Universidad de Cantabria

Departamento de Economía

TESIS DOCTORAL

Distribuciones de Renta y Curvas de Lorenz Multivariantes: Aspectos Probabilísticos e Inferencia con Información Completa y Limitada

Vanesa Jordá Gil

Santander, Noviembre 2013

Director: José María Sarabia Alegría

University of Cantabria

Department of Economics

DOCTORAL THESIS

Income distributions and multidimensional Lorenz curves: probabilistic aspects and inference with complete and limited information

Vanesa Jordá Gil

Santander, November 2013

Supervisor: José María Sarabia Alegría

Agradecimientos

Quiero expresar mi más sincero agradecimiento a todas aquellas personas que han contribuido de algún modo en la elaboración de esta tesis.

Mi mayor agradecimiento va dirigido a mi director, el profesor José María Sarabia. No hay palabras para agradecer la confianza que ha depositado en mí estos años y por impulsar mi carrera académica. Sus continuas palabras de aliento, sus comentarios y, por supuesto, su paciencia han enriquecido este trabajo de forma sustancial. Quiero dar las gracias también por el apoyo recibido por parte de mis compañeros del Área de Métodos Cuantitativos para la Economía y la Empresa, en especial a Carmen Trueba y Lorena Remuzgo. Su ayuda ha sido muy importante en el desarrollo de esta tesis. Han estado conmigo tanto en los buenos como en los malos momentos y me han brindado siempre su mano cuando lo he necesitado. Finalmente, agradezco a Faustino Prieto el continuo apoyo que he recibido por su parte a lo largo de estos años.

Agradecer también a todos los miembros del Departamento de Economía que me han ayudado durante este tiempo, y en especial a José Manuel Alonso, Julio Revuelta, Marcos Fernández, Ana Lara Gómez y Martin Rode.

Quiero expresar mi gratitud al profesor Geoff Meen por brindarme la oportunidad de realizar una estancia en Universidad de Reading. Agradecer también a los miembros del Departamento de Economía por sus valiosos consejos que contribuyeron a mejorar este trabajo. En especial, quiero dar las gracias al profesor Giovanni Razzu, por ofrecerme su ayuda durante el tiempo que viví en Inglaterra. Quiero agradecer también a los compañeros y amigos que encontré en Reading, en especial a Cristina, que me hicieron sentir como en casa. Los momentos que viví con ellos y las experiencias que compartimos me ayudaron a crecer como persona.

Reservo un lugar especial para mi familia y amigos, por quererme y respaldarme en todo momento. Nunca podré compensaros por el apoyo y amor incondicional que me demostráis cada día y que ha sido determinante para la elaboración de este trabajo. Una mención especial va dirigida a mi madre por enseñarme el valor del trabajo duro y a mi padre por estar ahí siempre pese a la distancia.

Mi agradecimiento más profundo va dirigido a Miguel. Él ha sido mi apoyo más fuerte, brindándome su ayuda en los momentos más difíciles. Siempre ha sido capaz de encontrar las palabras adecuadas para reavivar mi motivación en el trabajo. Gracias también por estar conmigo en los logros, por escuchar con entusiasmo mis ideas, por aconsejarme, por resolver mis dudas, en definitiva por impulsarme y ayudarme a sacar adelante en este proyecto más que cualquier otra persona.

Contents

List of Tables

List of Figures

Introducción

La evolución de los niveles de desigualdad es objeto de intenso debate tanto en el mundo académico como en los ámbitos político y social. Esta dualidad es inherente al propio concepto de desigualdad que, más allá de una noción teórica, representa un aspecto de relevancia social. A pesar de la oposición colectiva ante este fenómeno, las desigualdades se han ido incrementando con el tiempo. En 1820 el 10 por ciento de la población más rica poseía el 43 por ciento del ingreso total, ratio que asciende hasta el 53,4 por ciento en 1992 (Bourguignon y Morrison, 2002). Por otro lado, las medidas de desigualdad relativas muestran tendencias similares, concluyéndose que el índice de Gini se ha incrementado desde 0,553 hasta 0,646 en el último siglo (Morrison y Murtin, 2012).

Tradicionalmente la desigualdad en el bienestar se asociaba a diferencias en los niveles de renta, caracterizándolo como un fenómeno puramente económico. A partir de dicha concepción, el resultado anterior sugeriría que la desigualdad del bienestar se ha incrementado en los últimos siglos. No obstante, situar el crecimiento económico en el epicentro del bienestar ofrece una visión relativamente restringida de dicho proceso, el cual engloba otras dimensiones no monetarias e igualmente relevantes. Si bien es razonable suponer que el ingreso está correlacionado de forma positiva con aspectos sociales como la educación o la salud, en una contextualización como la actual dominada por el estado del bienestar, la relación anterior puede caracterizarse como débil, en función de las prestaciones sociales provistas por el sector público.

Numerosos trabajos investigan la evolución de la desigualdad de renta ya sea a nivel regional o desde una perspectiva global[1]. Sin embargo, el descontento con la hegemonía del PIB per cápita como indicador del bienestar ha ido ganando fuerza entre los académicos durante las últimas tres décadas. Actualmente, existe un consenso creciente de que el bienestar debe caracterizarse como un proceso multidimensional (Sen, 1985; Streeten, 1994; Stiglitz et al., 2009), que además de variables puramente económicas incluya también otro tipo de indicadores no monetarios de la calidad de vida.

En este sentido, en los últimos años se han llevado a cabo numerosos intentos para sintetizar los diferentes aspectos del bienestar en un índice compuesto, que proporcione una perspectiva más amplia de dicho proceso que la ofrecida por las variables estrictamente económicas (véase, entre otros, Alkire y Foster, 2010; Bilbao-Ubillos, 2013; Edgier y Tatlidil, 2006; Fakuda-Parr et al., 2009; Grimm et al., 2008; Morrison y Murtin, 2012). El indicador más popular es el Índice de Desarrollo Humano (IDH), publicado por primera vez por el Programa de Naciones Unidas para el Desarrollo (PNUD) en 1990. Siguiendo el *Enfoque de las Capacidades*, se creó un índice capaz de evaluar los aspectos más relevantes del desarrollo humano, siendo éste considerado como un proceso de ampliación del rango de opciones de los individuos:

> *"El desarrollo humano es un proceso de expansión de las capacidades* (…). *Las más importantes son tener una vida larga y saludable, acceso a educación y disfrutar de un nivel de vida digno"* (UNDP, 1990; 10).

Señalar que el IDH ha recibido una gran atención de los medios, así como numerosas críticas desde su lanzamiento[2]. Por otro lado, cabe destacar que la evaluación de los niveles de bienestar es una tarea ambiciosa y compleja. A pesar de sus limitaciones, el

---

[1] Para una revisión sobre estudios de desigualdad de renta véase Kleiber y Kotz (2003) y Johnson et al. (1995).

[2] El IDH ha sido criticado desde su lanzamiento respecto a su construcción (Grimm et al., 2008; Kelley, 1991), variables utilizadas (Srinivasan, 1994), dimensiones consideradas (Alkire, 2002), redundancia con sus componentes (Cahill, 2005; McGillivray, 1991, McGillivray y White, 1993; Ravallion, 1997) y la arbitrariedad de los pesos asignados a cada una de las dimensiones (McGillivray y White, 1993; Noorbakhsh, 1998). Para una revisión reciente de las críticas dirigidas a este indicador véase Kovacevic (2010b).

IDH representa uno de los mayores avances al respecto. Este indicador se ha convertido en una de las alternativas más adecuadas para realizar análisis a nivel internacional, dado que su construcción está basada en el uso de datos homogéneos durante periodos temporales más extensos que otros indicadores similares, incluyendo además un amplio abanico de países.

Bajo este nuevo paradigma del desarrollo, las disparidades deben evaluarse en un entorno multidimensional, contemplando de forma conjunta variables económicas e indicadores no monetarios. Nótese que no existe *a priori* ninguna razón para suponer que la distribución de los componentes sociales evolucione del mismo modo que la del ingreso (Bourguignon y Morrison, 2002). De hecho, mientras que las décadas de los ochenta y los noventa se caracterizaban por un proceso de divergencia en el ámbito económico, la desigualdad del bienestar disminuyó de forma paulatina (Konya, 2011; Martínez, 2012; McGillivray y Markova, 2010). En este contexto, el ingreso seguiría desempeñando un papel fundamental, aunque perdería la posición protagonista de la que gozaba en los análisis tradicionales de desigualdad.

El objetivo de la presente tesis es analizar la evolución de la distribución del bienestar utilizando tanto el enfoque tradicional centrado en variables de renta, como la nueva concepción multidimensional de este proceso que incluye a su vez aspectos no monetarios. La tesis se desarrolla a lo largo de cuatro capítulos, de modo que cada uno de ellos aborda el estudio de la desigualdad utilizando y desarrollando diferentes metodologías, haciendo uso de los datos disponibles en cada caso.

En el primer capítulo, el estudio de la desigualdad se aborda desde una perspectiva de modelización estadística utilizando variables puramente económicas. El desarrollo de distribuciones de renta ha generado numerosos trabajos, incrementándose de forma sustancial las alternativas para modelizar la distribución del ingreso desde una perspectiva paramétrica. Entre las familias clásicas, destacan la distribución de Pareto (Arnold, 1983), la distribución log-normal (Atchison y Brown, 1957), la distribución gamma (Salem y Mount, 1974), la distribución beta, la distribución de Singh-Maddala (Singh y Maddala, 1976) y la distribución de Dagum (Dagum, 1977), entre otras. En los últimos años se han propuesto nuevos modelos paramétricos como la distribución de Gompertz-Pareto (Moura y Ribeiro, 2009; Figueira et al., 2011) o la distribución Positiva Estable de Pareto (Sarabia y Prieto, 2009). Una de las principales ventajas de estos modelos paramétricos es que permiten obtener las medidas probabilísticas y los indicadores de desigualdad de forma cerrada en términos de unos pocos parámetros (Ryu y Slottje, 1996; Slottje, 1990).

Recientemente, se ha propuesto un nuevo modelo distributivo denominado *distribución Gaussiana modificada* (Guo y Gao, 2012). Se ha demostrado que esta familia ajusta datos de ingresos individuales de forma satisfactoria cuando la muestra disponible comprende un elevado número de observaciones. En este capítulo se plantea obtener los indicadores de desigualdad y las propiedades probabilísticas y estadísticas de esta nueva familia. Se describen a su vez dos métodos de estimación de los parámetros de la distribución, en concreto estimación por máxima verosimilitud y por el método de los momentos.

A modo de ilustración, se ajusta la distribución Gaussiana modificada a datos de ingresos individuales en España en tres momentos de tiempo 1993, 1996 y 1999. Los datos se obtienen del *Panel de Hogares de la Unión Europea* (PHOGUE), donde el número de observaciones es lo suficientemente elevado como para obtener estimaciones consistentes. Los resultados obtenidos ponen de manifiesto que la distribución Gaussiana modificada ajusta satisfactoriamente los datos de ingreso en España. Las estimaciones realizadas sobre varias medidas de desigualdad reflejan de forma unánime que no se han producido variaciones significativas en términos de desigualdad durante los años contemplados en el análisis.

En el segundo capítulo, las diferencias en los niveles de bienestar también se evalúan desde una perspectiva clásica utilizando variables de renta. En numerosas ocasiones, la disponibilidad de los datos referentes a ingresos individuales está restringida, de modo que la información libremente accesible se compone de estadísticos descriptivos de los datos primarios proporcionados por las encuestas. En este contexto, la estimación de distribuciones de renta a partir información de carácter parcial resulta esencial para analizar los patrones distributivos de la riqueza. La literatura presenta dos vías metodológicas para estimar la distribución del ingreso a partir de información limitada. Por un lado, se han utilizado técnicas no paramétricas basadas en kernels de tipo Gaussiano (Sala-i-Martin, 2006). Por otro lado, se han estimado numerosos modelos biparamétricos como es el caso de la distribución log-normal (Chotikapanich et al., 1997), la distribución gamma (Chotikapanich y Griffiths, 2008) o la distribución Weibull (Pinkovskiy y Sala-i-Martin, 2009; Chotikapanich y Rao, 1998), así como otras distribuciones paramétricas más flexibles caracterizadas por tres o más parámetros, como es el caso de la distribución Beta de segunda especie (Chotikapanich et al., 2007; 2009).

En este segundo capítulo se propone entonces estimar la distribución de ingresos mundial durante la década de los noventa a partir de información limitada. Asimismo, se adopta también una perspectiva regional, que permitirá investigar los patrones distributivos del ingreso de forma más desagregada. Para ello se utilizan datos referentes al índice de Gini y al ingreso medio, que viene dado por el PIB per cápita a precios constantes de 2005 expresado en paridades de poder de compra (PPP). Este indicador se obtiene de la base de datos Penn World Tables version 7.2 (Heston et al., 2012). Por otro lado, la última versión de la base de datos *Standardized World Income Inequality Database* (Solt, 2009) proporciona valores del índice de Gini comparables entre países para el periodo de estudio. En una primera etapa, se obtienen las distribuciones nacionales de ingreso utilizando un modelo distributivo biparamétrico. En concreto se propone utilizar las llamadas *distribuciones de Lamé* que representan dos versiones curvadas de las distribuciones clásicas de Singh-Maddala y de Dagum. La principal característica de esta familia es que incluye modelos parsimoniosos, capaces de ajustar distribuciones de renta con tan sólo dos parámetros y cuyas curvas de Lorenz vienen caracterizadas por un sólo parámetro (Sarabia et al., 2013). A partir de dichas estimaciones y haciendo uso de los diferentes pesos poblacionales se obtienen las distribuciones regionales, así como la distribución de la renta a nivel mundial.

Una vez que se ha estimado la distribución de la renta a nivel mundial y regional, se calculan diferentes medidas de desigualdad y pobreza, lo que permitirá estudiar la evolución de estos dos fenómenos durante la década de los noventa. Las estimaciones obtenidas, se comparan a su vez con los resultados de diversos estudios previos, concluyéndose que las medidas de desigualdad y pobreza muestran tendencias y valores muy similares a los obtenidos utilizando modelos más complejos. Asimismo, se analiza la validez de las estimaciones mediante un contraste de adecuación del modelo, que pone de manifiesto que las distribuciones nacionales estimadas ajustan adecuadamente los datos de renta en más de un 90 por ciento de los casos. Los resultados obtenidos en este análisis sugieren que los niveles de pobreza mundiales han decrecido durante los noventa, mientras que a nivel regional se observan diferentes tendencias. La desigualdad global, por otro lado, muestra un patrón decreciente derivado de la disminución de las diferencias entre países que tuvo lugar a lo largo del periodo de estudio que compensó el incremento en las disparidades internas de los países.

El Capítulo 3 aborda el estudio de la desigualdad en el bienestar desde una perspectiva multidimensional, de modo que se contemplan también aspectos no monetarios como la salud o la educación. Cabe destacar que el análisis de las disparidades en entornos multidimensionales presenta algunas dificultades y engloba a su vez un amplio abanico de posibilidades. Es por ello que la literatura recoge diferentes vías metodológicas para cuantificar la desigualdad en el bienestar concebido como un proceso multidimensional. Por un lado, algunos autores proponen construir un índice compuesto de bienestar (lo que requiere establecer juicios subjetivos acerca de sus componentes) y calcular medidas de desigualdad unidimensionales sobre dicho indicador (Pillarisetti, 1997; Martínez, 2012). Alternativamente, es posible medir las disparidades en cada variable del índice por separado lo que, por otro lado, ignoraría las relaciones entre las dimensiones incluidas en el análisis (McGillivray y Pillarisetti, 2004; Martínez, 2012; Hobin y Franses, 2001; Neumayer, 2003; McGillivray y Markova, 2010). La opción metodológica más adecuada parece ser el empleo de medidas multidimensionales de desigualdad (Decancq et al., 2009; Decancq y Lugo, 2012), que miden la desigualdad inherente a cada una de las dimensiones del bienestar teniendo en cuenta el grado de asociación entre variables.

Al igual que en el caso unidimensional, las medidas de desigualdad multidimensionales proporcionan información en términos agregados sobre la evolución de las disparidades en el bienestar. Cuando no es posible obtener conclusiones de dominancia estocástica, cabe la posibilidad de que ciertas partes de la distribución muestren tendencias opuestas a las obtenidas a partir de las medidas de desigualdad multidimensional. En este capítulo se desarrolla una nueva herramienta metodológica que permitirá estudiar estas dinámicas a partir de la extensión de la curva de Lorenz al plano multidimensional. En concreto, se obtienen expresiones cerradas para la curva de Lorenz bidimensional propuesta por Arnold (1983), utilizando la distribución de Sarmanov-Lee (Lee, 1996; Sarmanov, 1966) para modelizar la distribución bivariada subyacente. Asimismo, se obtiene una expresión del índice de Gini bidimensional que se puede descomponer en dos términos asociados a la equidad dentro de las variables y el grado de asociación entre ellas. Esta metodología se aplica a datos referentes a los componentes del IDH durante los últimos 30 años, lo que permite analizar la evolución de las diferencias en los niveles de calidad de vida bajo una perspectiva distributiva más amplia que la ofrecida por las medidas multidimensionales de desigualdad.

Las estimaciones obtenidas referentes al índice de Gini bivariado sugieren que la desigualdad bidimensional se ha reducido en todos los casos considerados. Sin embargo, este indicador proporciona información agregada sobre la evolución de las diferencias en los niveles de bienestar entre países y por tanto podría estar enmascarando ciertas dinámicas internas. De hecho, las estimaciones de las curvas de Lorenz bidimensionales muestran que los países más pobres, los que tienen niveles educativos más bajos y los que se caracterizan por una menor esperanza de vida, presentan una distribución más desigual al final del periodo de estudio. Por tanto, la nueva concepción del bienestar como un proceso multidimensional, hace que la extensión de la curva de Lorenz al caso multidimensional resulte esencial para analizar las dinámicas internas de su distribución y ofrecer a su vez una visión completa de la evolución de las disparidades en los niveles de calidad de vida.

El cuarto capítulo investiga patrones de β-convergencia en los niveles de bienestar entre países. El estudio de la hipótesis de convergencia ha generado numerosas investigaciones desde los trabajos de Solow (1956, 1957) y Swan (1956), adoptando por lo general un enfoque puramente económico. El argumento de que el bienestar no puede ser medido únicamente mediante variables monetarias ha dado lugar a diferentes trabajos que contrastan la hipótesis de convergencia β en otras dimensiones de la calidad de vida, como la salud o la educación (Mayer-Foulkes, 2003; Sab y Smith, 2001; Mazumdar, 2003).

Un enfoque alternativo consiste en estudiar la hipótesis de convergencia en un índice compuesto de calidad de vida que considere de forma conjunta factores sociales e indicadores de renta, lo que permitiría establecer conclusiones generales sobre la evolución de las diferencias en los niveles nacionales de bienestar. Varios estudios adoptan este enfoque para analizar la convergencia en el IDH (Konya y Guisan, 2008; Mayer-Foulkes, 2010; Noorbakhsh, 2006), concluyendo que los niveles de bienestar han convergido de forma lenta durante las últimas tres décadas. Algunos autores han cuestionado la linealidad de este proceso, especificando modelos paramétricos no lineales (Mazumdar, 2003) o regresión por cuantiles (Mayer-Foulkes, 2010). Sin embargo, las especificaciones paramétricas requieren establecer *a priori* supuestos sobre la velocidad de convergencia, por lo que dichos modelos pueden presentar un sesgo por error de especificación. Por otro lado, la regresión por cuantiles ofrece una panorámica escalonada de los patrones de convergencia en varias partes de la distribución del bienestar.

En este capítulo se opta por la utilización de modelos semiparamétricos, que permiten que sean los propios datos los que describan la dirección e intensidad que toma el proceso de convergencia. Se pretende por tanto reexaminar la hipótesis de convergencia en el bienestar a nivel global para el periodo 1980-2011. Para ello se considera el marco teórico del IDH como indicador de los niveles de calidad de vida.

Los resultados sugieren que la brecha entre los países desarrollados y los países en desarrollo ha disminuido para todos los indicadores utilizados. Sin embargo, la velocidad de convergencia ha sido relativamente baja durante los últimos 30 años. La utilización de contrastes de especificación pone de manifiesto que el proceso de convergencia en el bienestar ha sido lineal bajo el modelo de convergencia absoluta. Por otro lado, las estimaciones referentes a la hipótesis de convergencia condicional revelan que, en este contexto, el proceso de convergencia en los niveles de calidad de vida presenta no linealidades que serían ignoradas por los modelos clásicos. Asimismo, las estimaciones realizadas sugieren que, aun cuando la dimensión de ingreso presenta escasos avances en términos de convergencia, las dimensiones no monetarias han evolucionado positivamente. Este resultado pone de manifiesto la importancia de considerar variables no estrictamente económicas en el estudio de la convergencia en los niveles de bienestar, dado que éstas presentan patrones distributivos distintos a los de la renta.

