, introduces notation and sets up the analytic framework behind the continuous-time estimation procedure, emphasizing its differences with the discrete-time one. The second part of the paper, Section III, compares, by using European per capita wealth data as a test bank for our analysis, the results obtained from discrete and continuous-time estimation. Finally, some concluding remarks are given in the final section.

II. DISCRETE-TIME *VERSUS* CONTINUOUS-TIME ESTIMATION FOR TRANSITION PROBABILITIES

Empirical studies on intra-distribution dynamics using a transition matrix approach have traditionally assumed a discrete-time scale for estimation purposes. Thus, let suppose that the distribution under analysis is divided into an exhaustive finite set of m mutually exclusive states, denoted by S. Additionally, if changes between states occur at discrete times, let $X(t) \in S$ indicate the state occupied at time t $(t = 0,1,...,T)$. For a given time period $(0,t)$, let $P(0,t)$ be the $m \times m$ transition probability matrix with entries:

$$p_{ij}(0,t) = Pr\left[X(t) = j \mid X(0) = i\right] \qquad (1)$$

for all $i, j \in S$.

Finally, let assume that the hypothesis of stationary transition probabilities is adopted, so that transition probabilities can also be denoted as:

$$p_{ij}(0,t) = p_{ij}(t) \tag{2}$$

Under the above assumptions, the maximum likelihood estimator for the discrete-time transition probability from a state i to another j over a period t is given by:

$$\hat{p}_{ij}^{D}(t) = \frac{n_{ij}}{n_i} \quad , \tag{3}$$

where n_{ij} represents the number of countries in state j after a period t, having been initially in state i, and n_i is the number of countries initially in state i (Cox and Miller, 1965; Chung, 1967).

Despite its widespread use, it is worth noting that the discrete-time approach does not always provide the most appropriate framework for an intra-distribution dynamics analysis. According to some authors (Singer and Spilerman, 1973), for most social mobility processes changes in intra-distribution dynamics occur at continuous-time points. Then, why a discrete-time scale has been so much more successful than a continuous-time scale in modeling transition probabilities? One reason is obviously that such assumption is particularly attractive for its simplicity. In addition, the use of a discrete-time scale has found justification on the more extended use of flow rather than stock variables.

Accordingly in the cases, as here, where a stock variable is examined this second argument loses its force, leaving only the weak claim of simplicity as justification for applying a discrete-time framework. In these occasions, therefore, a continuous-time approach is mandatory. Thus, under the same finite set of states S, let suppose that $X(t) \in S$ is observed continuously in the time interval $[0,T]$, i.e. $\{X(t) \mid 0 \le t \le T\}$, and that transition probabilities are governed by the following system of ordinary differential equations (Cox and Miller, 1965; Chung, 1967; Singer and Spilerman, 1976):

$$dP(t)/dt = Q \cdot P(t) \tag{4}$$

where Q is a matrix with dimension $m \times m$. If Q is a matrix with entries q_{ij} satisfying:

$$q_{ij} > 0, \quad q_{ii} < 0, \quad q_{ii} = -\sum_{j \neq i} q_{ij} \,, \tag{5}$$

then, $P(t)$ comprises the continuous-time transition matrix with solution given by the exponential formula:

$$P(t) = \exp(tQ) = \sum_{k=0}^{\infty} \frac{Q^k t^k}{k!}, \quad t > 0 \tag{6}$$

with $\exp(\cdot)$ denoting the matrix exponential function.

Thus, the continuous-time transition matrix for every time horizon t is a function of the matrix Q, commonly known as the transition intensity matrix, whose elements q_{ij}, the so-called transition intensities, can be interpreted as the instantaneous rate of transition between pair of states, i and j:

$$q_{ij} = \lim_{dt \to 0} \frac{p_{ij}(dt)}{dt}, \quad i \neq j \tag{7}$$

Additionally, elements q_{ii} have an interesting probabilistic interpretation: $1/-q_{ii}$ gives the expected length of time for a country in state i to remain in that state (Chung, 1967).