Cada capítulo de la tesis doctoral contribuye a evaluar ciertos aspectos de la desigualdad en el bienestar adoptando distintos enfoques. Se han desarrollado varias herramientas para medir la desigualdad, empleándose diferentes metodologías e hipótesis en cada uno de ellos, los cuales se complementan entre sí ofreciendo un análisis profundo de las diferencias en los niveles de calidad de vida. Diferentes versiones de los cuatro capítulos han sido presentadas en diversos congresos especializados tanto nacionales como internacionales. Asimismo, los resultados obtenidos ya han sido publicados, aceptados para su publicación o han pasado la primera etapa del proceso de revisión en revistas académicas. En concreto, el Capítulo 1 ha sido publicado en *Physica A* (en colaboración). La parte teórica del Capítulo 2 ha sido aceptado para su publicación en *Comunications in Statistics: Theory and Methods* (en colaboración). El Capítulo 3 se ha presentado en el congreso de la Sociedad para el Estudio de la Desigualdad Económica (ECINEQ) celebrado en Bari (Italia) en Julio de 2013. Por último, una versión del Capítulo 4 se encuentra actualmente en proceso de revisión.

Introduction

The evolution of inequality levels remains a matter of intense debate among academics and has received an increasing amount of attention from social and economic spheres. Such duality is inherent to the concept of inequality, which beyond the theoretical notion, represents a socially relevant aspect. In spite of the collective aversion to this phenomenon, disparities have increased over time. Whereas in 1820, 10 percent of the wealthiest people had 43 percent of global income, this proportion rose to 53.4 percent in 1992. Relative inequality measures showed similar trends, concluding that the Gini index increased from 0.553 to 0.664 in the last century (Morrison and Murtin, 2012).

Traditionally, inequality in well-being was associated with differences in income levels, characterizing the quality of life as a purely economic process. Based on this conception, the previous result would imply that inequality in well-being increased in the past centuries. It is, however, argued that the assessment of well-being should include other non-income dimensions which are equally relevant. It is reasonable to assume that there is a positive relationship between income and other social aspects such as health or education, but in the current context dominated by the welfare state, this relationship can be characterized as weak, depending on the social services provided by the public sector.

Several papers investigate the evolution of income inequality at regional and global levels[3]. However, the discontent with the hegemony of per capita GDP as an indicator of well-being has gained prominence among academics in the last three decades. There is now almost a consensus that well-being is a multidimensional concept (Sen, 1985; Streeten, 1994; Stiglitz et al., 2009) which, in addition to income, should also consider non-income indicators of quality of life.

This line of argumentation has received an increasing amount of attention in the last years, thus resulting in many attempts to synthesize different aspects of well-being in a composite index which offers a more comprehensive perspective of such a process than per capita income alone (see *e.g.* Alkire and Foster, 2010; Bilbao-Ubillos, 2013; Edgier and Tatlidil, 2006; Fakuda-Parr et al., 2009; Grimm et al., 2008; Morrison and Murtin, 2012). Among them, the most popular is the Human Development Index (HDI), developed by the United Nations Development Program (UNDP) in 1990. This indicator was designed following the Sen's capability approach (Sen, 1988; 1989; 1999) which considers development as a process of enhancing individuals' choices. This new paradigm of development was presented in the first Human Development Report, which stated:

> "*Human development is a process of enlarging people's choices. In principle, these choices can be infinite and change over time. But at all levels of development, the three essential ones are for people to lead a long and healthy life, to acquire knowledge and to have access to resources needed for a decent standard of living*." (UNDP, 1990; p.10).

It should be emphasized that, while the HDI has received a large amount of attention from the media, several criticisms have been leveled at this indicator since it was launched[4]. On the other hand, it should be highlighted that the evaluation of quality of

[3] See Kleiber and Kozt (2003) and Johnson et al. (2005) for a review on the literature about economic inequality.

[4] The HDI has been criticized on the grounds of construction (Grimm et al., 2008; Kelley, 1991), selection of variables (Srinivasan, 1994), dimensions included (Alkire, 2002) arbitrary weighting scheme (McGillivray and White, 1993; Noorbakhsh, 1998), and redundancy with its components

life is complex, abstract and difficult to synthesize. Independently of its limitations, the HDI seems to be the most adequate alternative for carrying out international comparisons of well-being levels since it is constructed using homogeneous data for longer periods of time than other related indices, also including a wide range of countries.

Under the new paradigm of development, inequalities should be measured in multidimensional environments, considering jointly economic variables and non-income dimensions. Note that there is no reason to expect that the distributions of the social components will present similar patterns to that of income (Bourgignon and Morrison, 2002). In fact, while the decades of the eighties and nineties were characterized by a process of economic divergence, inequality in well-being was reduced substantially (Konya, 2011; Martínez, 2012; McGillivray and Markova, 2010). Therefore, in this context, income would play a fundamental role but it would have lost the predominant position that it enjoyed in the classical inequality analyses.

The aim of this thesis is to analyze the evolution of well-being distribution using the traditional approach that focuses on economic variables, as well as the new multidimensional conception of this process, which also includes non-income aspects. This work is developed in four chapters and each of them deals with specific methodologies to measure inequality in well-being, using the data available in each case.

In the first chapter, we study the evolution of inequality considering a statistical approach and using purely economic variables. The development of income distributions has generated several papers, thus increasing substantially the number of alternative parametric distributions to model income data. Among the classical families, we should highlight the distributions of Pareto (Arnold, 1983), log-normal (Atchison and Brown, 1957), gamma (Salem and Mount, 1974), beta, Singh-Maddala (Singh and Maddala, 1976), Dagum (Dagum, 1977) among others. In recent years, new models have been proposed, such as the Gompertz-Pareto (Moura and Ribeiro,

---

(Cahill, 2005; McGillivray, 1991; Ravallion, 1997). A review of the criticisms focused on the limitations of the HDI can be found in Kovacevic (2010b).

2009; Figueira et al., 2011) and the Pareto Positive Stable distribution (Sarabia and Prieto, 2009). In any case, one of the main advantages of these parametric models is that they allow us to derive probabilistic measures and inequality indicators in exact form (Ryu and Slottje, 1996; Slottje, 1990).

Recently, a new distribution has been proposed, called *modified Gaussian distribution*, which fits the data on individual income satisfactorily when the sample includes a large number of observations (Guo and Gao, 2012). In this chapter we obtain the probabilistic and statistical properties of this family. Two alternative estimation methods to obtain the parameters of the model are also described, namely maximum likelihood estimation and the method of moments.

In order to illustrate all the previous formulations, we have fitted individual incomes of Spain for three years, 1993, 1996 and 1999, using data from the *European Community Household Panel* survey. Our results point out that the modified Gaussian distribution fits data adequately on individual income in Spain over the study period. The performed estimates of the different inequality measures suggest that no changes are observed in terms of inequality during the years included in the study.

In the second chapter, differences in well-being levels are also evaluated in terms of income. On several occasions, the availability of data on individual income are restricted, while descriptive statistics of the primary information provided by surveys are freely accessible. In this context, the derivation of income distributions from the pieces of information readily available is essential to analyze national and regional patterns of wealth. There have been many attempts to develop regional and global estimates from limited data, mainly based on two distinct methodologies. On the one hand, non-parametric techniques based on kernel estimates have been applied (Sala-i-Martin, 2006; Minoiu, 2007). On the other hand, parametric models have been estimated using conventional inference techniques. Numerous functional forms of two parameters have been suggested in the literature, such as the log-normal (Chotikapanich et al., 1997), the gamma (Chotikapanich and Griffiths, 2008) and the Weibull (Pinkovskiy and Sala-i-Martin, 2009; Chotikapanich and Rao, 1998), as well as more flexible parametric models, characterized by three or more parameters as in the case of the beta distribution of second kind (Chotikapanich et al., 2007; 2009).

The second chapter aims to estimate the global income distribution during the nineties using limited information. To address this issue we combine two different approaches: regional analysis and country case studies. The methodology is applied using data on the Gini index and the mean income of each country. The latest version of the *Standardized World Income Inequality Database* (Solt, 2009) provides comparable Gini index values over the study period. However, mean income is represented by per capita GDP in constant international US dollars, which is drawn from Penn World Tables version 7.2 (Heston et al., 2012). In a first stage, we obtain national income distributions using a model with two parameters. In particular, we propose to use the so-called *Lamé distributions*, which are curved versions of the Sigh-Maddala and Dagum distributions. The main feature of these distributions is that they represent parsimonious models which can fit income data with just two parameters and whose Lorenz curves are characterized by only one parameter (see Sarabia et al., 2013). In a second stage, global and regional distributions are derived from a finite mixture of these families using population shares.

Once income distributions are estimated, inequality and poverty measures are computed, allowing us to investigate the evolution of these two phenomena during the nineties. We compare our estimates with the results obtained in previous studies, concluding that inequality and poverty measures show similar trends to those obtained using more complex models. We also investigate the validity of our estimations using the chi-square test of goodness of fit, which points out that the fitted national distributions are adequately modeled by the Lamé family in 90 percent of cases. Our results suggest that global poverty levels decreased during the nineties. However, we observe a variety of regional experiences. On the other hand, global inequality presents a decreasing pattern mainly driven by the fall of the differences across countries during the course of the study period that offsets the increase in disparities within the countries.

Chapter 3 studies inequality in well-being using a multidimensional approach, considering income variables and non-economic dimensions such as health and education. It should be worth noting that the analysis of disparities in multidimensional environments presents some difficulties and comprises a wide range of options. As a consequence, different methodological lines have been proposed in the literature to quantify inequality in well-being conceived as a multidimensional process. On the one hand, a composite index of quality of life is constructed (thus requiring subjective judgments about the variables of the index) and then unidimensional inequality measures are computed to assess the disparities in levels of well-being (Pillarisetti, 1997; Martínez, 2012). Alternatively, we can look at each variable of the index separately, thus ignoring the relationships between the dimensions included in the analysis (McGillivray and Pillarisetti, 2004; Martínez, 2012; Hobin and Franses, 2001; Neumayer, 2003; McGillivray and Markova, 2010). The most satisfactory option seems to be the use of multidimensional inequality measures (Decancq et al., 2009; Decancq and Lugo, 2012), which assess the disparities inherent to each dimension and also the degree of association among them.

As in the unidimensional case, multidimensional inequality measures only provide summarized information about the evolution of disparities in well-being. If no dominance relationships can be achieved, some parts of the distribution may present different trends than those obtained using inequality measures. In this chapter we develop a new tool extending the Lorenz curve to the multidimensional space, which allows us to study these dynamics. Using the definition proposed by Arnold (1983), we obtain closed expressions for the bivariate Lorenz curve, considering a flexible model for the underlying bivariate distribution. We study a relevant type of models based on a class of bivariate distributions with given marginals described by Sarmanov and Lee (Lee, 1996; Sarmanov, 1966). A closed expression for the bivariate Gini index (Arnold, 1987) is given in terms of the equality within dimensions and the degree of association between them. We apply the previous methodology to data on health, education and income over the last 30 years.

Our estimates of the bidimensional Gini indices point out that inequality has been reduced in all of the relationships considered. However, this indicator only provides summarized information of the evolution well-being differences across countries and hence some internal dynamics can be masked. In fact, our estimates of the bidimensional Lorenz curves show that the poorest, least educated and least healthy countries present a more unequal distribution at the end of the study period. Therefore, the new conception of well-being makes the multidimensional extension of the Lorenz curve essential to analyze the internal dynamics of well-being distribution and to offer a complete panorama of the evolution of disparities in levels of quality of life.

The last chapter investigates patterns of β-convergence in levels of well-being across countries. The study of the convergence hypothesis has led to numerous works since the presentation of the classical works of Solow (1956, 1957) and Swan (1956) but the majority of these papers focus solely on economic variables. The increasing concern that well-being cannot be assessed using only income variables has induced academics to test the convergence hypothesis in other dimensions such as health and education (Mayer-Foulkes, 2003; Sab and Smith, 2001; Mazumdar, 2003)

An alternative approach is based on testing the hypothesis of convergence in a composite index of quality of life, which considers jointly social factors and income indicators, thus providing aggregated information about the evolution of national levels of well-being. Several works use the HDI to study the convergence in quality of life (see *e.g*. Konya and Guisan, 2008; Mayer-Foulkes, 2010; Noorbakhsh, 2006), concluding that living standards have converged slowly over the last 30 years. Nevertheless, some authors have questioned the linearity of this process, specifying nonlinear parametric models (Mazumdar, 2002; 2003) and quantile regression (Mayer-Foulkes, 2010). Note, however, that the parametric approach requires making *a priori* assumptions about the evolution of convergence speed; thus the model might present misspecification bias. On the other hand, quantile regression offers a restricted panorama of the convergence patterns in different parts of the distribution.

In this chapter, we opt for a semiparametric specification which lets the data themselves show the intensity and direction of the convergence/divergence process. We aim to provide a reappraisal of the convergence process in terms of quality of life, using the Human Development Index (HDI) as an indicator of this phenomenon, for the period 1980-2011. Our results point out that the gap between developed and developing countries has been reduced for all indicators considered. However, the speed of convergence has been notably low over the last 30 years. The use of PLM models reveals that whereas the absolute convergence process in human well-being is adequately represented by a linear trend, under the conditional convergence model, this process shows nonlinear patterns that would be ignored using the classical specifications of the convergence hypothesis. Our results point out that, even when little advances have been achieved in income levels, significant improvements in non-income dimensions and human well-being have been accomplished. This conclusion highlights the relevance of considering non-income dimensions in the study convergence hypothesis, since their distributional patterns differ substantially from economic variables.

Each chapter contributes to evaluate particular aspects of inequality in well-being adopting different approaches. We have developed several tools to measure inequality, also considering different methodologies in each of them which complement each other, thus providing a comprehensive analysis of the differences in the levels of quality of life. Different versions of the four chapters have been presented in a variety of national and international conferences. The results obtained have been already published, accepted for publication or have passed the first stage of the review process in academic journals. In particular, Chapter 1 has been published in *Physica A* (in collaboration). The methodological part of the Chapter 2 has been accepted for publication in *Comunications in Statistics: Theory and Methods* (in collaboration). Chapter 3 was presented at the meeting of the Society for the Study of Economic Inequality that took place in Bari (Italy) in July 2013. Finally a version of the Chapter 4 is currently under review.

Chapter 1

About the modified Gaussian family of income distributions with applications to individual incomes

1.1. Introduction

The development of distributions for modeling data on income and wealth distributions has received an increasing amount of attention from different fields, including economics, statistics and econophysics (see *e.g*. Kleiber and Kotz, 2003; Johnson et al., 1995; Johnson et al., 1970; Sornette, 2004; Schinckus, 2009; Schinckus, 2010; Di Matteo et al., 2004).

Since Pareto's (1897) work, the list of probability distribution functions for modeling income and wealth distributions has increased considerably. This list includes classical distributions such as the log-normal, gamma, beta, Singh-Maddala, Mandelbrot, Pareto and generalized versions of each. A comprehensive survey of these distributions can be found in Arnold (1983) and Kleiber and Kotz (2003). Other relevant parametric models have also been recently proposed. These new models include the κ-generalized distribution (see Clementi et al., 2008), the Gompertz-Pareto income distribution (Moura and Ribeiro, 2009; Figueira et al., 2011) and the Pareto Positive Stable distribution (Sarabia and Prieto, 2009). Typically, economical systems (but also several physical systems) present power-law tails (see for instance Kaniadakis, 2009), and many of the previous families present this kind of tails.

One of the most important advantages of all these parametric models is that the main probabilistic measures (*e.g.* moments) and inequality tools (*e.g.* Gini index) are available in closed form. This fact provides a correct description of the parametric family of income and wealth distributions and allows us to compute all these indicators in an exact form (Ryu and Slottje, 1996; Slottje, 1990).

More recently, a new family of distributions for modeling individual incomes in China was proposed. This family is the so-called modified Gaussian (MG) distribution, which depends on two parameters (Guo and Gao, 2012). It has been evidenced that the MG distribution fits satisfactorily data on individual income for China between 1992 and 2009. However, for the practical use of this model, it is necessary to know its probabilistic and statistical properties, especially the corresponding inequality measures. In this chapter, probabilistic functions and inequality measures of the MG distribution are obtained in a closed form, including the normalizing constant, probability functions, moments and standard tools for inequality measurement. Several methods for parameter estimation are also discussed. In order to illustrate all the previous formulations, we have fitted individual incomes of Spain for three years using the European Community Household Panel survey. Our results point out a static pattern of inequality since the Gini index and other inequality measures remain constant over the study period.

The contents of this chapter are as follows. In Section 1.2 we present the probabilistic properties of the MG distribution: the normalizing constant, a simple interpretation in terms of weighted distributions, the cumulative distribution, survival and quantile functions, moments and related quantities, first-degree stochastic dominance conditions and the relationships with other families of distributions (chi-square, stretched exponential and Weibull distributions). The different tools for inequality measurement (Lorenz curve, generalized Lorenz curve, Gini index, Donaldson-Weymark-Kakwani index and Pietra index) are obtained in Section 1.3. Estimation methods (moments and maximum likelihood) are discussed in Section 1.4. An empirical application with individual incomes of Spain for three years using the European Community Household Panel survey is included in Section 1.5. Finally, some conclusions are given in Section 1.6.

1.2. The modified Gaussian distribution

According to Guo and Gao (2012), their distribution is composed of two factors. The first factor is the variable factor (x - μ) if x $\geq \mu$ and the second factor is related to the planned economic system income, which is $\exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}$. Then, the modified Gaussian distribution (see Guo and Gao, 2012) is defined in terms of the probability density function (PDF) by,

$$f(x;\mu,\sigma) = K(x-\mu)\exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}, \tag{1.1}$$

and $f(x;\mu,\sigma) = 0$ if $x < \mu$, where μ, $\sigma > 0$ are parameters and K is the normalizing constant.

1.2.1 The normalizing constant

Making the change of variable $\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2 = t$,

$$\int_{-\infty}^{\infty} f(x;\mu,\sigma)dx = K\int_{0}^{\infty}(x-\mu)\exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}dx =$$

$$= K\sigma^2\int_0^{\infty}\exp(-t)dt = K\sigma^2.$$

Then, the value of the normalizing constant is $K = \frac{1}{\sigma^2}$.

1.2.2 Interpretation of the MG distribution

The PDF of the MG distribution defined in Equation (1.1) can be seen as a weighted distribution of the form

$$f_{\omega}(x) = \frac{\omega(x) f(x)}{E_f[\omega(X)]},$$

where $f(x)$ is the PDF of the classical Gaussian distribution with mean μ and standard deviation σ, and $\omega(x)$ is the weighted function defined as $\omega(x) = (x - \mu)$ if $(x \geq \mu)$ and $\omega(x) = 0$ if $(x < \mu)$.

The new PDF $f_{\omega}(x)$ is called the weighted version of X, and its distribution in relation to that of X is called the weighted distribution with weight function ω. In our case, because $\omega(x)$ is linear $f_{\omega}(x)$ is called the length-biased or size-biased version of f, and the corresponding observational mechanism is called length- or size-biased sampling (Patil et al, 1988; Patil, 2002). In the case of income distributions, this mechanism provides different weights to the different incomes.

1.2.3 Cumulative distribution, survival and quantile functions

The cumulative distribution function (CDF) is defined by $F(x) = \Pr(X \leq x)$. Then, $F(x) = 0$ if $x < \mu$. Instead, if $x \geq \mu$, we have

$$\begin{aligned} F(x) &= \Pr(X \leq x) \\ &= \int_{\mu}^{x} \left(\frac{t-\mu}{\sigma^2} \right) \exp\left\{ -\frac{1}{2} \left(\frac{t-\mu}{\sigma} \right)^2 \right\} dt \qquad (1.2) \\ &= 1 - \exp\left\{ -\frac{1}{2} \left(\frac{t-\mu}{\sigma} \right)^2 \right\}. \end{aligned}$$

On the other hand, the survival function $S(x) = \Pr(X > x)$ is,

$$S(x) = \exp\left\{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2\right\}, \quad \text{if } x \geq \mu,$$

and $S(x) = 1$ if $x < \mu$.

If $0 < p < 1$, the quantile function is defined as $X(p) = F_X^{-1}(p)$, where

$$F_X^{-1}(y) = \inf\{x : F_x(x) \geq y\}. \tag{1.3}$$

For the MG distribution (1.1) we have,

$$X(p; \mu, \sigma) = \mu + \sigma[-2\log(1-p)]^{1/2}. \tag{1.4}$$

Figure 1.1 represents the PDF, the CDF, the survival function and the survival functions in scale log-log, for some selected values of the parameters.

1.2.4 Moments and related quantities

Again making the change of variable $\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2 = t$, we obtain

$$E(X^r) = \frac{1}{\sigma^2}\int_\mu^\infty x^r (x-\mu)\exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\} dx$$

$$= \int_0^\infty (\mu + \sigma\sqrt{2}t^{1/2})^r \exp(-t)\, dt$$

$$= \sum_{k=0}^{r} \binom{r}{k} \mu^{r-k}\sigma^k 2^{k/2} \int_0^\infty t^{k/2} \exp(-t)\, dt,$$

and using the definition of the Gamma function we obtain the rth-moment about the origin of the MG distribution:

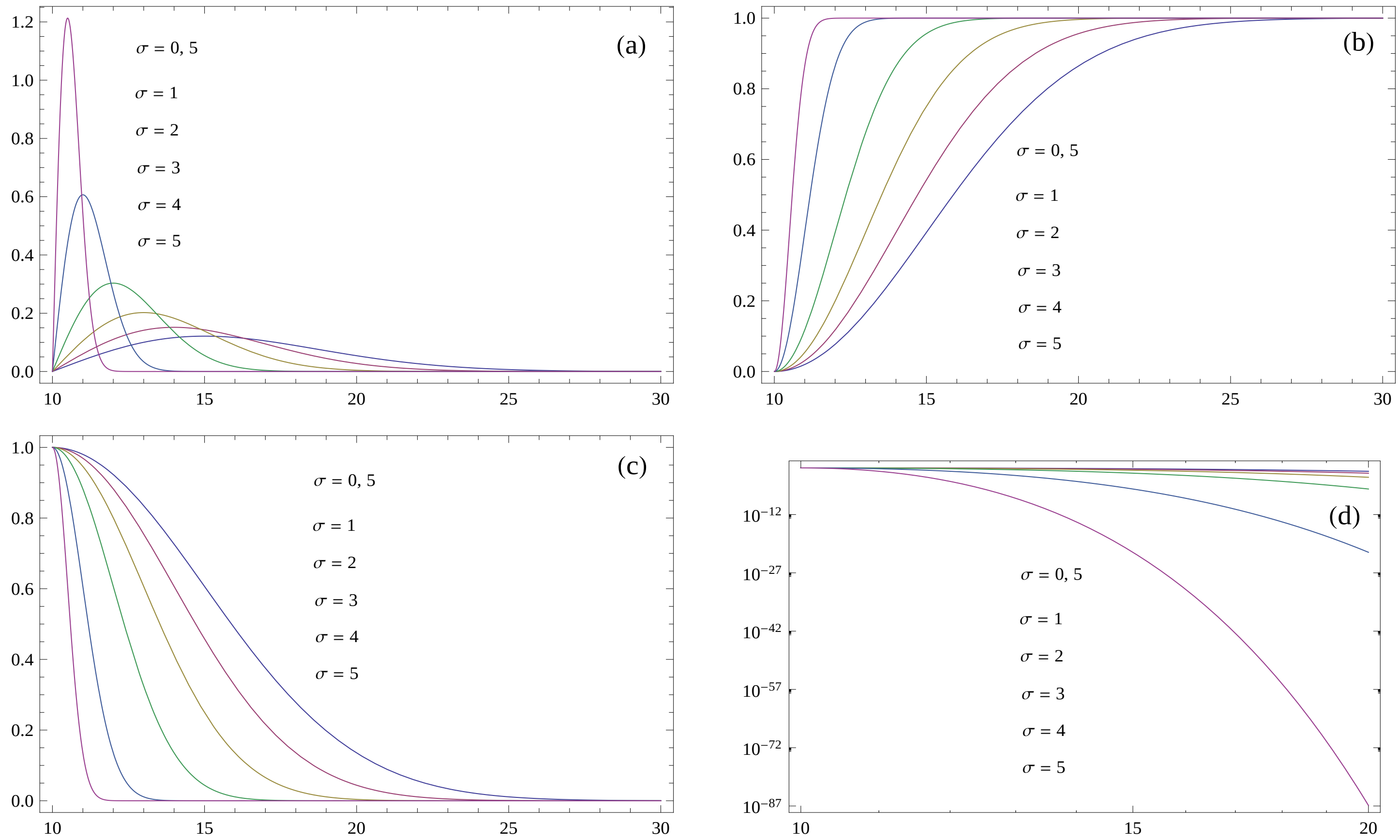

Figure 1.1. Graphics of the MG distribution: (a) Probability density functions; (b) Cumulative distribution functions; (c) Survival functions; (d) Survival functions in log-log scale with $\mu = 10$

$$E(X^r) = \sum_{k=0}^{r} \binom{r}{k} \mu^{r-k} \sigma^k 2^{k/2} \Gamma\left(\frac{k}{2}+1\right),$$

where $\Gamma(x)$ denotes the Gamma function defined as $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$. In particular, the first two moments are,

$$E(X) = \mu + \sqrt{\frac{\pi}{2}}\sigma, \tag{1.5}$$

$$E(X^2) = \mu^2 + \sqrt{2\pi}\mu\sigma + 2\sigma^2. \tag{1.6}$$

Using Equations. (1.5) and (1.6), the value of the variance is

$$\mathrm{var}(X) = \frac{4-\pi}{2}\sigma^2. \tag{1.7}$$

Other kind of moments can be obtained using the relation between the MG distribution and the chi-square distribution. Using Equation (1.9) we conclude

$$E\left[(X-\mu)^{2r}\right] = 2^r \sigma^r \Gamma(r+1)$$

if $r > -1$.

1.2.5. First-degree stochastic dominance

In this section we discuss first-degree stochastic dominance (Marshall et al., 2011) for two MG distributions. First-degree stochastic dominance holds in situations where one distribution provides a Pareto improvement compared to another distribution. A random of variable X_1 is said to be stochastically less than (or equal to) X_2 in the first-degree sense (FSD) if

$$F_1(x) \le F_2(x), \quad \forall\, x,$$

and we will represent $X_1 \leq_{FSD} X_2$. Let X_1 be a MG distribution with parameters (μ_1, σ_1) and a second MG distribution with parameters (μ_2, σ_2). Then it is verified that

$$X_1 \leq_{FSD} X_2 \Leftrightarrow \mu_2 \leq \mu_1, \quad \sigma_2 \leq \sigma_1. \tag{1.8}$$

1.2.6 Relationships with other families of distributions

In this section we include some simple relationships with other usual distributions commonly used in econophysics and economics. Particularly we stress its relationship with the chi-square, the stretched exponential and the Weibull distributions.

The modified Gaussian distribution can be related to the classical chi-square distribution in the following way. It is verified that,

$$\left(\frac{X-\mu}{\sigma}\right)^2 \sim \chi_2^2, \tag{1.9}$$

where χ_p^2 represents a classical chi-square distribution with *p*-degrees of freedom. If $X_1,\ldots, X_n$ is a set of i.i.d. modified Gaussian distributions, we have

$$\frac{1}{\sigma^2}\sum_{i=1}^{n}(X_i-\mu)^2 \sim \chi_{2n}^2.$$

On the other hand, the stretched exponential distribution has been found to be a useful and versatile intermediate distribution between “thin tail” (exponential, Gaussian, etc.) and very “fat tail” distributions (Sornette, 2009). Laherrere and Sornette (1999) have found some examples of fat tail distributions (in natural and social sciences) which were considered good examples of power laws, but could be modeled as well as or even better by a stretched distribution.

A simple relation between the MG distribution and the stretched exponential distribution can be found. Let *X* be a MG distribution and let *Y* be a stretched exponential distribution with exponent two and CDF,

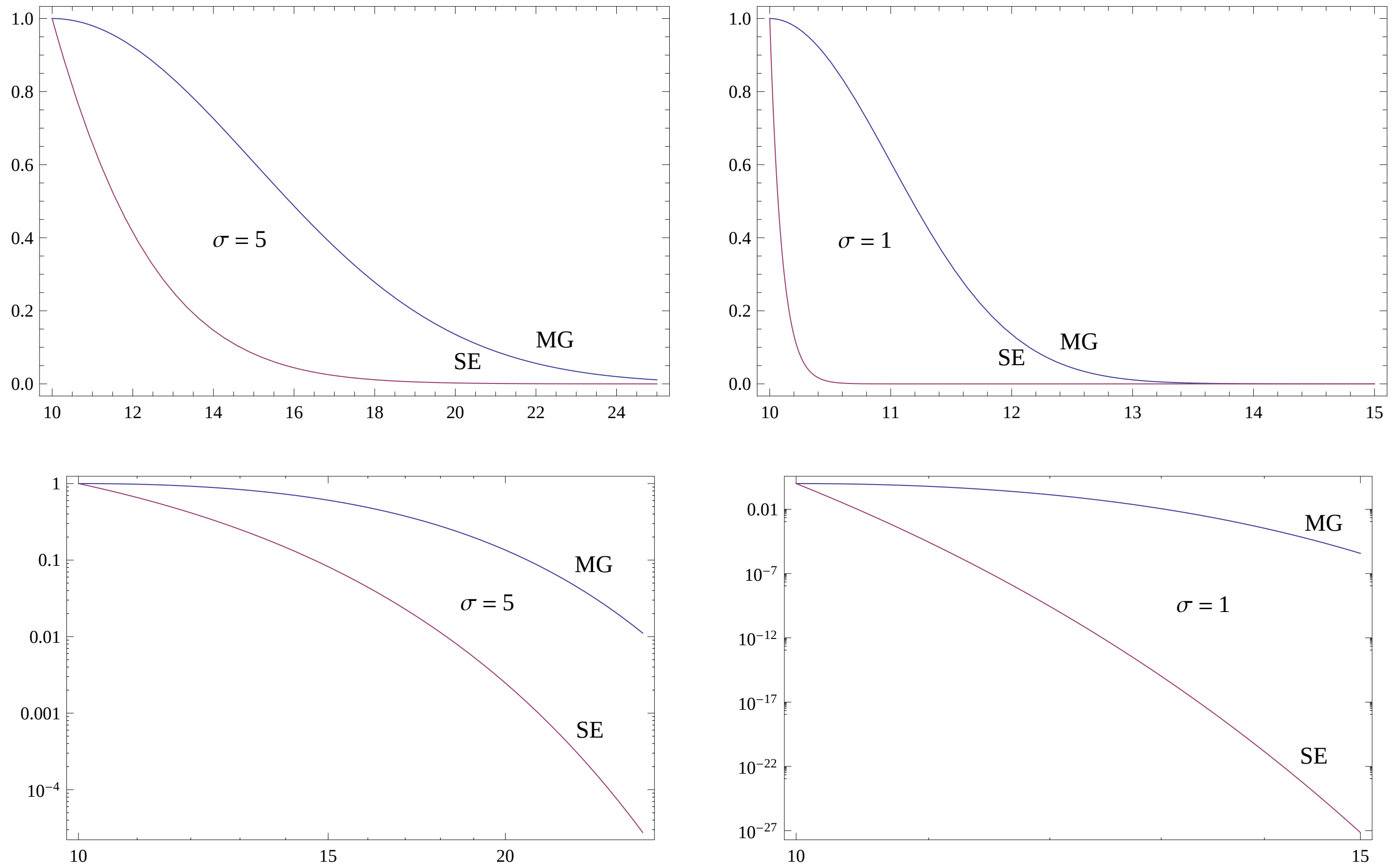

Figure 1.2. Tails of the MG distribution and the stretched exponential distribution with shape parameters equal to two, in standard and log-log scale, with $\mu = 10$

$$F_Y(y) = 1 - \exp\left(-\frac{y^2}{2\sigma^2} + \frac{\mu^2}{2\sigma^2} \right), \quad y \geq \mu,$$

and $F_Y(y)=0$ if $y < \mu$. Since $\left(-x^2 + \mu^2\right) - \left(-(x-\mu)^2\right) = 2\mu(\mu - x) < 0$ if $x \geq \mu > 0$, we have $\exp\left(-\frac{x^2}{2\sigma^2} + \frac{\mu^2}{2\sigma^2} \right) > \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma} \right)^2 \right)$, and then

$$\Pr(Y > x) < \Pr(X > x),$$

that is, $Y <_{FSD} X$ and we conclude that the modified Gaussian distribution has heavier tails than the stretched exponential distribution.

The previous result is clearly observed from Figure 1.2 which shows the tails of two distributions, MG and stretched exponential, with the same parameters. We observed that, irrespective to the parameter value, the MG distribution presents fatter tails than the stretched exponential. As a consequence, the MG distribution is more appropriate than the stretched exponential distribution for modeling data with heavy tails.

The Weibull distribution is a parametric family of common use in reliability and engineering (see Castillo et al., 2005). Recently, a unified physics of stretched exponential relaxation and Weibull fracture has been considered in Mauro and Smedskjaer (2012). If we include a location parameter μ in the classical two-parameter Weibull distribution, we can obtain the MG distribution, assuming a value of two for the shape parameter and a scale parameter equals to $\sigma\sqrt{2}$.

1.3. Inequality measures

In this section we consider closed expressions for some important inequality measures for the MG distribution.

1.3.1 Lorenz curve

The Lorenz curve is defined by (p, $L(p)$), where p represents the cumulative proportion of income-receiving units and $L(p)$ the cumulative proportion of incomes, when the incomes are arranged in ascending order of magnitude. Let $\mathcal{L}$ be the class of all non-negative random variables with positive finite expectations. For a random variable X in $\mathcal{L}$ with cumulative distribution function F_x, the mathematical expectation of X is $\mu_X = \int_0^1 F_X^{-1}(y)dy.$ According to the Gastwirth definition (Gastwirth, 1971) the Lorenz curve L_X corresponding to X is defined by,

$$L_X(p) = \frac{1}{\mu_X}\int_0^p F_X^{-1}(y)dy, \quad 0 \le p \le 1, \tag{1.10}$$

where $F_X^{-1}(x)$ is defined in (1.3). Then, by considering Equation (1.4),

$$\int_0^p F_X^{-1}(u)du = \mu p + \sigma\int_0^p \left[-2\log(1-u)\right]^{1/2}du = \mu p + \sigma\sqrt{2}\gamma(-\log(1-p), 3/2),$$

where $\gamma(x,a)$ denotes the incomplete gamma function defined as $\gamma(x,a) = \int_0^x t^{a-1}e^{-t}dt.$ Now, using Equations (1.10) and (1.5) we have

$$L(p;\mu,\sigma) = \frac{\mu p + \sigma\sqrt{2}\gamma(-\log(1-p), 3/2)}{\mu + \sqrt{\frac{\pi}{2}}\sigma}, \quad 0 \le p \le 1. \tag{1.11}$$

1.3.2 Generalized Lorenz curve

In this section the generalized Lorenz curve (GLC) introduced by Shorrocks (1983) is obtained. The Lorenz curve is scale invariant and then it is an indicator of relative inequality. As a consequence, it does not provide a complete basis for making social welfare comparisons. The generalized Lorenz curve is defined as,

$$GL_X(p) = \mu_X L_X(p) = \int_0^p F_X^{-1}(y)dy, \quad 0 \le p \le 1.$$

Note that $GL_X(0) = 0$ and $GL_X(1) = \mu_X$. Using previous definition, the GLC for the MG distribution is,

$$GL(p;\mu,\sigma) = \mu p + \sigma\sqrt{2}\gamma(-\log(1-p), 3/2), \quad 0 \leq p \leq 1. \tag{1.12}$$

A distribution with a dominating GLC provides greater welfare according to all concave increasing social welfare functions defined on individual incomes (see Kakwani, 1984; Davies et al., 1998).

1.3.3 Gini index

The two best measures of inequality which are related to the Lorenz curve are the Gini and Pietra indices. Both indices can be viewed as alternative ways of measuring the distance between the Lorenz curve and the egalitarian line. The Gini index is defined as twice the area between the egalitarian line and the Lorenz curve. One of the expressions to compute the Gini index is (see Sarabia, 2008),

$$G_X = 1 - \frac{E(X_{1:2})}{E(X)} = 1 - \frac{1}{E(X)}\int_0^\infty [1 - F_X(x)]^2\, dx, \tag{1.13}$$

where $X_{1:2}$ represents the smaller of a sample of size two coming from the CDF F_X.

Given that the distribution of the minimum $X_{1:v}$ of a sample of size v is given by $F_{X_{1:v}}(x) = 1 - [1 - F_x(x)]^v$. Using (1.2), it can be shown that the distribution of the minimum in a MG distribution is again a MG distribution with new parameters $\tilde{\mu} = \mu$ and $\tilde{\sigma} = \frac{\sigma}{\sqrt{v}}$. Consequently, using (1.5) we have that

$$E(X_{1:v}) = \mu + \frac{\sigma}{\sqrt{v}}\sqrt{\frac{\pi}{2}}. \tag{1.14}$$

The Gini index corresponds to the choice $v = 2$, thus we have

$$\int_0^\infty [1 - F_X(x)]^2\, dx = \mu + \sigma\frac{\sqrt{\pi}}{2}.$$

As a consequence, the expression of the Gini index is:

$$G_X = 1 - \frac{\mu + \sigma \frac{\sqrt{\pi}}{2}}{\mu + \sigma \sqrt{\frac{\pi}{2}}} \tag{1.15}$$

From Equation (1.15) we can deduce that if μ goes to infinity, G_X goes to zero, if σ goes to infinity $G_X \approx 0,2929$ and if σ goes to zero, G_X goes to zero.

1.3.4 Donaldson-Weymark-Kakwani index

One relevant generalization of the Gini index was proposed by Donaldson and Weymark (1980) and studied by Yitzhaki (1983). These authors proposed the generalized Gini index defined as

$$G_v = 1 - v(v-1)\int_0^1 (1-p)^{v-2} L_X(p)dp, \tag{1.16}$$

where $v > 1$. If $v = 2$ we obtain the Gini index. When v increases, higher weights are attached to small incomes. The limit case when v goes to infinity depends on the lowest income, expressing the judgment introduced by Rawls that social welfare depends only on the poorest society member.

On the other hand, it can be proved that Equation (1.16) is equivalent to (see Muliere and Scarsini, 1989),

$$G_v = 1 - \frac{E(X_{1:v})}{\mu_X},$$

which can also be seen as a generalization of Equation (1.13), where $X_{1:v}$ represents the minimum random variable in a random sample of size v. Using Equation (1.14), it can be derived that the Donaldson-Weymark-Kakwani index for the MG distribution is expressed as,

$$G_v = 1 - \frac{\mu + \frac{\sigma}{\sqrt{v}}\sqrt{\frac{\pi}{2}}}{\mu + \sigma\sqrt{\frac{\pi}{2}}}. \tag{1.17}$$

1.3.5 Pietra index

The Pietra index is defined as the maximal vertical deviation between the Lorenz curve and the egalitarian line, that is,

$$P_X = \max_{0 \le p \le 1} \{p - L_X(p)\}.$$

If we assume that F_X is strictly increasing on its support, the function $p - L_X(p)$ will be differentiable everywhere on (0, 1) and its maximum will be reached when $1 - F_X^{-1}(p)/\mu_X$ equals zero, that is, when $x = F_X(\mu_X)$. The value of $p - L_X(p)$ in this point is given by,

$$P_X = F_X(\mu_X) - L_X(F_X(\mu_X)).$$

In our case since $F(\mu_X) = 1 - e^{-\pi/4}$ and using Equation (1.11) we obtain the formula,

$$P_X = 1 - e^{-\pi/4} - \frac{\mu(1 - e^{-\pi/4}) + \sigma\sqrt{2}\gamma(\pi/4,\ 3/2)}{\mu + \sqrt{\frac{\pi}{2}}\sigma}. \tag{1.18}$$

1.4. Parameter estimation

Let $x_1, \ldots, x_n$ be a sample of size n drawn from a MG distribution. We propose two estimation methods: moments and maximum likelihood. The first estimation method leads to simple estimators, which can be used as initial estimators in the maximum likelihood method.