When information on all movements is available over a time interval $[0,T]$, estimates for the transition intensities can be found by maximum-likelihood estimation from the expression (Billingsley, 1961; Küchler and Sørensen, 1997; Lando, 2004):

$$q_{ij} = \frac{n_{ij}(T)}{\int_0^T n_i(s)ds}, \quad i \neq j, \tag{8}$$

where $n_{ij}(T)$ represents the total number of transitions between states i and j over the whole period $[0,T]$, and $n_i(s)$ is the number of countries in state i at time s. Estimation for diagonal elements would be, accordingly, $q_{ii} = -\sum_{j \neq i} q_{ij}$. Next, applying the exponential function, given in expression (6), to transition intensities estimates appropriately scaled by the chosen time horizon t, we obtain the continuous-time transition probability estimates $\hat{p}_{ij}^{C}(t)$.

Bearing in mind the ultimate aim of this paper, it is convenient to notice that there are two significant differences between the discrete and continuous-time approaches that are behind the different maximum likelihood estimates for transition probabilities. Firstly, as pointed out by Lando and Skodeberg (2004), while the continuous-time case counts through $n_{ij}(T)$ all transitions from i to j over the entire period of observation (see equation (8)), the discrete-time case only considers, as defined by n_{ij}, the number of transitions from i to j between the initial and final year of any period of duration t (equation (3)), thus deliberately ruling out any transition from i to j occurring in-between years. Secondly, if we take a look to the denominator of equation (8), we can see that the continuous-time estimation is more informative as it considers all countries staying at a state i "at any time" during the time interval $[0,T]$, being these countries weighted by the exact time spent at this state. For the discrete case, however, only countries originally staying in state i are considered, and all exactly in the same way (see denominator of equation (3)), that is, without consideration of the time spent in such state.

III. AN APPLICATION TO THE EUROPEAN PER CAPITA WEALTH DISTRIBUTION

As an illustration of differences in results obtained by discrete and continuous-time estimation, in this section we examine the dynamics of the European wealth distribution over the period 2000-2010. To the best of our knowledge, this is the first time that a continuous-time approach is applied to the long-studied topic of European disparities. Data employed in this study consist of annual relative per capita wealth of 40 European countries. These data have been drawn from the publication "Credit Suisse Global Wealth Databook 2010", published by the Credit Suisse Research Institute.[5] As is usual in the literature, national per capita wealth is normalized by the European average (Europe=100) in order to take out from the analysis the effect of absolute changes over time and, thus, to pave the way for comparisons.

Anyway, and before proceeding to the analysis, some remarks must be made because, as it is well-known, the transition matrix results depend critically on the number and length of the intervals considered. Following the criteria suggested by Quah (1993), the overall wealth distribution is divided into five exhaustive and mutually exclusive wealth states of equal size at t representing poor, middle-poor, middle, middle-rich and rich countries.[6] In addition, the results also depend on the transition period length. In this case, and as is also common in many applications on distribution dynamics, we opt for estimating a five-year transition probability matrix (that is, $t=5$ in previous equations).[7] Based on the above considerations, we begin by estimating the discrete-time transition matrix, reported in Table 1, and so following with the intensity matrix (Table 2a) and the continuous-time transition matrix (Table 2b).

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[5] Wealth data are computed from Household Balance Sheet (HBS) data and theoretical models estimated by standard econometric techniques. For more information see Shorrocks et al. (2010).

[6] Magrini (1999) suggests an alternative method for the discretization of the distribution based on the minimization of an error measure. However, this method of boundary selection may lead to having a disproportionate number of states, some of them, as indicated by Bosker (2009), containing very few observations.

[7] Whereas a one-year transition period, for example, would imply a very low degree of mobility and emerging patterns would be really difficult to detect, a longer transition period, 10 or 15 years, would lead, in the case of discrete-time estimation, to a noteworthy loss of information.

[Table 1 around here]

[Table 2 around here]

Given the reasons mentioned in the previous section, it is expected that the continuous-time estimation provides a more precise description of wealth distribution dynamics than its static counterpart. In any case, in order to assess it properly we calculate the accuracy of both estimations using, as in Hierro (2009), the Root Mean Square Error (RMSE), the results (0.101 for the continuous case and of 0.118 for the discrete one) confirming our previous intuition. On the other hand, we should also question how different are the discrete and continuous-time matrices. In order to gain an overall view of these differences, we use the metric proposed by Jafry and Schuermann (2004), namely M_{SVD}, that is simply the average of the singular values of the so-called mobility matrix $\tilde{P}$, defined as the estimation of the original transition matrix – either the discrete-time matrix ($\hat{P}_D$) or the continuous-time matrix ($\hat{P}_C$) – minus the identity matrix of the same dimension.[8] We obtain $M_{SVD}(\hat{P}^C(5)) = 1.4421$ and $M_{SVD}(\hat{P}^D(5)) = 0.2841$, so that $M_{SVD}(\hat{P}^C(5)) - M_{SVD}(\hat{P}^D(5)) = 1.158$. This makes clear that differences for the whole matrix are apparently large. In fact, by comparing both matrices in a more meticulous way we appreciate the existence of significant differences between some discrete transition probabilities and their counterpart. Thus, for example,

[8] Unlike other metrics proposed in the literature for comparing matrices, that proposed by Jafry and Schuermann (2004) is particularly appealing as it verifies, among other properties, *monotonicity*, that is, larger off-diagonal transition probabilities imply larger values of the metric, and *distribution discriminatory*, that is, the metric is sensitive to the distribution of off-diagonal probability mass.

the third diagonal entry is 0.958 for the discrete-time setting and 0.827 for the continuous-time one. The transition probability from middle-rich to rich countries and that from poor to middle-poor ones are other cases for which estimations are markedly different.

Leaving disparities between absolute values aside, another distinctive difference between both estimations is that most transition probabilities equal to zero in the discrete case vanish in the continuous counterpart. This fact maybe is not very remarkable from an economic point of view, but reflects a different concept of transition. The explanation is simple. Let consider as example the transition probability from poor to middle wealth countries. It is clear that the discrete-time method estimates its value to be zero, as there are no sample records of any country moving "directly" between these states for any five-year period. In the continuous case, however, considering the fact that the probabilities of moving from poor to poor-middle countries and, subsequently, from this last state to middle wealth countries (namely "indirect transitions") are positive, an strictly positive probability is obtained. Thus, the continuous estimation assigns a positive transition probability to a transition, even if there was no country that experienced that transition directly, insofar as that transition is possible to occur indirectly through other intermediate states. In this regard, and as pointed out by Gómez-González and Kiefer (2009), the continuous-time formulation is more realistic and offers the advantage of taking into account not only "direct transitions", but also "indirect transitions" between states, overcoming the problem of underestimating the probability of infrequent transitions.

Finally, in order to address which implications these differences may have on the hypothetical long-term equilibrium distribution, the last row in Tables 1 and 2b displays the ergodic distribution in either case (discrete- and continuous-time).The results show that differences in the ergodic distribution are much more salient than in the transition probabilities. This is especially so when comparing the third value of each ergodic distribution, as the discrete-time estimation gives the middle wealth state a proportion in the long-term almost two times larger than the continuous-time estimation. In other words, our results reveal that the discrete-time approach highly overestimates the share of middle wealth European countries in the long term.

IV. CONCLUSION

Formulation of discrete-time transition matrices has been a constant in empirical analysis on intra-distribution dynamics so far. Some attempt to justify this practice, apart from that of simplicity, has been made on the extended use of flow variables which, by its nature, can be measured in discrete time. However, in those cases in which the analysis is carried out on the base of a stock variable, consideration of a continuous-time process seems mandatory. This being so, the present paper highlights the main advantages of the continuous-time transition matrix approach compared with the conventional one when analysing stock variables and, using cross-national European data on per capita wealth for the 2000-2010 period as a benchmark dataset, illustrates the differences between discrete-time and continuous-time transition probabilities. As main conclusions, firstly the analysis revealed that the continuous-time approach yields higher accuracy in the estimation than the discrete-time one. Secondly, it showed the existence of significant differences between discrete-time and continuous-time transition probabilities, these being even more significant in the case of the ergodic distribution comparison.

REFERENCES

Archibugi, D. and Filippetti, A. (2011). 'Is the Economic Crisis Impairing Convergence in Innovation Performance across Europe?', *Journal of Common Market Studies*, 49 (6), pp. 1153-1182.

Bickenbach, F. and Bode E. (2003). 'Evaluating the Markov Property in Studies of Economic Convergence', *International Regional Science Review*, 26, pp. 363–392.

Billingsley, P. (1961). 'Statistical Inference for Markov Processes', Chicago: The University of Chicago Press.

Bosker, M. (2009). 'The spatial evolution of regional GDP disparities in the 'old' and the 'new' Europe', *Papers in Regional Science*, 88 (1), pp. 3-27.