1.4.1. Moments estimates

Let $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $s_x^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2$ the sample mean and variance respectively. If we equate the sample mean and variance to the corresponding theoretical moments given by Equations (1.5) and (1.7) and we solve for μ and σ, we then obtain moment estimates

$$\hat{\mu} = \overline{x} - \sqrt{\frac{\pi}{4-\pi}}\, s_x \,, \tag{1.19}$$

and

$$\hat{\sigma} = \sqrt{\frac{2}{4-\pi}}\, s_x \,. \tag{1.20}$$

1.4.2. Maximum likelihood estimates

The log likelihood function is given by,

$$l(\mu,\sigma) = -2n\log\sigma + \sum_{i=1}^{n}\log(x_i - \mu) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i - \mu)^2 .$$

Taking partial derivatives with respect to μ and σ and equating them to zero we obtain the normal equations:

$$\frac{\partial l(\mu,\sigma)}{\partial\mu} = -\sum_{i=1}^{n}\frac{1}{(x_i - \mu)} + \frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i - \mu) = 0 \tag{1.21}$$

$$\frac{\partial l(\mu,\sigma)}{\partial\sigma} = -\frac{2n}{\sigma} + \frac{1}{\sigma^3}\sum_{i=1}^{n}(x_i - \mu)^2 = 0 \tag{1.22}$$

If we eliminate σ in Equation (1.22), we obtain

$$\sigma = \left\{ \frac{1}{2n} \sum_{i=1}^{n} (x_i - \mu)^2 \right\}^{1/2} , \tag{1.23}$$

and substituting in Equation (1.21) we have

$$\frac{\sum_{i=1}^{n} (x_i - \mu)}{\frac{1}{2n} \sum_{i=1}^{n} (x_i - \mu)^2} = \sum_{i=1}^{n} \frac{1}{(x_i - \mu)} , \tag{1.24}$$

which only depends on μ. The previous equation can be solved using the Newton-Raphson methods, taking as the initial value the moment estimates defined in Equations (1.19) and (1.20). Finally, the maximum likelihood estimate of σ is given in Equation (1.23).

1.5. Empirical application

1.5.1. Data

To analyze the properties of the MG distribution developed in this study we have used data on individual incomes from the European Community Household Panel survey. This survey was developed by the Statistical Office of European Communities (EUROSTAT) and national data collection units. It contains data on socio-economic factors for all current members of the European Union. Available data correspond with eight waves concerning the eight years of the period 1994-2001. It is important to note that the concept of income refers to disposable household income in the previous year to the interview at constant 1992 prices. Disposable, in this context, regards total income received from all sources, *i.e.* after tax and with transfers added. Personal information of all members over sixteen years of age of a household is also presented, including the sources of the total income.

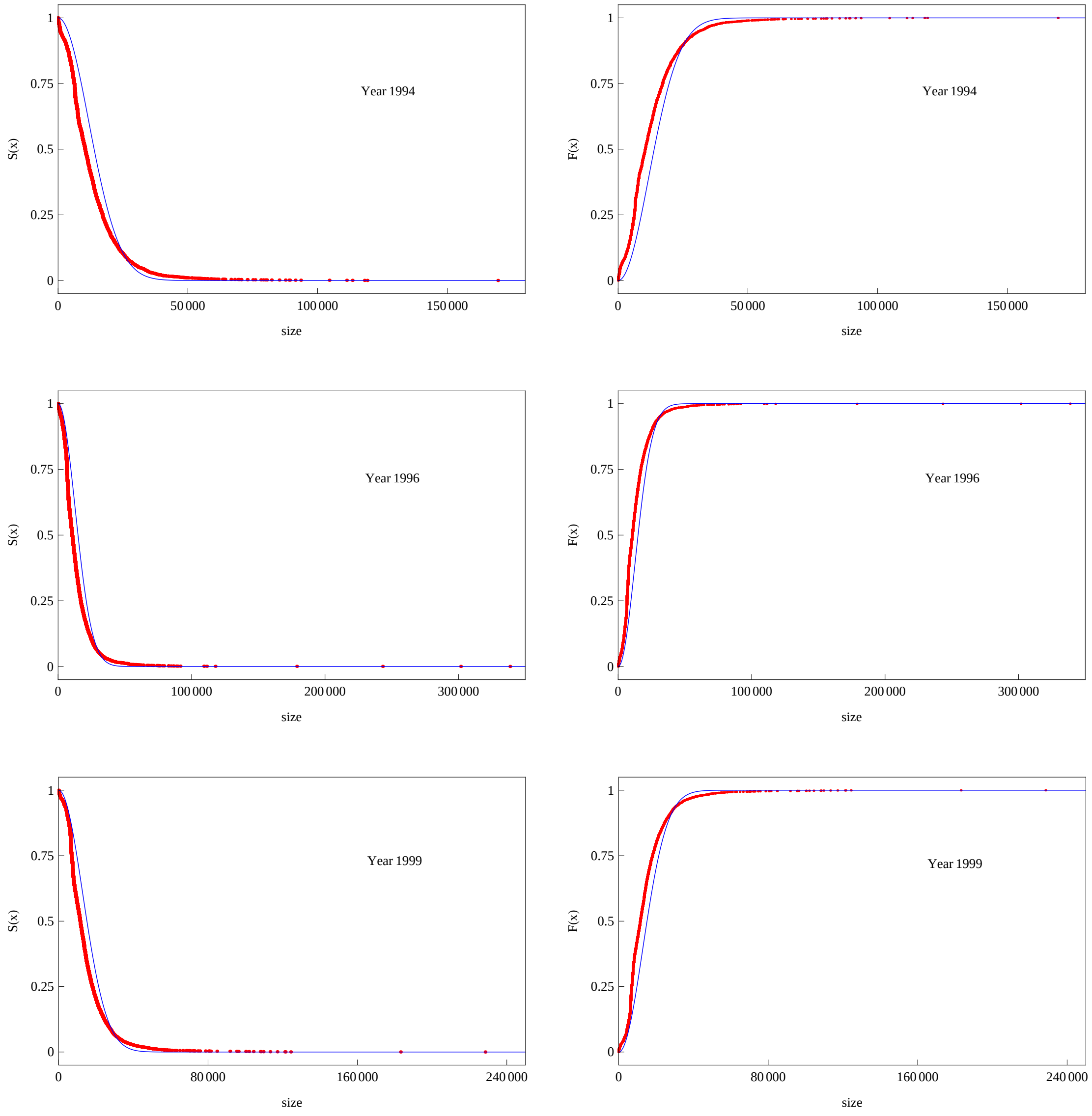

Figure 1.3. Plot of the CDF F(x) (right) and the complementary of the CDF S(x) (left) of the modified Gaussian distribution (solid lines), with the observed data for the years 1994, 1996 and 1999

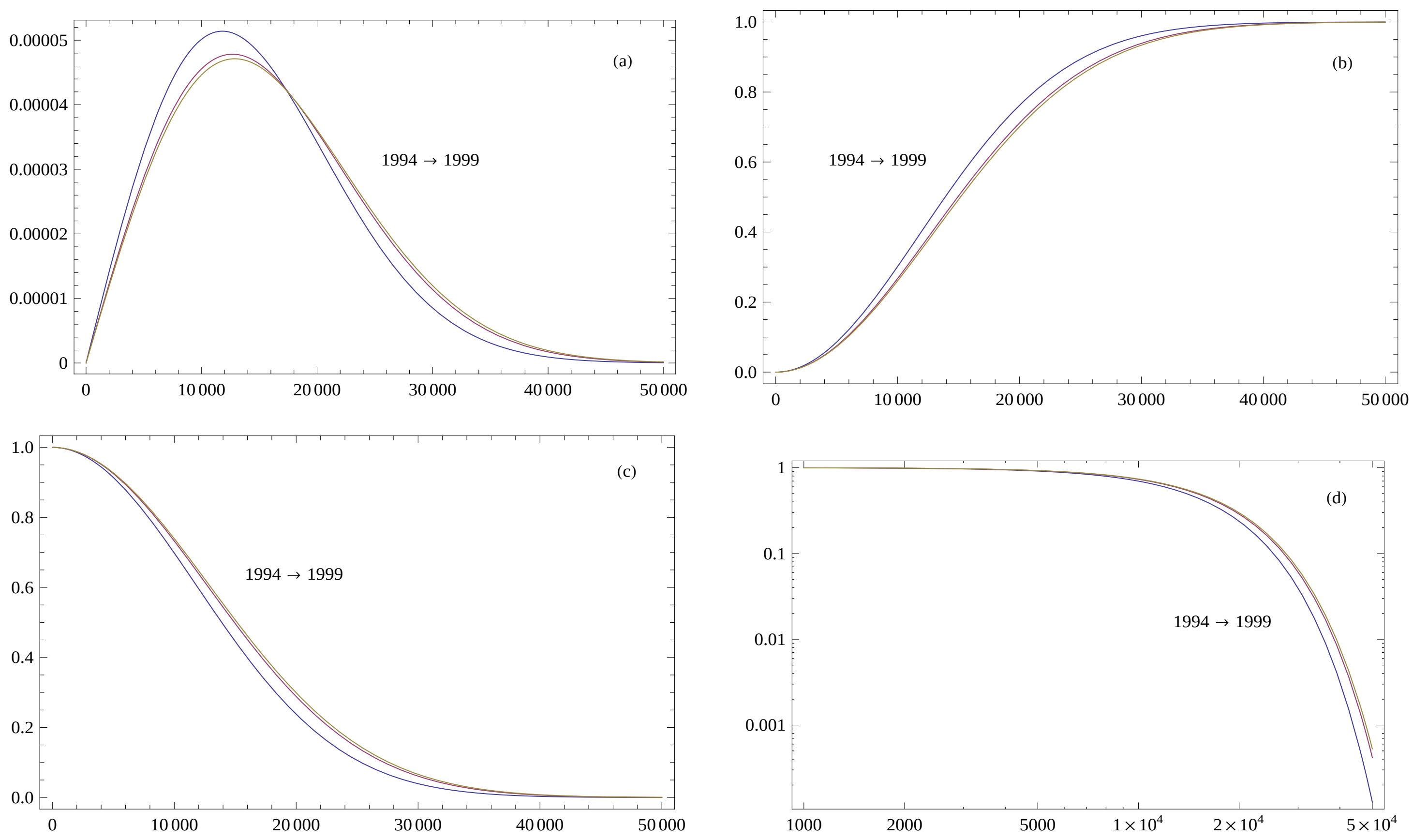

Figure 1.4. Graphics of the (a) PDF, (b) CDF, (c) Survival function and (d) Survival function in log-log scale, of the MG distribution for the ECHP individual income data for the years 1994, 1996 and 1999

Table 1.1. Parameter estimates obtained from the fitting of the MG distribution to individual incomes data by maximum likelihood for the years 1994, 1996 and 1999

Year	μ	σ	Mean	Std. Dev.	$X(0.25)$	$X(0.5)$	$X(0.75)$
1994	2.8	11800	14789.1	7730.6	8950.6	13893.4	19648.3
1996	3.6	12679.6	15891.5	8306.9	9617.8	14929.1	21112.9
1999	1.8	12870.3	16130.5	8431.8	9762.5	15153.6	21430.5

The corresponding means are estimated using Equation (1.5), standard deviations are obtained from Equation (1.7) and first, second and third cuartiles using Equation (1.4) with p = 0.25, 0.5 and 0.75 respectively.

We have worked with Spanish microdata (see Sarabia et al., 2007; Arnold et al., 2006), including only individuals whose information was available in waves 1, 3 and 6, to make incomes comparable over time and to eliminate the effect of missing observations (which would bias our estimations to zero). Therefore, we are considering individual incomes for the years 1994, 1996 and 1999. In wave 1 the survey contains information about 7206 households and 17893 individuals, wave 2 is made up of 6297 households and 15640 individuals and, finally, the last wave includes 5418 households and 13104 individuals. However, our sample comprises 6378 individuals whose information is available over the whole period. In order to transform household incomes to individual incomes, we use the OECD equivalence scale, and the equivalent income is assigned to each member of a particular household, assuming that all members have the same level of welfare, thus resulting in individual incomes.

1.5.2. Results

We have fitted the Modified Gaussian distribution to the data by maximum likelihood method using Equations (1.23) and (1.24), for μ and σ parameters, respectively, taking as initial values the moment estimates given in Equations (1.19) and (1.20). The results are included in Table 1.1. We also present the corresponding theoretical mean, standard deviation and the first, second and third quartiles for each year. Given that Spanish income distribution is left skewed (Oliver-Alonso et al., 2001), modified Gaussian distribution seems to be a suitable proposal. For the graphical validation of the model, we have plotted the CDF and the complementary of the CDF of the MG distribution joint with the observed data (Figure 1.3).

Figure 1.4 shows the evolution of Spanish individual incomes for the years 1994, 1996, 1999 in different format graphics: PDF, CDF, survival and survival in standard and log-log scale. Figure 1.5 shows the GLC for these years. According to the results presented in Table 1.1, the first-degree stochastic dominance given by Equation (1.8) is not verified in all the years. The year 1994 dominates the other two ones, but the couple of years 1996 and 1999 are not comparable in first-degree stochastic dominance terms. This fact can also be deducted from a graphical inspection of Figure 1.4. Note that stochastic dominance for the GLC in these years is also confirmed.

In addition to the estimations performed, we have computed several income inequality measures that have been previously derived in the chapter (Section 1.3). We have calculated the Gini index given by Equation (1.15), the generalized Gini indices for ν values 5 and 10 (Equation (1.17)) and the Pietra index (Equation (1.18)). The four inequality measures considered show an analogous behavior during the study period, characterized by a stable pattern. Note that the Gini index is around 0.29 in the three years considered, whereas the Gini-5, the Gini-10 and the Pietra indices are about 0.55, 0.68 and 0.21 respectively.

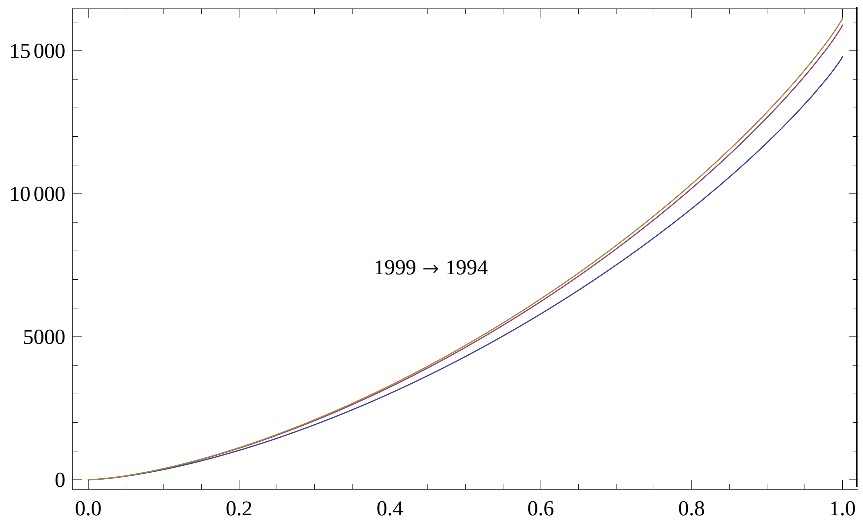

Figure 1.5. Graphics of the GLC of the MG distribution (Equation (1.12)) for the ECHP individual income data for the years 1994, 1996 and 1999

1.6. Summary and conclusions

In this chapter, probabilistic and statistical properties of the modified Gaussian distribution (Guo and Gao, 2012) have been studied. We have obtained closed formulas for the normalizing constant, the different probability functions (PDF, CDF and survival), moments, first-degree stochastic dominance conditions and the relationships with other families of distributions (chi-square, Weibull and stretched exponential distributions). A simple interpretation of the MG distribution in terms of weighted distributions has been proposed. Different tools for studying inequality have been obtained, including Lorenz curve, generalized Lorenz curve, Gini index, Donaldson-Weymark-Kakwani index and Pietra index. Two estimation methods (moments and maximum likelihood) have been proposed. In order to illustrate all the previous formulation, we have fitted individual incomes of Spain for three years using the European Community Household Panel survey, concluding a static pattern of inequality since the Gini index and other inequality measures remain constant over the study period.

Chapter 2

On the estimation of the global income distribution using a parsimonious approach

2.1. Introduction

Inequality, poverty and growth, as well as the links between these concepts have received an increasing amount of attention from economists, analysts and policymakers around the world. The exponential growth of China over the last 20 years, the relationship between globalization and inequality or the financial crisis and its consequences to the real economy, are contexts where these variables play a crucial role for assessing the new economic and social context and for evaluating the efficacy of the policies implemented.

Even when there is nearly consensus that per capita GDP is not an adequate measure of well-being, it is the most widely used indicator to make international comparisons as well as intertemporal evaluations of living standards. National incomes inform about the mean and hence about economic progress, but say nothing regarding other intrinsic features of this distribution. Consequently, aspects such as poverty and inequality would be completely ignored.

Despite their relevancy, global income distribution has not been investigated intensely until the nineties mainly due to the scarcity of data on individual incomes for a wide number of countries over reasonably long periods of time. However, there has been a great effort in the last twenty years to collect data on individual incomes (Deininger and Squire, 1996; Trasmonee, 1999; LIS, 2000; UNU-WIDER, 2008), which come from surveys conducted every five years in most cases. Therefore, it is required long time to complete the collection of the database, which is resource intensive. Consequently, the access to individual data is restricted, but instead, descriptive statistics of surveys such as mean, mode or income shares are freely available.

In this context, the derivation of income distributions from the pieces of information readily available is essential to analyze national and regional patterns of wealth. This estimation is relatively simple for individual countries, however, the estimation of regional and global distributions requires a more complex methodological procedure. The problem arises when we are interested in investigating the evolution of regional or global income distributions whose summary statistics are not accessible. In that sense there have been many attempts to develop regional and global estimates from limited data, mainly based on two distinct methodologies.

On the one hand, non-parametric techniques have been applied to estimate income distributions based on kernel estimates (Sala-i-Martin, 2006; Minoiu, 2007). On the other hand, parametric models of the global income distribution have been estimated using conventional inference techniques. Numerous functional forms have been suggested in the literature. Chotikapanich et al. (1997) assumes a lognormal distribution for modeling national income distributions, which is the most commonly used family along with the gamma (Chotikapanich and Griffiths, 2008) and Pareto distributions. Other two-parameter distributions have been extensively used, including the beta, the Fisk and the Weibull distributions (Pinkovskiy and Sala-i-Martin, 2009; Chotikapanich and Rao, 1998). Three-parameter distributions have also been proposed. In this line, we find special cases of the generalized beta distribution: the beta of second kind (Chotikapanich et al., 2007; 2009), the Singh-Maddala and the Dagum distributions.

As the number of parameter increases, the model becomes more flexible, thus improving the goodness of fit. It should also be noted that more information is needed and highly complex expressions are employed to estimate the parameters and other characteristics of the distribution. However, it is possible to model national income distributions based on a more parsimonious approach, using two-parameter families without a significant loss of reliability. Specifically, we propose to use the so-called Lamé family of distributions, which are two versions of the Singh-Maddala and the Dagum distributions. The adequacy of these distributions has been studied in Sarabia et al. (2013), concluding that the inclusion of an additional parameter does not significantly improve the goodness of fit. Thus, this framework offers simple instruments to construct global and regional distributions as well as to report reliable estimates of inequality and poverty measures.

The objective of this chapter is to determine global and regional income distributions using limited information. To address this issue we combine two different approaches, regional analysis and country case studies. The methodology is applied using data on Gini indices and per capita GDP of each country. The latest version of Standardized *World Income Inequality Database* (Solt, 2009) provides comparable Gini indices using gross income definition for 153 countries over the period 1960-2009. However, most countries have no available data before 1985, especially the developing nations. For this reason our study focuses on the period from 1990 to 2000 with three point estimates in 1990, 1995 and 2000. On the other hand, per capita GDP is drawn from Penn World Tables version 7.2 (Heston et al., 2012) which includes per capita GDP in constant international US dollars[5].

The proposed methodology to estimate regional income distributions from country level data is made up of two steps. In a first stage, we estimate individual distributions of 127 countries included in the sample. We compute several inequality measures and poverty indicators for the countries included in the sample in three points of time

[5] It should be emphasized that we are merging limited information from surveys with national accounts to estimate the regional distribution, but note that it is a common practice and an accepted procedure in this field (Bhalla, 2002; Bourguignon and Morrison, 2002; Chotikapanich et al., 1997; 2007; Grüen and Klasen, 2008).

1990, 1995 and 2000. Thereafter, global and regional distributions are derived from a finite mixture of these families using population shares. We provide different poverty and inequality indices for each region to assess the evolution of poverty and inequality over the nineties. We also compare our results with previous studies, concluding that in most cases two-parameter distributions offer similar estimates to those obtained from three-parameter families, such as the beta-2 in Chotikapanich et al. (2009). Our results point out that, whereas world poverty has declined over the study period, a mix of regional experiences is observed. Global inequality presented a decreasing pattern derived from the convergence process that took place during the nineties, which offset the increase in disparities within countries.