Bulli, S. (2001). 'Distribution Dynamics and Cross-Country Convergence: A New Approach', *Scottish Journal of Political Economy*, 48, pp. 226–243.

Cavenaile, L. and Dubois, D. (2011). 'An empirical analysis of income convergence in the European Union', *Applied Economics Letters*, 18 (17), pp. 1705-1708.

Chung, K.L. (1967). *Markov chains with stationary transition probabilities*, Berlin: Springer.

Cox, D. R. and Miller, H. D. (1965). *The theory of stochastic processes,* London: Metheun.

Cuadrado-Roura, J. R., Mancha, T. and Garrido, R. (2002). 'Regional dynamics in the EU: winners and losers', in: Cuadrado, J. R. and Parellada, M. (eds), *Regional Convergence in the European Union: Facts, Prospects and Policies*, Springer, pp. 23-52.

Desli, E. (2009). ‘Convergence and efficiency: evidence from the EU-15’, *Journal of Post Keynesian Economics*, 31 (3), pp. 403-430.

Ertur, C., Le Gallo, J. and Baumont, C. (2006). ‘The European regional convergence process 1980-1995: Do spatial regimes and spatial dependence matter?’, *International Regional Science Review*, 29 (1), pp. 3-34.

Ezcurra, R., Gil, C. and Pascual, P. (2006). ‘Regional Mobility in the European Union’, *Applied Economics*, 38 (19), pp. 2237-2253.

Ezcurra, R., Gil, C., Pascual, P. and Rapún, M. (2005). ‘Inequality, polarisation and regional mobility in the European Union’, *Urban Studi*es, 42 (7), pp. 1057-1076.

Fingleton, B. (2003). *European Regional Growth*, Berlin: Springer.

Fotopoulos, G. (2006). ‘Nonparametric analysis of regional income dynamics: The case of Greece’, *Economics Letters*, 91 (3), pp. 450-457.

Fotopoulos, G. (2008). ‘European Union Regional Productivity Dynamics: A Distributional Approach’, *Journal of Regional Science*, 48, pp. 419-454.

Gandolfo, G. (1997). ‘Economic Dynamics’, Berlin: Springer-Verlag.

Geppert, K. and Stephan, A. (2008). ‘Regional disparities in the European Union: Convergence and agglomeration’, *Papers in Regional Science*, 87 (2), pp. 193-217.

Gómez-González, J. E. and Kiefer, N. M. (2009). ‘Evidence of non-Markovian behaviour in the process of bank rating migrations’, *Cuadernos de Economía*, 46, pp. 33-50.

Hierro, M. (2009). ‘Modeling the dynamics of internal migration flows in Spain’, *Papers in Regional Science*, 88 (3), pp. 683–692.

Hierro, M. and Maza, A. (2009). ‘Non-stationary transition matrices: An overlooked issue in intradistribution dynamics’, *Economics Letters*, 103, pp. 107–109.

Jafry, Y. and Schuermann, T. (2004). ‘Measurement, estimation and comparison of credit migration matrices’, *Journal of Banking and Finance*, 28, pp. 2603–2639.

Johnson, P. A. (2000). ‘A nonparametric analysis of income convergence across the US state’, *Economics Letters*, 69, pp. 219–223.

Küchler, U. and Sørensen, M. (1997). *Exponential Families of Stochastic Processes*, New York: Springer.

Lando, D. (2004). *Credit risk modeling*, Princeton University Press.

Lando, D. and Skodeberg, T. (2004). ‘Analyzing rating transition and rating drift with continuous observations’, *Journal of Banking and Finance*, 26, pp. 423-444.

Le Gallo, J. (2004). ‘Space-time analysis of GDP disparities among European regions: A Markov chain approach’, *International Regional Science Review*, 27, pp. 138-163.

Le Gallo, J. and Chasco, C. (2008). ‘Spatial analysis of urban growth in Spain, 1900–2001’, *Empirical Economics*, 34 (1), pp. 59-80.

López-Bazo, E. (2003). ‘Growth and Convergence across Economies: The Experience of the European Regions’, In: Fingleton, B., Eraydin, A. and Paci, R. (eds), *Regional Economic Growth, SMEs and the Wider Europe*, Ashgate Publishing, pp. 49-74.

López-Bazo, E., Vayá, E., Mora, A. and Suriñach, J. (1999). ‘Regional economic dynamics and convergence in the European Union’, *Annals of Regional Science*, 33, pp. 343-370.