The balance of this chapter is as follows. Section 2.2 describes the methodology used to estimate global and regional distributions. Inequality measures are derived and its decomposition in within- and between-country components is also presented. The sources and the construction of the database are detailed in Section 2.3. Section 2.4 presents the estimates of inequality and poverty measures both globally and for each of the regions considered. The main conclusions of the chapter are included in Section 2.5.

2.2. Methodology

In this section, we describe the methodology used to derive regional and global income distributions as well as the estimates of poverty and inequality measures. As a starting point, we compute national distributions of 127 countries for each benchmark year. It should be recalled that a large number of probability distributions have been proposed in the literature for modeling income and wealth distributions (for a review, see Kleiber and Kotz (2003)).

Despite the long list of candidates[6], the possible choice is limited by the nature and availability of income data. In this case, we work only with the per capita GDP and the value of the Gini index[7]. This implies that we need a simple two-parametric distribution with scale and shape parameters. Notwithstanding its simplicity, the proposed distribution must be sufficiently flexible to fit zero-mode – characteristic of income distributions in developing countries – and one-mode data. Moreover, it should be derived from a simple economic model and, at the same time, the distribution should be also connected with the previous economic literature about income distributions.

2.2.1. Modeling national income distributions

We propose the following family of income distributions, composed by two models called Lamé distributions of first and second class respectively (see Sarabia et al., 2013), which are described in terms of the cumulative distribution functions (CDFs) as,

$$F_1(x;a,\mu)=1-\frac{1}{\left[1+(x/\mu)^{a/(1-a)}\right]^{1/a}},\ \ 0\leq x<\infty, \tag{2.1}$$

and $F_1(x;a,\mu)=0$ if $x<0$, where $0<a\leq 1$ and $\mu>0$, and

$$F_2(x;a,\mu)=\frac{1}{\left[1+(x/\mu)^{a/(a-1)}\right]^{1/a}},\ \ 0\leq x<\infty, \tag{2.2}$$

[6] These families include the classical Pareto distribution (Arnold, 1983), the lognormal, gamma (Salem and Mount, 1974), beta, Singh-Maddala (Singh and Maddala, 1976), Dagum (Dagum, 1977), Weibull and the different generalized versions of each. Many of the above distributions can be embedded in the generalized gamma and generalized beta of first (GB1) and second kind (GB2) distributions, which were proposed by McDonald (1984). McDonald and Xu (1995) developed a generalized beta distribution with five parameters and the generalized beta-exponential (EGB), as extensions of the previous families.

[7] Other studies used data on income shares which are the points associated with the Lorenz curve (See *e.g.* Chotikapanich, 2009; Grüen and Klasen, 2008). In all cases five or ten data points are available, making possible to estimate distributions with several parameters. However, the availability of this data is substantially limited with respect to the mean and the Gini index, thus restricting notably the sample of countries and the possibility to consider long temporal periods.

and $F_2(x;a,\mu)=0$ if $x < 0$, where $a \geq 1$ and $\mu > 0$. In both cases μ represents the mathematical expectation of the population.

The quantile functions associated with the previous distributions (Equations (2.1) and (2.2)) are respectively:

$$X_1(p_i;a,\mu) = \mu(1\text{-}(1\text{-}p)^a)^{-1+\frac{1}{a}}(1-p)^{a-1}, \tag{2.3}$$

$$X_2(p_i;a,\mu) = \mu\,(1\text{-}p^a)^{-1+\frac{1}{a}}p^{a-1}. \tag{2.4}$$

This family is connected with the GB2 distribution (McDonald, 1984), given by the following relationships:

$$X_1 \sim GB2\left(A=\frac{a}{1-a}, P=1, Q=\frac{1}{a}, \mu\right),$$

$$X_2 \sim GB2\left(A=\frac{a}{a-1}, P=\frac{1}{a}, Q=1, \mu\right).$$

Note that Equations (2.1) and (2.2) are curved versions of the Singh-Maddala and Dagum distributions respectively.

On the other hand, Equations (2.1) and (2.2) are derived from two well-known economic theories, namely trickle-up and trickle-down effects (Henle et al., 2008). The first approach assumes that an increase in income of the lower-middle class would be more advantageous for the economy given that they spend their wealth faster than the upper class. On the other hand, trickle-down theory assumes that if the income of the upper class rises, this fact would stimulate the investment, thus resulting in a fall of unemployment and hence benefiting the society as a whole. Then, the trickle-up effect is related to the following expression:

$$\frac{\partial I}{\partial t} = \frac{(A/N)L}{N(1-r)}, \tag{2.5}$$

whereas the trickle up effect verifies that,

$$\frac{\partial I}{\partial t} = \frac{(A/N)(1-L)}{Nr}, \tag{2.6}$$

where L is the Lorenz curve, I is the income of a family at rank r. As a consequence, $(A/N)L$ is the aggregate income of poorer citizens, and $N(1-r)$ is the number of wealthy individuals. Conversely, $(A/N)(1-L)$ is the income of the wealthiest citizens and Nr is the number of individuals at lower rank. Assuming that L is adequately represented by the so-called Lamé Lorenz curves (see Equations (2.9) and (2.10)), the CDFs (2.1) and (2.2) are the solutions of Equations (2.5) and (2.6). According to Equation (2.5), an increase in income of any individual of the society can be achived with an improvement of the economic situation of poorer citizens. In contrast, Equation (2.6) states that an increase in the income of the upper class would lead to economic progress of the whole population.

To study the evolution of income inequality, the Lorenz curve is an extremely usefull resource, which relates the income share ($\eta(x)$) with its respective population share ($F(x)$). For the Lamé income distributions, income shares are given by,

$$\eta_1(x;a,\mu) = \frac{1}{\left[1+(x/\mu)^{-a/1-a}\right]^{1/a}},\ 0 \le x \le \infty, \tag{2.7}$$

with $0 < a < 1$ and

$$\eta_2(x;a,\mu) = \frac{1}{\left[1+(x/\mu)^{a/1-a}\right]^{1/a}},\ 0 \le x \le \infty, \tag{2.8}$$

if $a > 1$.

Accordingly, Lamé Lorenz curves associated with (2.1) and (2.2) are expressed respectively, as follows (see Sarabia et al., 2013):

$$L_1(p;a) = \left[1-(1-p)^a\right]^{1/a},\ 0 \le p \le 1, \tag{2.9}$$

$$L_2(p;a) = 1-(1-p^a)^{1/a},\ 0 \le p \le 1, \tag{2.10}$$

and the corresponding Gini index of each curve is given by:

$$G_1(a) = 1 - \frac{\Gamma(1/a)^2}{a\Gamma(2/a)},\ 0 \leq a \leq 1, \tag{2.11}$$

$$G_2(a) = \frac{\Gamma(1/a)^2}{a\Gamma(2/a)} - 1,\ a \geq 1. \tag{2.12}$$

The study of Lorenz ordering is a crucial aspect in the analysis of income and wealth distributions. For the family used in this chapter, simple relationships are obtained from the value of the parameter *a*. Let $\mathcal{L}$ be the class of all non-negative random variables with positive finite expectation. The Lorenz partial order $\geq_L$ on the class $\mathcal{L}$ is defined by,

$$X \leq_L Y \Leftrightarrow L_X(p) \geq L_y(p), \quad \forall\ p \in [0,1].$$

If $X \leq_L Y$, then *X* exhibits less inequality than *Y* in the Lorenz sense. The families (2.1) and (2.2) are ordered with respect to the parameters. Focusing on $L_1(p;a_2)$, if $a_1 \leq a_2$ then $L_1(p;a_1) \leq L_1(p;a_2)$ for $0 \leq p \leq 1$. On the other hand, for $L_2(p;a_2)$, if $a_1 \leq a_2$ then $L_2(p;a_1) \geq L_2(p;a_2)$ for $0 \leq p \leq 1$.

2.2.2 Estimation with limited information

In this section we expose the procedure to estimate the parameters of the distrtribution for each country and year when few pieces of information are available. In particular, we expose the case that the sole available information for the estimation is the mean income of the country distribution and the Gini index. A plausible estimation method for the parameters which provides consistent estimates, consists of solving the following system:

$$\mu = \bar{X};$$

$$G_k(a) = g;\ k = 1, 2,$$

where $\overline{X}$ and g represent the per capita GDP and Gini index values, respectively and $G_k(a)$, k = 1, 2, are the theoretical Gini indices given, respectively, by (2.11) and (2.12). Therefore, the estimated values of μ and a are given by,

$$\hat{\mu} = \overline{X};$$
$$\hat{a} = G_k^{-1}(g);\ k = 1, 2. \tag{2.13}$$

It is worth noting that, as the right hand side of (2.13) is a monotonic function of parameter g, this system has only one solution. The standard errors of the parameters can be estimated using a parametric bootstrap, thus providing a measure of the accuracy of the estimation.

2.2.3. Modeling regional and global income distributions

Once the parameters of each country are estimated, it is possible to derive the regional and global income distributions as a mixture of the national distributions using population weights. Assume that the region under study comprises $M^{(r)}$ countries, being this region the world or any of the subsamples considered in this study (see Appendix 1). The number of countries varies across regions but, for simplicity, we remove the superscript r, assuming that the rest of the methodology applies for each territory specifically.

Each country has an associated density function (PDF) and a cumulated distribution function (CDF) previously defined as $f_k(x)$ and $F_k(x)$ respectively, k = 1, 2,…, $M^{(r)}$. The demographic weigths are given by $\pi_k = N_k/N$, where N_k is the population of the k-th country and $N = \sum_{k=1}^{M} N_k$ is the total population of the region under study. Then, the regional income distribution is given by the finite mixture:

$$f_X(x) = \sum_{k=1}^{M} \pi_k f_k(x)\,.$$

Assuming that $f_k(x)$ follows a Lamé I distribution (Equation (2.1)), the regional PDF is given by,

$$f_X(x) = \sum_{k=1}^{M} \pi_k \frac{(x/\mu_k)^{(2a_k-1)/(1-a_k)}}{(1-a_k)\mu_k \left(1+(x/\mu_k)^{a_k/(1-a_k)}\right)^{1/a_k+1}}. \tag{2.14}$$

Conversely, the regional CDF follows the expression[8],

$$F_X(x) = \sum_{k=1}^{M} \pi_k \left\{ 1 - \frac{1}{\left(1+(x/\mu_k)^{a_k/(1-a_k)}\right)^{1/a_k}} \right\}. \tag{2.15}$$

Note that, from Equations (2.14) and (2.15), it is straightforward to compute poverty rates, since they are directly calculated integrating the area below the specified poverty line.

On the other hand, regional cumulative income shares are also a weigthed average of the country income shares, given by the following expression:

$$\eta_X(x) = \sum_{k=1}^{M} \frac{\pi_k \mu_k}{\mu} \eta_{x_k}(x),$$

where $\eta_{X_k}(x)$, k = 1, …, n, are defined in Equations (2.7) and (2.8) for each family and μ denotes the regional mean which is expressed as a population weighted average of country means. Once we have derived income shares and their corresponding population shares from $F_X(x)$, the regional Lorenz curve can be graphed using a grid of values of x to calculate the points $(F_X(x), \eta_X(x))$.

To analyze the evolution of inequality, we propose to use generalized entropy (GE) measures given that they allow us to decompose total inequality in two different components, namely within-country inequality and between-country inequality. The first term informs about the level of inequality in each country, assuming that there are no differences across nations. Conversely, the second component states the level of inequality that would be if countries were totally equal, only presenting differences in the mean income of nations.

[8] Similar expressions of the CDF and the PDF are obtained for the Lamé II distribution.

The regional GE measure can be specified in a simple form using the decomposability of this index. Accordingly, if *X* represents the regional income distribution we have:

$$I_X^{(\theta)} = \sum_{k=1}^{M} \pi_k^{1-\theta} s_k^{\theta} I_{X_k}^{(\theta)} + \frac{1}{\theta(\theta-1)}\left(\sum_{k=1}^{M} \pi_k^{1-\theta} s_k^{\theta} - 1\right),$$

where θ is a parameter that states the weight attached to the top of the distribution. High positive values of this parameter yield indices more sensitive to changes in the upper tail, whereas negative values make this measure more sensitive to redistributions at the lower tail. s_k is the proportion of the mean income of each country in the regional mean, $s_k = \pi_k \mu_k \Big/ \sum_{k=1}^{M} \pi_k \mu_k$, $j = 1, 2, ..., M$, and $I_{X_k}^{(\theta)}$ is the GE index of the *k*-th country, expressed as follows for the Lamé I and Lamé II distributions respectively:

$$I_{X_k}^{(\theta)} = \frac{\Gamma\left(1 + \frac{\theta(1-a_1)}{a_1}\right)\Gamma\left(\frac{1-\theta(1-a_1)}{a_1}\right) - 1}{\theta(\theta-1)},$$

$$I_{X_k}^{(\theta)} = \frac{\Gamma\left(\frac{1-\theta(a_2-1)}{a_2}\right)\Gamma\left(1 - \frac{\theta(a_2-1)}{a_2}\right) - 1}{\theta(\theta-1)},$$

with $\theta \neq 0,1$. These results are obtained using the raw moments developed in Sarabia et al. (2013).

GE measures are additively decomposable in two terms – inequality within and between countries – given, respectively, by the following expression:

$$I_W^{(\theta)} = \sum_{k=1}^{M} \pi_k^{1-\theta} s_k^{\theta} I_{X_k}^{(\theta)},$$

$$I_B^{(\theta)} = \frac{1}{\theta(\theta-1)}\left(\sum_{k=1}^{M} \pi_k^{1-\theta} s_k^{\theta} - 1\right).$$

We also have considered the limit case $\theta = 1$, that is the so-called Theil index, which can be decomposed in the form,

$$T_X = T_W + T_B,$$

where,

$$T_W = \sum_{k=1}^{M} \pi_k T_k\,; \;\; T_B = \sum_{k=1}^{M} \pi_k \log\left(\frac{\mu}{\mu_k}\right).$$

T_k is the Theil index of the k-th country which, for the Lamé I and Lamé II, is given respectively by:

$$T(X) = \frac{1-a_1}{a_1}\left\{\psi\left(\frac{1}{a_1}\right) - \psi(1)\right\}, \;\; 0 < a_1 < 1,$$

$$T(X) = \frac{a_2-1}{a_2}\left\{\psi(1) - \psi\left(\frac{1}{a_2}\right)\right\}, \;\; a_2 > 1,$$

where $\psi(z) = \Gamma(z)/\Gamma(z)$ is the digamma function.

2.3. Data and sources

The data used in this analysis come from two main data sources, national accounts to obtain data per capita GDP and surveys which provide Gini index values. Per capita GDP in constant international US$ and population is drawn from Penn World Tables 7.1 (Heston et al., 2012). These data are used to construct income and population shares for deriving regional distributions.

The World Bank has been a major provider of income data on for cross-country comparisons. In fact, recent works that examined global income distributions used the *World Income Inequality Database* (WIID) compiled by the World Bank (Milanovic, 2002; Chotikapanick et al., 2007). However, the data contained in WIID come from a variety of sources, including Deininger and Squire (1996), Trasmonee project

(Trasmonee, 1999); Luxembourg Income Study (LIS, 2000) and national statistical offices. Therefore, this database reports a mix of observations regarding unity of analysis, income concept and coverage population, thus resulting in a lack of comparability.

Due to the high heterogeneity that characterizes the World Bank data, we opt for using the Standardized World Income Inequality Database (Solt, 2009), which provides comparable Gini indices based on gross income definition for 153 countries over the period 1960-2009. This database homogenizes the observations of WIID, taking LIS observations as a benchmark. After this procedure, more than 4500 observations are provided, although the majority of them are concentrated in the period from 1985 to 2000, especially for developing countries. Our sample comprises 127 countries for three benchmark years in the nineties, 1990, 1995 and 2000. When there is no available data for the exact year, the closest observation is used only if it is within the previous two years or in the two following ones[9]. The percentage of coverage for each region in the three years considered is presented in Table 2.1.

Table 2.1. Regional and global population coverage

	1990			1995			2000		
	Total	Included	Covered	Total	Included	Covered	Total	Included	Covered
EAP	1821.36	1568.78	0.8613	1940.03	1675.65	0.8637	2045.00	1761.20	0.8612
EECA	842.51	451.75	0.5362	855.78	458.68	0.5360	862.19	461.51	0.5353
LAC	442.07	430.69	0.9743	481.54	470.06	0.9762	520.37	507.06	0.9744
MENA	253.29	190.46	0.7520	283.43	210.74	0.7435	312.51	230.05	0.7361
SSA	512.73	392.91	0.7663	585.95	444.52	0.7586	666.68	502.24	0.7534
SA	1146.78	1088.11	0.9488	1272.29	1198.09	0.9417	1398.31	1315.70	0.9409
WENAO	852.53	852.17	0.9996	884.55	884.18	0.9996	912.10	911.71	0.9996
World	5296.21	4974.88	0.9393	5714.72	5341.91	0.9348	6118.05	5689.48	0.9299

[9] For the benchmark year 1990 the Gini indices of Burundi, Central African Republic, Gambia, Niger, Laos, Vietnam, Guyana, Nicaragua, Yemen and Iceland corresponds to 1992. This statistic in the case of Guinea, Guinea-Bissau, Senegal and Bosnia and Herzegovina corresponds to 1991. For the benchmark year 2000, the Gini index of Guyana corresponds to 1999.

For the whole period we cover nearly 93 per cent of the world population. Note, however, that this percentage is not achieved for each of the regions. About 90 percent of the population is covered in East Asia and the Pacific (EAP), Latin America and the Caribbean (LAC), South Asia (SA) and Western Europe, North America, Oceania (WENAO). Nearly 75 percent of the population in the Middle East and North Africa (MENA) and Sub-Saharan Africa (SSA) is included in this study. Finally, the coverage for Eastern Europe and Central Asia (EECA) is only about 53 percent for all years.

Therefore, despite the large coverage of world population, many African and Eastern European countries are not included in the sample due to the scarcity of data. Given that practically all absentees are developing countries, our estimates can be biased downward. Consequently, the conclusions derived from this analysis should be cautiously interpreted, especially in the regions with low levels of representativeness.

2.4. Results

We present the discussion of the results obtained using the methodology developed in previous sections to estimate regional income distributions from country-level data. The proposed methodology includes two steps. First we estimate national income distributions for 127 countries, considering two versions of the Sing-Maddala and the Dagum distributions. The estimates of national distributions for the most populous country in each region and their evolution over the study period are presented in Section 2.4.1. In a second step, regional and global distributions are derived from a mixture of these families using population weights, whose evolution is studied in Section 2.4.2. Thereafter, a description of the poverty patterns in each territory is provided in Section 2.4.3. Finally, an analysis of the evolution of inequality and its decomposition in between-country and within-country components is detailed in Section 2.4.4.

2.4.1. National income distributions

In this section we present the results for the most populous countries in each region. Specifically, we investigate the evolution of the income distribution in China, India, Nigeria, United States, Russia, Brazil and Iran. It should be emphasized that a detailed description of this information for all the countries included in the sample would require a large amount of space[10]. Note that through the analysis of the selected countries, we investigate their influence on the respective regional distributions.

Table 2.2 presents the estimated parameters for the two Lamé distributions: Lamé I, related to trickle-up effect, and Lamé II, associated with trickle-down effect (Equations (2.1) and (2.2) respectively) in three moments of time: 1990, 1995 and 2000. Estimated parameters (a_1 for the Lamé I, and a_2 for the Lamé II), have been obtained from summary statistics, specifically in this case, the mean and the Gini index which are also included in the table. At this point, it should be recalled that, given the nature of the families proposed to estimate national income distributions, these parameters determine Lorenz dominance relationships. In that sense, denoting $\succ_L$ as Lorenz dominance, it would be concluded that for the first and the last year of the study period we have:

$$\text{Russia} \succ_L \text{India} \succ_L \text{China} \succ_L \text{United States} \succ_L \text{Iran} \succ_L \text{Nigeria} \succ_L \text{Brazil}$$

$$\text{India} \succ_L \text{China} \succ_L \text{Iran} \succ_L \text{United States} \succ_L \text{Nigeria} \succ_L \text{Russia} \succ_L \text{Brazil}$$

These relationships state an ordering in terms of inequality, indicating that Russia and Brazil are, respectively, the most equal and unequal countries among the nations considered at the beginning of the nineties. In 2000, the deterioration of the income distribution in Russia during the course of the decade relegates it as the second most unequal nation, while in the past, this country has been characterized to have lower inequality levels than the rest of the countries included in this analysis. The other countries remain in so far as their relative position in terms of Lorenz orderings.

[10] The mean and the Gini index used for the estimation, parameter estimates and inequality measures for each country are presented in Appendix 3.

To complete these results, Figure 2.1 presents the evolution of the density function for three benchmark years considered during the nineties, 1990, 1995 and 2000. To reflect how poverty has evolved over the study period, we also include a vertical line which corresponds to the official poverty line of $1.25 a day in 2005 prices, stated by the World Bank.

China is the most populous country in the world and its influence on the global distribution of income is a well-documented fact in the literature (see *e.g.* Sala-i-Martin, 2006; Milanovic, 2005; Chen and Ravallion, 2004). The outstanding economic growth of China during the nineties is clearly observed from Figure 2.1A. The distribution has shifted completely to the right over the study period, thus indicating that the whole population has improved their economic situation at the end of the decade. It is also worth noting that the income distribution in this country presents a fatter right tail at the end of the period and less proportion of population is concentrated around the mode, indicating an increase in the percentage of the medium and high classes. In fact, the mean has almost tripled in just ten years and poverty has been reduced substantially. It should also be emphasized that the economic progress came with the price of inequality, which has increased by 18 percent in just one decade.

Russian income distribution has worsened considerably over the course of the nineties. The liberalization process and the transition to the market economy have deeply affected the shape of the income distribution in this country. It became more peaked in 2000, which indicates an increase in the concentration of the distribution. This result is also confirmed by the value of the Gini coefficient which has increased by slightly over 50 percent. Moreover, the income distribution has shifted to the left and consequently, the mean has decreased dramatically also increasing considerably the proportion of Russian citizens living in extreme poverty.

Derived from the uneven economic development that actually favored the southern regions (Baer, 1995) and protectionism which was characteristic (Castilho et al., 2011) of the lost decade, Brazil is regarded as the most unequal country in the world but more alarming is the fact that no progress has taken place in terms of inequality. The high disparities of this country give a characteristic leptokurtic distribution, although this pattern is becoming less pronounced given that a fatter right tail is observed at the end of the period and a flatter shape is adopted. As a result, a decrease in poverty rates is appreciable along with an increase in the mean income.

A similar evolution is observed for the Iranian case. The income distribution became less peaked and it moved to the right, which led to a considerable expansion of the mean along with a small reduction of poverty. Note, however, that the levels of inequality remain constant over the whole period since the Gini index decreases for the first half of the period and then started to rise again. This evolution of the shape of the Iranian distribution suggests that the economic growth achieved by this country over the nineties is at expense of higher levels of inequality.

India is the second most populous country in the world. This fact makes the analysis of its distribution along with China's essential to understand the global patterns of income distribution. In response to the orthodox economic reforms implemented during the nineties (Nagaraj, 2000), some signs of economic growth are observed, a fact indicated by the translation of the distribution to the right. From Figure 2.1D, it is concluded that the income distribution also adopts a less peaked shape. As a result, the area below the poverty line was substantially reduced and per capita GDP levels increased considerably during the studied decade. On the other side of the coin, we found that disparities increased slightly, since the right part of the distribution shifts more than the lower tail. This result is confirmed by the value of the Gini index. Even when it is a residual increase, the alarming fact is that this country does not seem to be able to reduce its high levels of inequality.

Table 2.2. Parameter estimates, mean and Gini indices for selected countries

Country	1990				1995				2000			
	a_1	a_2	Gini	Mean	a_1	a_2	Gini	Mean	a_1	a_2	Gini	Mean
China	0.7091 (0.0289)	1.4649 (0.0832)	34.99	1154.30	0.6750 (0.0345)	1.5581 (0.1097)	39.87	1931.26	0.6656 (0.0345)	1.5864 (0.1190)	41.25	2822.38
Russia	0.7314 (0.0283)	1.4107 (0.0691)	31.89	12607.88	0.5985 (0.0421)	1.8274 (0.1875)	51.34	8084.51	0.6213 (0.0385)	1.7367 (0.1564)	47.86	8521.85
Brazil	0.5536 (0.0459)	2.0411 (0.2870)	58.30	6144.66	0.5605 (0.0448)	2.0049 (0.3095)	57.23	6646.40	0.5603 (0.0466)	2.0058 (0.2868)	57.25	6839.00
Iran	0.6369 (0.0369)	1.6804 (0.1480)	45.51	5809.50	0.6398 (0.0363)	1.6703 (0.1415)	45.08	6351.69	0.6431 (0.0359)	1.6589 (0.1492)	44.58	7334.96
India	0.7230 (0.0283)	1.4306 (0.0824)	33.05	1430.57	0.7074 (0.0329)	1.4691 (0.0941)	35.22	1611.27	0.7136 (0.0301)	1.4535 (0.0800)	34.35	1921.90
Nigeria	0.6318 (0.0384)	1.6983 (0.1747)	46.27	1167.91	0.6037 (0.0409)	1.8060 (0.1927)	50.54	1047.76	0.6217 (0.0404)	1.7353 (0.1625)	47.80	1107.70
United States	0.6522 (0.0346)	1.6287 (0.1269)	43.23	31388.79	0.6324 (0.0380)	1.6962 (0.1549)	46.19	33560.13	0.6258 (0.037/)	1.7202 (0.1529)	47.19	39668.69

Standard deviation of parameters in parenthesis (based on 999 simulations)

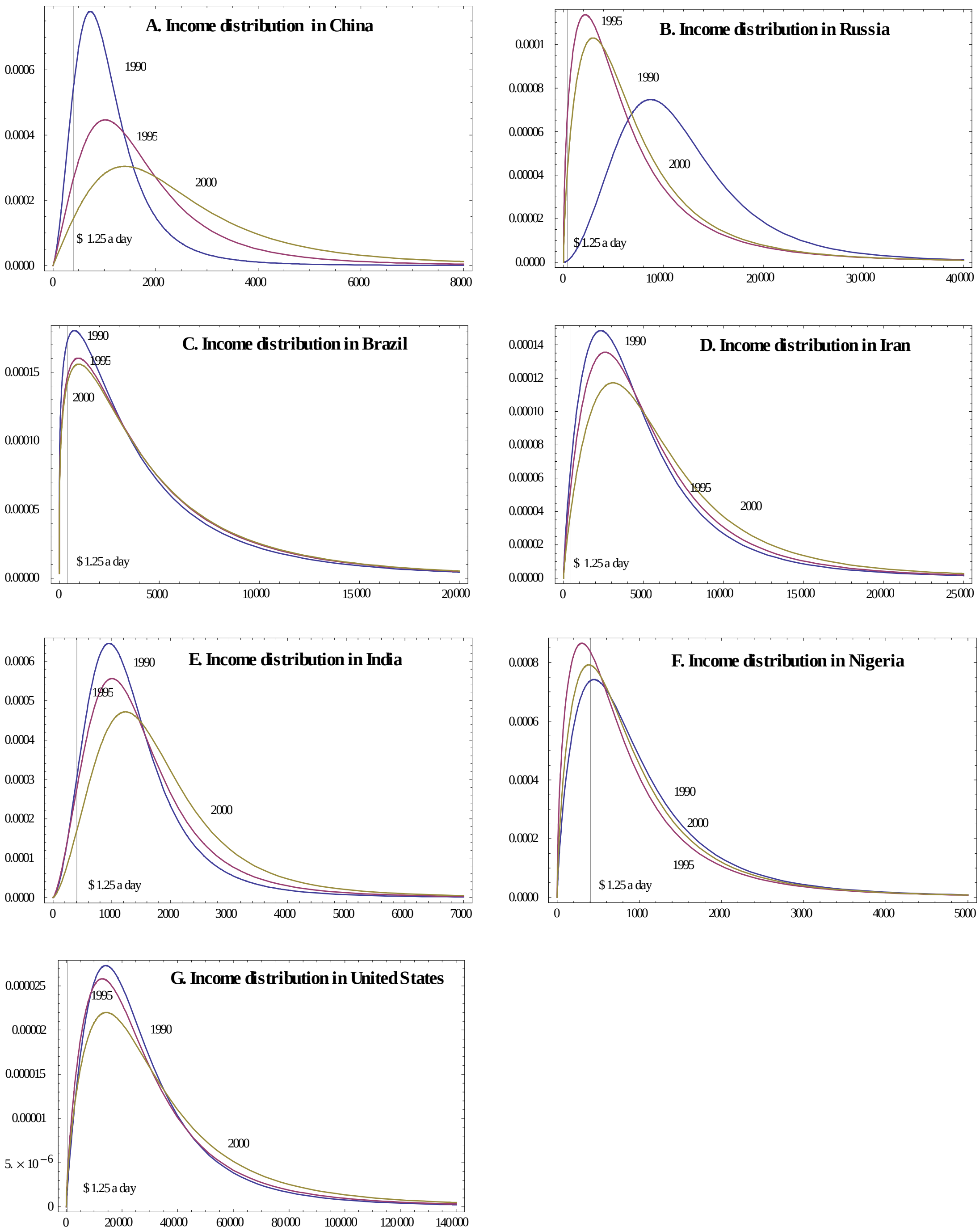

Figure 2.1. Lamé I density functions for selected countries

One of the most dramatic cases is Nigeria. In 1990, the mode of the distribution was placed close to the official poverty line. There was a considerable area below the poverty line defined as $1.25 a day, suggesting that a large proportion of Nigerian population lived in extreme poverty. The situation became even worse over the study period. The distribution has shifted to the left, thus decreasing the mode income and the per capita GDP. Despite the numerous poverty alleviation programs implemented in Nigeria during the nineties, poverty rates increased significantly (Aigbokhan, 2008). At the same time, inequality has increased, attenuating the problems of this country.

Finally, we analyze the evolution of the most populous advanced country and the third nation in population size worldwide. The influence of United States on the global economy is not only consequence of its population size, being one of the most powerful nations in the world. Figure 2.1G reveals signs of economic growth given that a flatter distribution with a fatter right tail is observed. In fact, the mean increased by slightly over 25 percent. However, inequality increased significantly during these years. The value of the Gini index rose from 43.23 to 47.8, which seems to be consequence of long-term effects of the Tax Reform Act of 1986 (Piketty and Saez, 2003). Finally, we recall that, as expected from a developed country, poverty rates are zero in all years.

Having reached this point, it should be recalled that previous conclusions are based on the assumption that income distributions are adequately represented by the Lamé family. However, it can be questionable to estimate the parameters using the sole information provided by the mean and the Gini index. Moreover, it can be argued that two parameter distributions are not flexible enougth to model national income distributions. To evaluate the performance of the proposed families to fit income data, we have compared the empirical and the estimated income shares for the countries included in the sample when this information is available[11]. The empirical shares,

[11] The estimated income shares along with the income shares reported by the World Bank extracted from the WIID database (UNU-WIDER, 2008) are presented in Appendix 2.

obtained from WIID, are available in the form of five or ten points of the Lorenz curve depending on the country.

Using the parameter estimates from Equation (2.13), we estimate the corresponding number of shares based on the following expression:

$$s_i = \eta(x_i; a, \mu) - \eta(x_{i-1}; a, \mu) \quad \forall i = 1,2,..., k \tag{2.16}$$

Where k = 5, 10 depending on the number of points that we estimate. x_i are the estimates of the theoretical quantiles derived using (2.3) and (2.4) for the Lamé I and Lamé II distributions respectively. Substituting x_i and the parameter estimates in $\eta(x; a, \mu)$ given by Equation (2.7) and (2.8), respectively, for Lamé I and Lamé II, we obtain the estimated income shares.

With the previous information, we perform a chi-square test of goodness of fit to investigate the hypothesis that the income distribution of the country i follows a Lamé distribution. The empirical and estimated shares along with the results of the test statistic and its associated p-value are presented in Appendix 2. It is worth noting that the data on income shares are available for a lower number of countries than the Gini and the mean. We have performed 39 tests for the year 1990, 54 for 1995 and 61 for 2000. In aggregate terms, we have performed 154 chi-square tests among which the null hypothesis is rejected only for 10 percent of the cases. This result points out that the distributions proposed in this chapter fit reasonably well in the 90 percent of the countries included in the sample.

Note that the previous result reinforces the validity of our estimates. We have proposed to use a two parameter distribution to estimate poverty and inequality trends. This parsimonious approach gives us the opportunity to include a large number of observations in our sample given that the availability of income shares is not as wide as the Gini and the mean thus restricting our sample considerably. In recent years the most commonly used families comprise three- or four-parameter distributions, which are estimated using income shares. The rationale behind the use of this type of models is that two-parameter families are no flexible enough to model income distributions.

However, the previous test demonstrates that the estimations computed in this study perform reasonably well in more than 90 percent of the countries.

2.4.2. Regional and global income distributions

The estimation of the national distributions allows us to derive regional and global distributions from a finite mixture of these families using the population weight of each country. The evolution of regional and global density functions over the study period is presented in Figure 2.2. Before going any further, it should be emphasized that, even when the Lamé I and Lamé II distributions can only represent unimodal and zero modal distributions, a mixture of them is not necessarily characterized by these distributional features.

In fact, the distribution of EECA shows two different modes in 1990, corresponding with 1700 and 7000 dollars in international prices. This distribution turns into a unimodal at the end of the decade, coinciding its mode with the lower mode at the beginning of the period. Therefore, the distribution has moved to the left, also exhibiting more peaked shape with thinner tails. It should be emphasized that these distributional dynamics are highly influenced by the effects of the transition from planned to market economy in communists countries. The consequences of adopting this new politic and social context on national living standards have been catastrophic, increasing substantially the number of people living in extreme poverty.

EAP has experienced a period of astonishing economic growth which has shifted the distribution to the right, making it less peaked and characterized by a fatter right tail. The other Asian region shows a similar behavior over the nineties although less pronounced changes are observed. Note that these regions include the so-called *Asian Dragons* and the emerging economies of China and India, characterized by growth rates ranged from 5 to 10 percent (Bosworth and Collins, 2008). The outstanding growth in these countries potentiates the economic progress of the two Asian regions that resulted in improving the living standards of most population in Asia. These dynamics are reflected in the shape of the income distribution, in the sense that fatter tails are observed in 2000, thus indicating that the proportion of population that belongs to the medium class has increased.

The positive economic trends observed in Asia contrast with the tragedy experienced by SSA. In 1990 this region was characterized by a highly skewed distribution, which reflected that an important proportion of the population lived in extreme poverty. As if this situation was not alarming enough, the evolution of the income distribution all over this territory seems to be even worse at the end of the decade. Note that this deterioration of the economic situation is not sharply pronounced because this region presents three different phases over the nineties. It is observed that the mode income moved to the left over the first half of the period, thus relocating the distribution and increasing the poverty rates of the region. During the second half of the nineties, the SSA income distribution moved again to the right, reaching the mode almost the same value than in 1990. However, more peaked distribution is observed, which has led to an increase in the area below the poverty line.

The nineties in LAC were characterized by a period of macroeconomic stability and recovery before the phases of volatility and stagnation characteristic of the eighties (Londoño and Székely, 2000). However, no remarkable progress is observed in the shape of the income distribution. From Figure 2.3, we observe two opposite forces. On the one hand, the mode has slightly moved to the left, thus indicating an increase in poor population in Latin America. On the other hand, the shape of the income distribution of this region in 2000 presents fatter right tails, hence indicating that a proportion of the population previously situated around the mode has improved its economic background. The strongest tendency would determine the evolution of inequality in this region.

The long-term effects of the oil-fired economic boom are also present during the course of the nineties (Adams and Page, 2003). After two decades of intense economic progress with annual growth rates close to 4.5 percent, the economic context of this region improved slightly over the study period. Income distribution in MENA presents a less peaked distribution at the end of the nineties, also characterized by a fatter right tail. The mode has moved slightly to the right in this case, thus showing a sign of economic progress. Even when no significant advancements have been achieved in terms of poverty, this region stands as the developing region with lowest levels of poverty.

Finally, the region of advanced economies, WENAO, shows a less skewed distribution with a fatter right tail at the end of the nineties. As expected for the developed countries, poverty rates are almost zero over the whole period. From Figure 2.2, it is observed that the mode has vaguely moved to the right, reflecting that this region has experienced a decade of economic progress, but at expense of increasing inequality levels, given that the left tail sifts less than the right tail of the distribution.

To analyze how the changes experienced by each region have affected the relative position of their distributions in terms of income, Figure 2.3 plots regional and global density functions in 1990 and 2000. Probably, the most relevant case is SSA which is relegated to the last position and characterized by the lowest income mode. The stagnation phase that characterizes this territory during the nineties makes it difficult to improve its economic situation with respect to other regions. The African tragedy contrasts with the outstanding progress achieved by EAP. This territory has a peaked distribution in 1990 whose mode was similar to that of global. Ten years later, the distribution has shifted to the right, also relocating the mode and diminishing the probability mass around it, thus improving its economic situation considerably. The terrible performance of EECA over the nineties is patent from Figure 2.3. Its distribution has worsened until the point to be really close to Latin American, while at the beginning of the period it was closer to the distribution of the advanced economies. For the rest of the regions, their relative situation has not changed considerably over the study period.

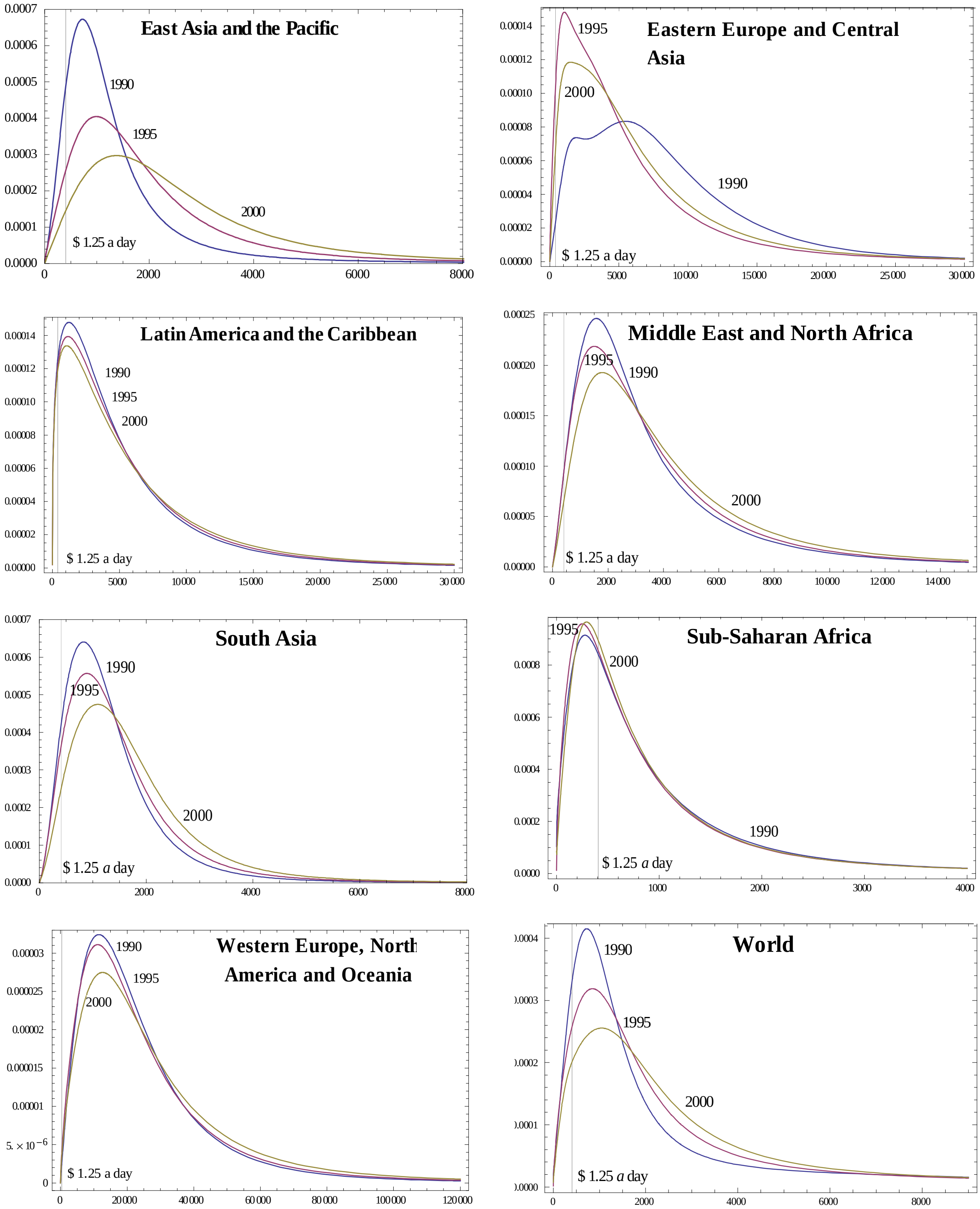

Figure 2.2. Intertemporal evolution of regional and global density functions. Lamé I