Magrini, S. (1999). ‘The evolution of income disparities among the regions of the European Union’, *Regional Science and Urban Economics*, 29 (2), pp. 257-281.

Magrini, S. (2004). ‘Regional (Di)Convergence’, in: Henderson V. J. and Thisse J. F. (eds), *Handbook of Regional and Urban Economics*, Amsterdam, Elsevier B.V, vol. 4, pp. 2742–2796.

Maza, A., Hierro, M. and Villaverde, J. (2010). ‘Measurement of intra-distribution dynamics: An application of different approaches to the European regions’, *Annals of Regional Science*, 45 (2), pp. 313-329.

Petrakos, G. and Artelaris, P. (2009). ‘European Regional Convergence Revisited: A Weighted Least Squares Approach’, Growth and Change, 40 (2), pp. 314-331.

Quah, D. (1993). ‘Empirical cross-section dynamics in economic growth', *European Economic Review*, 37, pp. 426-434.

Quah, D. (1996). ‘Empirics for economic growth and convergence’, *European Economic Review*, 40, pp. 1353–1375.

Quah, D. (1997). ‘Empirics for growth and distribution: stratification, polarization, and convergence clubs’, *Journal of Economic Growth*, 2, pp. 27-59.

Rummel, O. (2009). ‘Distribution dynamics and measurement error’, in: Binner J. M., Edgerton D. L. and Elger T. (eds), Measurement Error: Consequences, Applications and Solutions, Emerald Group Publishing, vol. 24, pp. 251-279.

Sakamoto, H. and Islam, N. (2008). ‘Convergence across Chinese provinces: An analysis using Markov transition matrix’, *China Economic Review*, 19 (1), pp. 66-79.

Singer, B. and Spilerman, S. (1973). ‘Social Mobility Models for Heterogenous Populations’, in: Costner, H. (ed), *Sociological Methodology*, San Francisco, Jossey-Bass, pp. 356-401.

Shorrocks, A., Davies, J.B., Lluberas, R. (2010). *Credit Suisse Global Wealth Databook 2010*, Zurich.

Tortosa, E., Pérez, F., Mas, M. and Goerlich, F.J. (2005). ‘Growth and convergence profiles in the Spanish provinces’, *Journal of Regional Sci*ence, 45 (1), pp. 147-182.

TABLE 1

Discrete-time estimation of the five-year transition matrix, $\hat{P}^{D}(5)$ *, and ergodic distribution for the period 2000-2010*

(Number)	States	(1)	(2)	(3)	(4)	(5)
(48)	(1)	0.521	0.479	0.000	0.000	0.000
(48)	(2)	0.021	0.688	0.292	0.000	0.000
(48)	(3)	0.000	0.021	0.958	0.021	0.000
(48)	(4)	0.000	0.000	0.104	0.771	0.125
(48)	(5)	0.000	0.000	0.000	0.104	0.896
Ergodic distribution		0.002	0.047	0.660	0.132	0.158

Note: The numbers in parentheses on the left are the number of country/year pairs beginning in a particular state.

TABLE 2

Intensity matrix, $\hat{Q}$, continuous-time estimation of the five-year transition matrix, $\hat{P}^C(5)$, and ergodic distribution for the period 2000-2010

a) *Intensity matrix*

States	(1)	(2)	(3)	(4)	(5)
(1)	-1.424	1.424	0.000	0.000	0.000
(2)	0.489	-1.344	0.856	0.000	0.000
(3)	0.000	0.367	-0.489	0.122	0.000
(4)	0.000	0.000	0.256	-1.023	0.767
(5)	0.000	0.000	0.000	0.494	-0.494

b) *Continuous-time transition matrix*

(Number)	States	(1)	(2)	(3)	(4)	(5)
(80)	(1)	0.562	0.358	0.079	0.002	0.000
(80)	(2)	0.123	0.603	0.267	0.007	0.001
(80)	(3)	0.012	0.115	0.827	0.041	0.006
(80)	(4)	0.000	0.007	0.085	0.657	0.251
(80)	(5)	0.000	0.001	0.010	0.162	0.828
Ergodic distribution		0.055	0.159	0.367	0.166	0.253

Note: The numbers in parentheses on the left are the number of country/year pairs beginning in a particular state.