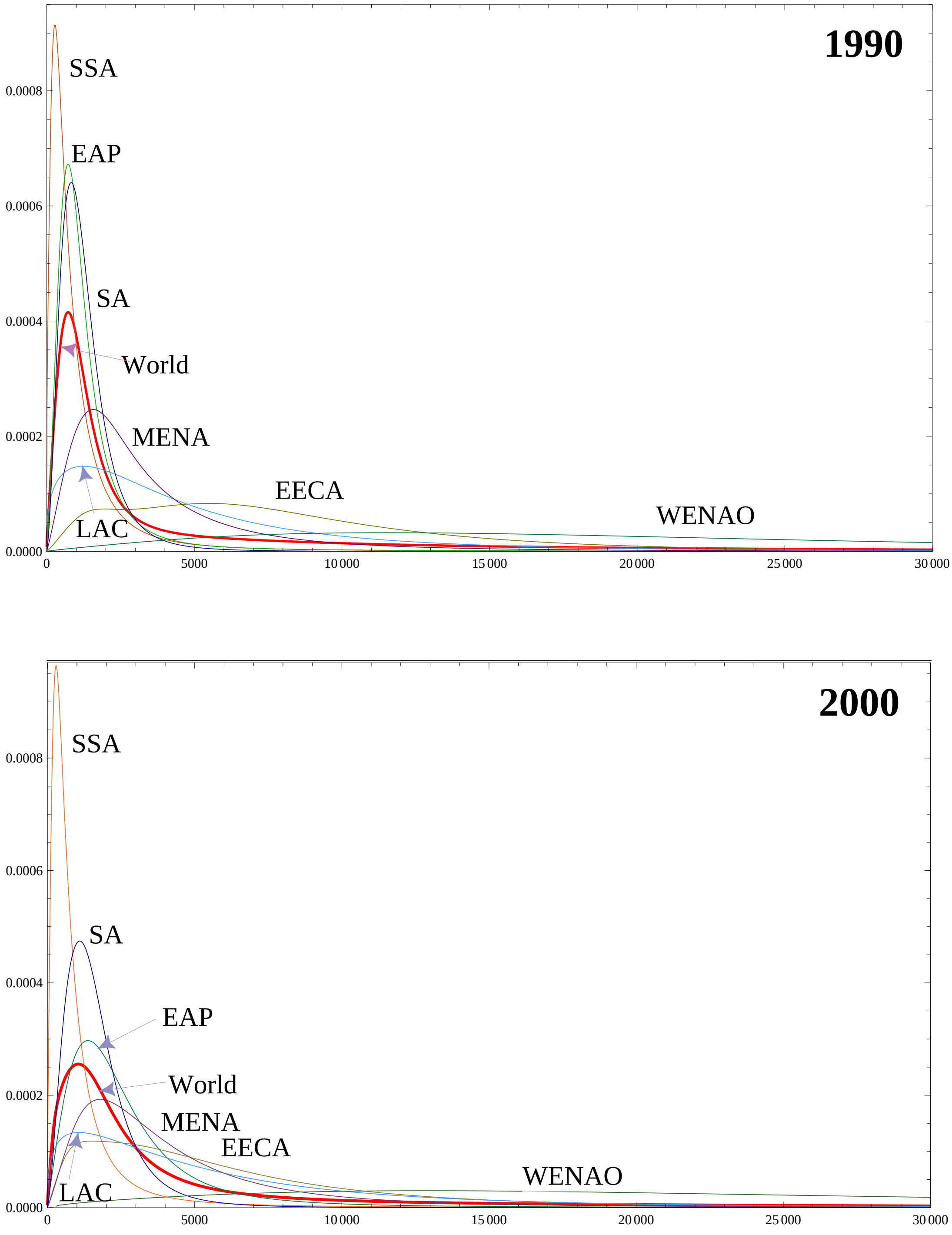

Figure 2.3. Regional and global density functions in 1990 and 2000. Lamé I

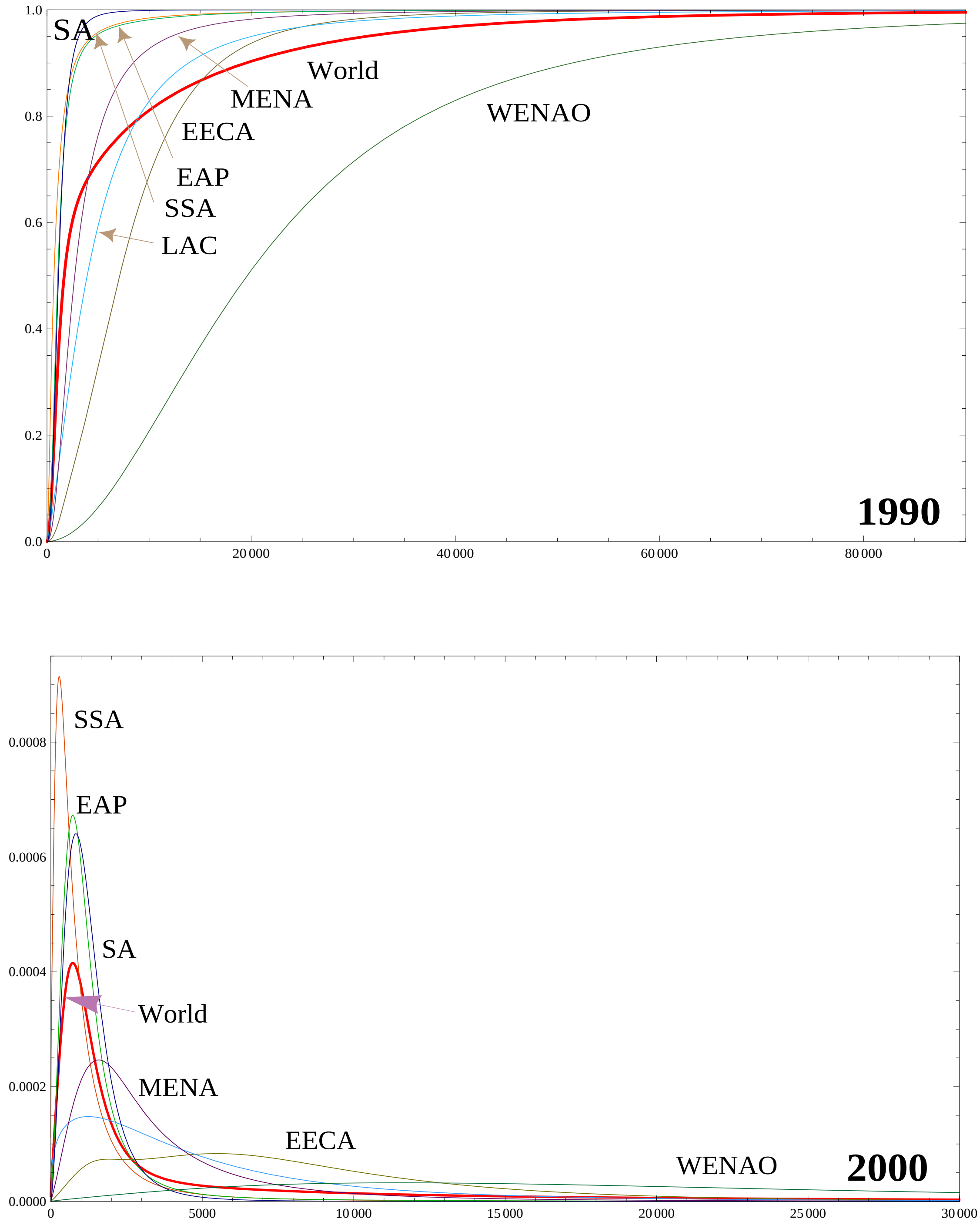

Figure 2.4. Regional and global cumulative distribution functions in 1990 and 2000. Lamé I

The world distribution of income was placed between SA, EAP and LAC, which presented similar modes around 1500 dollars in international and constant prices. This place of the global distribution reveals that the high levels of per capita GDP characteristic of WENAO and EECA are completely offset by low income levels of the most populous countries such as China, India and Nigeria. The relative position of the global distribution is also confirmed in Figure 2.4 which shows regional and global distribution function. Even when no conditions of stochastic dominance have been developed in this chapter for supranational distributions, it is possible to establish it graphically using the estimated PDFs. Denoting the existence of first order stochastic dominance as $\succ_{FSD}$, it is possible to determine the following relationships for the first and the last year of the study period respectively:

$$\text{WENAO} \succ_{FSD} \{\text{LAC, EECA}\} \succ_{FSD} \text{MENA} \succ_{FSD} \{\text{SA, EAP, SSA}\}$$

$$\text{WENAO} \succ_{FSD} \{\text{LAC, EECA}\} \succ_{FSD} \text{MENA} \succ_{FSD} \text{EAP} \succ_{FSD} \{\text{SA, SSA}\}$$

It is clear the WENAO dominates the rest of the regions over the whole decade, thus positioning it as the richest territory. Overall, no notable changes have happened during this period. The sole improvement is the situation of EAP which in 1990 was dominated by all regions, whereas in 2000, it dominates the regions SSA and SA. This progress is mainly driven by the economic growth of China, given that its regional population weight is greater than 0.7. In fact, if China is removed from the sample, an analogous ordering to that of 1990 is obtained.

2.4.3. Regional and global poverty rates

Directly related to the shape of the CDF, poverty rates constitute an important characteristic of the income distribution, especially in developing countries. Once global and regional income distributions have been estimated, it is straightforward to calculate the proportion of people living in a situation of extreme poverty, given that it is the result of integrating the density function of such a region from zero to a certain threshold determined by the poverty line. Having reached this point, a brief discussion about the subjectivity of the poverty lines is in order. Regarding the definition of poverty, this topic has been investigated in a large number of studies. As an example, the World Bank changed its conception of this phenomenon, modifying the line from $1.02 a day in 1985 prices to $1.08 in 1993 prices. The most recent definition is $1.25 a day in 2005 prices and currently, the $2 a day (2005 prices) line is also considered in the statistics provided by this organization. Obviously, these three indicators are not equivalent. Staying in the middle, Bhalla (2002) uses a poverty line of $1.5 a day. A battery of poverty indicators are considered in Chen and Ravallion (2010) who discussed the adequacy of each one, pointing out that all of these measures are reasonable but somewhat arbitrary (Sala-i-Martin, 2006).

To ensure the comparability across countries, we have considered three international poverty lines set in 2005 purchasing power parity (PPP) exchange rates. First we calculate $1.25 a day, which is the official poverty rate stated by the World Bank and corresponds with the mean poverty line of the poorest 15 countries. As other measures of poverty, some criticisms have been attached to this indicator (Deaton, 2010). Therefore we also consider $1.45 in 2005 prices a day which is the result of updating the previous official poverty line of $1.08 a day in 1993 prices, and finally, $2 a day in 2005 prices.

In Figure 2.5 the global PDF of income is presented for the three benchmark years considered in the study. In order to facilitate the visualization of poverty trends, we only present two of the lines considered, which correspond to the thresholds stated by the World Bank. It should be recalled that the area below the poverty line represents the poverty rate associated with that level of income. The first interesting feature that should be noted is that the year 2000 stochastically dominates 1995. Even when not clear dominance orderings can be established for the whole income distribution in 1990, it is observed that there is first order dominance up to the poverty lines, implying that all inequality measures would rank these distributions identically. Our estimates suggest that poverty unambiguously fell over the nineties.

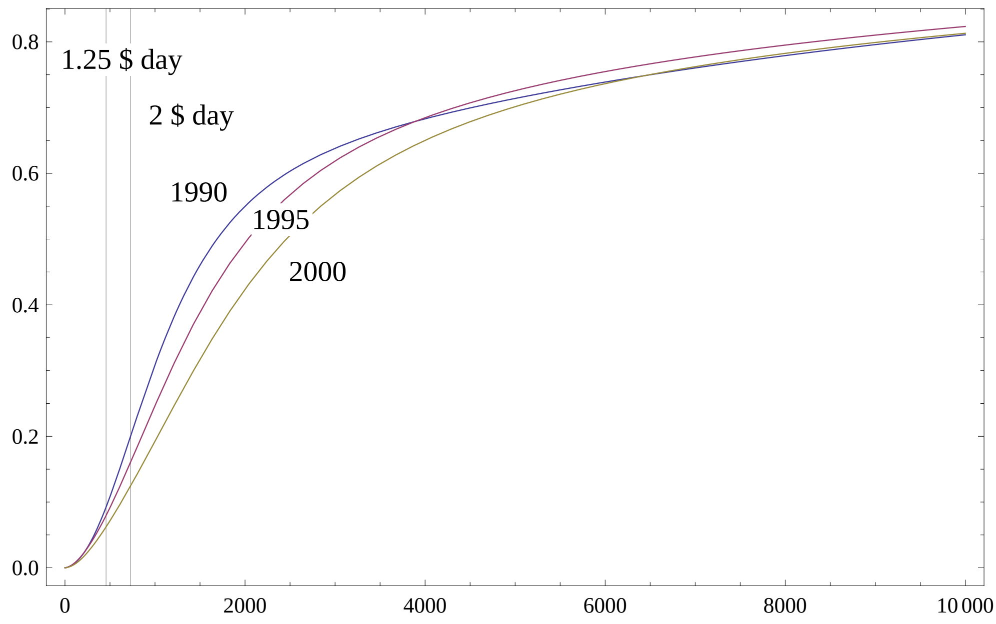

Figure 2.5. Global cumulative distribution function of income

To quantify the decline in poverty at global and regional levels, in Table 2.3 we present the poverty rates related to the three definitions stated previously over the nineties. It is observed that, consistently with the result stochastic dominance, global poverty rates fell quite dramatically over the study period. Using the $1.25 a day definition, a decline of 32 percent is observed, falling the rate from 9 to 6 percent. Our estimates suggest even more progress in getting above $2 a day, which has fallen from 20 to 12 percent, corresponding with a decrease in the poverty rate of 38 percent.

Having reached this point, it should be emphasized that our estimates are substantially lower than the official estimates reported by the World Bank, which states that the poverty rate was about 20 percent in 2002 according to $1.25 definition. Note, however, that the World Bank quantifies poverty using the definition of household consumption, whereas this study uses national accounts to measure mean income. It seems to be complex to compare estimates that come from so different sources. In fact, our estimates are really similar to those obtained by Sala-i-Martin (2006) who also uses national accounts. Another study that estimates poverty using national accounts is Holzmann et al. (2007), reporting that poverty rates were 5.9 and 14.8 for

$1 and $2 a day poverty lines respectively. Note that these estimates are really similar to the results obtained in this study. To compare estimates based on consumption and income concepts, it is argued that the line specified for the first concept should be doubled for the income definition (Chen and Ravallion, 2004). This result was confirmed by Sala-i-Martin (2006), whose estimates of poverty headcounts were considerably close to those of World Bank when using $3 a day definition. Following this line of reasoning, we calculate poverty rates for $3 a day line, revealing that the world population that lives in extreme poverty was 32.71, 26.22 and 20.52 for the years 1990, 1995 and 2000 respectively, which are close to the estimates reported by the World Bank.

Let us move on the decomposition of global poverty by regions. Table 2.3 presents the estimates of poverty rates in East Asia and the Pacific, South Asia, Sub-Saharan Africa, Latin America and the Caribbean, Eastern Europe and Central Asia and Middle East and North Africa for the three benchmark years. As aggregate trends certainly hide a variety of country patterns, we also highlight the experiences of some relevant countries in each territory[12].

EAP is the most populous among the regions considered, including almost one third of the world population. Note that poverty rate for the $1.25 definition was 12 per cent in 1990, which has been reduced by a factor of three, being lower than 4 percent in 2000. The trend followed by moderate poverty rates (based on the definitions $1.45 and $2 a day) is very similar. As China accounts for more 70 percent of the regional population, it is argued that this tremendous achievement of EAP is motivated principally by the progress of this country against absolute poverty, which has seen a decline of its poverty rates from 13 percent in 1990 to 3 percent in 2000. However, almost all the countries in the region have declined their poverty levels being remarkable the case of Vietnam and Philippines which have halved the proportion of extremely poor people.

[12] National estimates of poverty rates for the three lines considered in this study are included in the Appendix 3.

Table 2.3. Global and regional poverty rates

	$1.25 a day				$1.45 a day				$2 a day			
	1990	1995	2000	Change	1990	1995	2000	Change	1990	1995	2000	Change
World	0.0922	0.0794	0.0622	-0.3254	0.1176	0.0987	0.0770	-0.3453	0.2007	0.1609	0.1248	-0.3781
EAP	0.1208	0.0697	0.0384	-0.6818	0.1603	0.0903	0.0503	-0.6861	0.2936	0.1620	0.0933	-0.6821
EEAC	0.0057	0.0296	0.0172	2.0115	0.0077	0.0376	0.0225	1.9394	0.0152	0.0645	0.0415	1.7299
LAC	0.0464	0.0449	0.0440	-0.0518	0.0555	0.0537	0.0525	-0.0533	0.0838	0.0808	0.0788	-0.0587
MENA	0.0234	0.0235	0.0164	-0.3005	0.0313	0.0313	0.0219	-0.3000	0.0612	0.0603	0.0431	-0.2962
SA	0.0970	0.0874	0.0586	-0.3957	0.1317	0.1169	0.0795	-0.3958	0.2529	0.2197	0.1565	-0.3811
SSA	0.3461	0.3643	0.3465	0.0012	0.3980	0.4169	0.4014	0.0085	0.5254	0.5451	0.5345	0.0173

The other Asian region also brings down its poverty levels significantly, from 10 percent in 1990 to 6 percent in 2000. The nineties were also characterized by a decrease in population living in a situation of moderate poverty. It is fair to say that all the countries in the region have declined their levels of poverty, with the exception of Nepal. However, the efforts of India in reducing poverty have played a main role in the advances of this region. The case of Pakistan should be also emphasized given that it has reduced its poverty by more than 75 percent.

The third region in population size, SSA has not shown positive signs of poverty eradication. At the beginning of the nineties, 35 percent of the population lived in extreme poverty. 10 years later, the situation remains unaltered, with a similar proportion of people living with less than $1.25 a day. Our results suggest that, taking the period as whole and irrespective to the definition used, poverty rates have remained almost constant. The stagnation of poverty hides uneven performance at national level which balanced in the aggregate. While poverty has been reduced in the majority of countries, the most populous nations, such as Nigeria, report an increase in poverty rates. These dynamics relegate SSA as the poorest region in the world.

MENA is the region that presents the lowest levels of poverty among developing regions. In 1990, 2.3 percent of the population of this region lived in absolute poverty, whereas this percentage has fallen to 1.6 in 2000. For the other lines used in this study poverty is also trending downward, resulting in a decrease by 1 percent. According to Adams and Page (2003), this reduction of the poverty is mainly due to international migration remittances and public employment along with the enhancement of the mean income levels. Our estimates indicate that poverty rates have been declined in the majority of the countries, being remarkable the performance of Yemen and Jordan. Conversely Morocco and Israel show positive trends in poverty rates, although the increase has not been significant.

Despite the significant growth rate of the mean income, Latin America does not present considerable progress in the eradication of poverty. The proportion of people living in extreme poverty decreased at slow rate, resulting in a reduction of 5.5

percent. Accordingly, the proportion of moderately poor individuals repots similar dynamics with reductions ranged from 5 to 6 percent. In fact, its poverty rate was three times lower than this indicator in EAP at the beginning of the period. In 2000, EAP outperforms Latin America which reported higher levels of poverty.

Finally, the economic situation in EECA was getting worse over the course of the studied period, showing little sign of possible reversal. While the transitional economies of this region began the nineties with low poverty levels, the extreme contraction in incomes that followed the fall of communism doubled the proportion of extremely poor people in 2000. It is worth noting that, despite this terrible economic performance, the poverty levels were so low in 1990 that the rate in 2000 is lower than 2 percent.

2.4.4. Regional and global income inequality

In this section we analyze how the changes in the shapes of regional and global distributions have affected the disparities within each territory. The study of income inequality has risen to prominence among academics and policy makers in recent years. In fact there is growing consensus that inequality within-countries, which is a weighted average of the internal disparities of each nation, increased during the postwar period (see *e.g.* Milanovic, 2005; Sala-i-Martin, 2006; Pinkovskiy and Sala-i-Martin, 2009; Anand and Segal, 2008). On the other hand, the between country component is the amount of inequality that would exist if there are no internal disparities, in the sense that all citizens of a particular country have the same level of income which would coincide with the per capita GDP. This is the so-called weighted inequality[13], whose decreasing trend over the past decades is a well-documented fact (see *e.g.* Decancq et al., 2009; Decancq, 2011; Milanovic, 2005; World Bank, 2006).[14]

[13] Another type of inequality is the so-called cross-country inequality which takes each nation as a unit of observation not considering the size of population of each country. It is obviously questionable that a country such as China, which represents one third of world population, counts the same as Brunei, one of the smallest countries in the world. Therefore we do not consider this type of inequality.

[14] A recent review on this topic can be found in Anand and Segal (2008).

Numerous inequality indices have been proposed in the literature[15]. Note that different measures attach different weights to some sections of the distribution. As a consequence, it is worth checking the robustness of the results to the choice of inequality index. In this study, we focus on the Generalized Entropy (GE) measures since they are additively decomposable in the two components described before, namely within and between country inequality. Such decomposition would provide additional information about the distributional dynamics of income during the nineties. At this point, it should be recalled that these measures have a sensibility parameter which attaches different weight to the top of the distribution depending on its value, thus providing infinity of possibilities to be considered. In this study, we have set the parameter to 0.5, 0.75, 1.5 and 1 which corresponds with the limit case of the GE measures, namely the Theil index which assigns the same weight to all parts of the distribution. To save in space, we present the complete results for the global distribution, whereas only the Theil index is reported for the regions[16].

Table 2.4. Global income inequality

	1990		1995		2000		Change
	Value	%	Value	%	Value	%	
Mean	7013.06		7298.6		8242.43		17.53
Theil	1.10766	100	1.0646	100	1.0292	100	-7.09
Within	0.28956	26	0.3374	32	0.3273	32	13.02
Between	0.81810	74	0.7271	68	0.7019	68	-14.20
Generalized Entropy (θ=0.5)	0.97010	100	0.9584	100	0.9379	100	-3.32
Within	0.23887	25	0.2836	30	0.2803	30	17.35
Between	0.73123	75	0.6748	70	0.6576	70	-10.08
Generalized Entropy (θ=0.75)	0.98048	100	0.9846	100	0.9697	100	-1.10
Within	0.26202	27	0.3107	32	0.3093	32	18.04
Between	0.71846	73	0.6740	68	0.6604	68	-8.09
Generalized Entropy (θ=1.5)	1.40511	100	1.5161	100	1.5348	100	9.23
Within	0.61974	44	0.7494	49	0.7709	50	24.40
Between	0.78537	56	0.7667	51	0.7639	50	-2.74

15 See Cowell (2011) for a detailed description of a broad range of inequality measures and the properties attributable to each one.
16 The values of the different inequality measures considered in the study for regions can be found in the Appendix 3.

The estimates of the different GE measures considered and their components for the three years are presented in Table 2.4 for the global distribution of income. The evolution of the sample mean suggests that the nineties are characterized as a period of economic growth at global level, given that the world per capita GDP has increased by nearly 18 percent. With regard to global inequality, we observe that if we attach larger weight to wealthy individuals (θ = 1.5), inequality tends to increase over the study period. In contrast, if we value more the transfers at the bottom of the distribution, inequality decreases by 1 to 7 percent. The different trends observed for different values of the sensitivity parameter suggest that no unambiguous conclusions can be achieved regarding the evolution of global inequality. This result is derived from the fact that we cannot establish a complete stochastic ordering for the income distributions in 1990 and 2000 (see Figure 2.5) and therefore the inequality measures not necessarily present the same dynamics.

It is observed that, irrespective to the value of the parameter, within-country inequality tends to increase over the course of the decade. This result is consistent with previous studies which point out that the internal disparities of the countries have soared since 1960 (Bourguignon and Morrison, 2002; Sala-i-Martin, 2006; Pinkovskiy and Sala-i-Martin 2009; Milanovic, 2005; World Bank, 2006). It should be noted that this trend has not been monotonic. In fact, the value of this component rose sharply during the first half of the period, peaks in 1995 – reaching values from 0.33 to 0.77– and then declines slightly in the second half of the nineties. Conversely, a continuously decreasing trend is observed for inequality between-countries.

It is worth noting that the intensity of the variation of both components depends on the measure considered, given the different weights assigned to the top of the distribution. Regarding within-country inequality, it is found that the increase has been remarkably higher when the parameter is set to 1.5 (close to 25 percent), while the lowest rate is achieved by the Theil index with an increase of 13 percent. For the between-country component, the greatest decline is observed for the Theil index, which reports a reduction of 14 percent, whereas the lowest fall, around 3 per cent, is achieved when we set the sensitivity parameter to 1.5. Therefore, in general terms, the decline of the between-country inequality has offset the increase in within-country disparities, thus resulting in a reduction of overall inequality. Note, however that, when we attach a high weight to the top incomes, the increase in within country inequality is so intense and the convergence process so weak that it is not enough to avoid the increase in global inequality.

The opposite evolution of both components has also affected the proportion of total inequality that is represented by each one. Note that, at the beginning of the period, inequality across countries accounted for nearly 75 percent of the global disparities in all cases, except for GE (θ = 1.5). Ten years later, the within-country component has gained weight at expense of between-country inequality but it continues playing the predominant role in the evolution global inequality, except when a high weight is attached to developed countries, where both components present an equal weighting scheme.

Let us focus on regional inequality patterns. The values of the Theil index and its components within and between are presented in Table 2.5, for each of the regions and years considered. Our results point out that SSA is the most unequal region in the world and its inequality levels have been continuously increasing, thus attenuating the unequal distribution of this region. As a positive sign, we can highlight that the mean income has increased by 2 percent over the nineties. Note however that, even when the most populous country in the region – Nigeria – has increased its inequality levels by 7 percent, the fall of disparities in other highly populated nations such as Ethiopia, Tanzania and Kenya with reductions of 18, 38 and 32 percent leads to the decrease in inequality within countries, hence reducing its share in overall inequality in favor of the disparities across countries. In contrast, the differences in income levels across countries have sharply increased over the study period, being the increase in African inequality a direct consequence of this process of divergence in mean income levels across countries.

Similar dynamics are observed for the second most unequal region, LAC. Despite the economic recovery that followed the lost decade and the macroeconomic stability, inequality levels soared during the course of the nineties. Such an upward trend of disparities was mainly due to the increase in differences in mean income across countries. This trend was reinforced by the rise of disparities within countries, which was driven by the rise of inequality in Colombia, Argentina and Peru. Note that the described behaviors of both components have not altered the proportions of each component significantly, being the within-country inequality the predominant factor in the overall disparities in LAC, which accounts for the 90 percent of inequality, while the differences between countries are seen as a residual component. This result emphasizes the fact that the rise of inequality in Latin America over the nineties is derived from important redistributions of wealth occurring within the borders of each nation.

EAP is characterized as the third most unequal region in 1990. Over the study period, its inequality levels have declined due to the convergence among countries that took place during the nineties. This strong decrease offset the increase in disparities within countries mainly driven by the substantial rise of inequality in China. The reduction of Asian inequality along with the increase in disparities in the rest of the regions has positioned this territory in the same level of MENA, with even better position than EECA. As a consequence of the aforementioned opposite trends of both terms, the proportion of between-country inequality declined substantially. It was 30 percent in 2000 whereas at the beginning of the period it accounted for almost 50 percent. Therefore overall inequality in this region is principally determined by internal disparities of the nations.

The other Asian region (SA) is characterized as the most equal territory over the whole period. Its level of inequality has remained constant, not experiencing significant changes in the weighting scheme of both inequality components. According to our estimates, most of inequality in SA is due to internal inequalities of the countries, whereas only around 9 percent comes from differences in the mean income levels of the countries in the region.

MENA has increased its inequality slightly due to the increase in the disparities within and between countries. It should be emphasized that in 1990, this region presented higher disparities than those of WENAO and EECA. Ten years later, this region reported inequality levels close to the disparities found in WENAO and lower than in EECA, given that these territories increased sharply their inequality levels. Note that this small variation has not affected the proportion of each of the components of inequality, which is dominated by the differences within countries.

One of the most dramatic cases is EECA, whose levels of inequality in 1990 were close to the disparities that we found in other developed regions such as WENAO. The fall of the communist and the subsequent transition to the market economy led to a sharp increase in disparities in the first half of the nineties, from 0.31 to 0.51, while the second part of the period was characterized by a decrease in inequality levels in this region, finally getting the value 0.45. Note however, that this evolution has resulted in a growth rate of 50 percent in just ten years. The remarkable increase in disparities is driven by large income redistributions within each nation that has led to the expansion of within-country inequality by 58 percent. In fact, only four out of 29 countries have reduced their internal inequality. It should be also noted that differences across countries have risen during the nineties (by slightly over 17 percent), also playing an important role in the increase in inequality in EECA. According to our estimates, overall inequality in this region mainly comes from within-country differences that represent the 70 percent of total disparities.

Finally, WENAO, the region that comprises most of the advanced economies, is characterized by low levels of inequality. Differences between countries have been reduced by 16 percent, though not sufficient to offset the increase in inequality within each nation, thus resulting in an increase in disparities in this region. The upward trend presented by the within-country term is mainly derived from the rise in internal disparities in some of the most populous countries such as United States and Germany. Note that these trends have not affected the proportion of total inequality represented by each component, which is strongly dominated by inequality within-countries.

Table 2.5. Regional income inequality. Theil index

		1990		1995		2000		Change
		Value	%	Value	%	Value	%	
East Asia and the Pacific	**Mean**	1757.43		2619.27		3350.72		90.66
	Theil	0.4679	100	0.4779	100	0.4319	100	-7.70
	Within	0.2525	54	0.3095	65	0.3027	70	19.89
	Between	0.2154	46	0.1684	35	0.1292	30	-40.04
Eastern Europe and Central Asia	**Mean**	8792.88		6427.31		7195.83		-18.16
	Theil	0.3129	100	0.5095	100	0.4480	100	43.20
	Within	0.1983	63	0.3815	75	0.3133	70	58.01
	Between	0.1146	37	0.1281	25	0.1347	30	17.59
Latina America and the Caribbean	**Mean**	6539.32		7038.05		7699.79		17.75
	Theil	0.5938	100	0.6066	100	0.6334	100	6.67
	Within	0.5302	89	0.5492	91	0.5557	88	4.80
	Between	0.0636	11	0.0574	9	0.0777	12	22.25
Middle East and North Africa	**Mean**	4200.2		4586.12		5247.04		24.92
	Theil	0.4183	100	0.4394	100	0.4317	100	3.19
	Within	0.2802	67	0.2955	67	0.2899	67	3.45
	Between	0.1381	33	0.1439	33	0.1418	33	2.66
South Asia	**Mean**	1362.21		1526.47		1783.03		30.89
	Theil	0.2235	100	0.2412	100	0.2302	100	3.02
	Within	0.2040	91	0.2200	91	0.2052	89	0.58
	Between	0.0195	9	0.0212	9	0.0250	11	28.59
Sub-Saharan Africa	**Mean**	1448.93		1393.25		1479.74		2.13
	Theil	0.7483	100	0.7521	100	0.7573	100	1.21
	Within	0.4574	61	0.4443	59	0.4336	57	-5.20
	Between	0.2909	39	0.3077	41	0.3238	43	11.28
Western Europe, North America and Oceania	**Mean**	26393.7		28194		32326.5		22.48
	Theil	0.3434	100	0.3899	100	0.4025	100	17.22
	Within	0.3186	93	0.3703	95	0.3818	95	19.85
	Between	0.0248	7	0.0196	5	0.0207	5	-16.50

2.5. Conclusions

In this study we derive income distributions for 127 countries in three benchmark years 1990, 1995 and 2000, using two versions of Singh-Maddala and Dagum distributions. We use limited data on mean income, extracted from national accounts, and the Gini index, which comes from microeconomic surveys. In a second stage, global and regional distributions are computed using a mixture of the two families considered in this study.

The objective of this chapter is twofold. On the one hand, we try to shed more light on the study of world distribution of income and its dynamics, using two-parameter families that are able to represent its main features adequately. Secondly, we analyze the evolution of poverty and inequality during the nineties, pointing out some interesting findings.

We have calculated three different poverty lines for each benchmark year, namely \$1.25 a day, \$1.45a day and \$2 a day. Our estimates of the cumulative distribution function show that the distribution in 2000 stochastically dominates the distribution in 1990 up to the higher poverty line. This result would indicate that irrespectively of the threshold considered, poverty declined at global level. According to our estimates, the proportion of people that live in extreme poverty has been reduced by 32 percent over the nineties.

Note however that the sharp decrease in global poverty rates hides uneven regional dynamics. On the one hand, the two Asian regions present an astonishing performance that has declined their poverty rates more than two thirds. The Asian success contrasts with the African tragedy which has failed in the eradication of poverty, showing poverty rates over 36 percent. The proportion of poor people in EECA has been doubled in only ten years, although its situation is not alarming since its initial levels were significantly low. As a consequence, poverty has become basically an African phenomenon, a region that comprises most of the global poor population. Therefore, the efforts to decrease the number of people living in extreme poverty need to be focused in this region. It should be also emphasized that, if poverty trends in EECA are not reversed, it will become a serious concern in few years.

The proposed methodology allows us to analyze different inequality measures. In particular, we compute generalized entropy measures, considering different values for the sensitivity parameter. Our results show a reduction of between-country inequality derived from the convergence in income experienced by some of the most populous countries, such as China and India, which present an astonishing growth of their per capita GDP over de study period. This dynamic has offset the terrible performance of African economies which diverged with respect to the leading economies. However, inequality within countries increased notably during the nineties although not enough to eclipse the improvements cross-country disparities. Consequently, our findings suggest a reduction in global inequality.

The global patterns, however, cannot be extrapolated to all regions, which are characterized by a mix of experiences. In fact, EAP is the only region that brought down its inequality over the study period, mainly due to the decrease in the inequality between countries. WENAO also presented a process of convergence among countries but it was too weak to offset the increase in internal disparities of nations. Conversely, SSA shows a modest decrease in the within-country component, which was not enough to bring down the inequality levels driven by the divergence process among African economies. The rest of the regions presented an increase in overall inequality derived from the increase in within-county component, a trend that was reinforced by the enhancement of disparities between countries.

Chapter 3

Modeling multidimensional Lorenz curves with applications to inequality in well-being

3.1. Introduction

The interest of academics in assessing country levels of well-being has shifted from an evaluation of solely economic aspects to a more comprehensive conception of such a process, which has an intrinsic multidimensional nature. In fact, there is nearly consensus that income is not an adequate indicator of well-being (see *e.g.* Sen, 1988; 1989; 1999), thus other factors need to be taken into account to evaluate this phenomenon effectively. The present economic research has stressed the importance of using more than one attribute in the study of inequality in well-being. The different works of Atkinson (2003), Atkinson and Bourguignon (1982), Kolm (1977), Maasoumi (1986), Slottje (1987) and Tsui (1995, 1999) move in this direction.

In multidimensional environments, the assessment of inequality across countries can present some difficulties and a wide range of options. Different approaches have been proposed to analyze multidimensional inequality in well-being. An intuitive procedure would be the construction of a composite index and then compute inequality measures of such an indicator (Pillarisetti, 1997; McGillivray and Pillarisetti, 2004; Martínez, 2012). This approach provides overall conclusions about the evolution of well-being although some problems can arise. First, it is supported that this methodology "*sweeps the multidimensional nature of well-being under the carpet*" (Decancq et al., 2009, pp. 14). Moreover, even when it is accepted that the composite index satisfies some qualitative properties which define it as a well-behaved index, there are a whole array of well-being indicators. As a consequence, subjective judgments play an important role and the arbitrariness of this choice comes with criticisms and disagreements.

To avoid making arbitrary decisions about the functional form of the index, the simplest alternative is to calculate inequality in each dimension independently (Hobin and Franses, 2001; Neumayer, 2003; World Bank, 2006; McGillivray and Markova, 2010). Obviously, this method provides additional insights rather than a sole focus on income, thus allowing us to extract conclusions about the existing disparities within each dimension, which is the so-called *distribution sensitive inequality* (Kolm, 1977). Note, however, that, when some indicators have worsened their inequality levels and others have reduced their disparities, it is not possible to draw integral conclusions about the evolution of inequality using the dimension-by-dimension approach. Furthermore, this methodology ignores the relationship between dimensions, more specifically the degree of association among them which is known as *association sensitive inequality* (Atkinson and Bourgignon, 1982).

The fact that inter-dimensional association has a strong influence in the assessment of disparities has been repeatedly argued in the literature (Tsui, 1995; 1999; 2002; Bourguignon and Chakravarty, 2003). Consequently, among the different approaches proposed in the literature to measure inequality in well-being, the most satisfactory one seems to be the use of multidimensional inequality measures since this methodology takes into account inequality within each dimension and the degree of association among them[17]. However, as in the unidimensional case, these measures only offer overall conclusions about the evolution of well-being distribution.

[17] See McGillivray and Shorrocks (2005) for a review on this topic.

At this point, it should be recalled that greater values of a particular inequality measure, *e.g.* the Gini index, does not imply that the whole distribution has worsened. It could be possible that poor countries have a more unequal situation while the wealthiest economies were found more equally distributed. To draw some conclusions about these dynamics multidimensional stochastic dominance conditions have been developed in the literature (Atkinson and Bourguignon, 1982; Duclos et al., 2011; Muller and Trannoy., 2011), since it is considered as a preliminary task before the estimation of multidimensional inequality indices (Muller and Trannoy, 2011). In fact, concluding stochastic dominance implies that the whole distribution is less unequal and hence calculating inequality measures provides integral conclusions about distributional dynamics of well-being. However, if no dominance relationships are observed, we could only obtain summarized information about the evolution of disparities in well-being levels. In that case, additional tools are needed to state which parts of the distribution are more equal and which ones have worsened in terms of inequality.

In the unidimensional case, the Lorenz curve provides relevant insights about the evolution of different parts of the distribution and it has been widely used for studying economic inequality as well as the distribution of non-income variables[18]. However, the extension of the univariate Lorenz curve to higher dimensions is not an obvious task. The three existing definitions were proposed by Taguchi (1972a, b), Arnold (1983) and Koshevoy and Mosler (1996), who introduced the concepts of Lorenz zonoid and Gini zonoid index.

In this chapter, using the definition proposed by Arnold (1983), closed expressions for the bivariate Lorenz curve are given, using a flexible model for the underlying bivariate distribution. We study a relevant type of models based on a class of bivariate distributions with given marginals described by Sarmanov and Lee (Lee, 1996; Sarmanov, 1966). This specification presents several advantages. In particular, the expression of the bivariate Lorenz curve can be easily interpreted as a convex linear combination of Lorenz and concentration curves. A closed expression for the bivariate

[18] For a review, see Anand and Segal (2008).

Gini index (Arnold, 1987) is given in terms of the Gini and concentration indices of the marginal distributions. This index is especially useful, and can be decomposed in two factors, corresponding to the equality within and between variables. Closed expressions of these statistics are provided using the beta distribution as a theoretical model for marginal distributions. We apply the previous procedure to data on health, education and income in order to assess the evolution of multidimensional inequality in well-being over the last three decades.

The contents of the chapter are the following. In Section 3.2 we present preliminary results including the definition of univariate Lorenz and concentration curves and the general definition of bivariate Lorenz curve proposed by Arnold (1987). In Section 3.3 we introduce the bivariate Sarmanov-Lee Lorenz curve and we obtain a nice closed expression for the curve and its corresponding bivariate Gini index. A decomposition of this index in two factors is given, which correspond to the equality within and between variables. A suitable model of bivariate Lorenz curves for a class of well-being indicators and the estimation procedure are presented in Section 3.4. An application to measurement of multidimensional inequality in well-being is given in Section 3.5. Finally, Section 3.6 concludes the chapter.

3.2 An extension of the univariate Lorenz curve

We start defining the concepts of Lorenz curve and concentration curve (Kakwani, 1977) for the univariate case. Denote the class of univariate distribution functions with positive finite expectations by $\mathcal{L}$ and denote by $\mathcal{L}+$ the class of all distributions in $\mathcal{L}$ with $F(0) = 0$ corresponding to non-negative random variables. We use the following definition by Gastwirth (1971).

Definition 3.1. *The Lorenz curve L of a random variable X with cumulative distribution function $F \in \mathcal{L}$ is*

$$L(u;F) = \frac{\int_0^u F^{-1}(y)\,dy}{\int_0^1 F^{-1}(y)\,dy} = \frac{\int_0^u F^{-1}(y)\,dy}{E[X]}, \quad 0 \le u \le 1, \tag{3.1}$$

where

$$F^{-1}(y) = \sup\{x : F(x) \le y\}, \qquad 0 \le y \le 1,$$

$$= \sup\{x : F(x) < 1\}, \qquad y = 1,$$

is the right continuous inverse distribution function or quantile function corresponding to F.

Definition 3.2. *Let g(x) be a continuous function of x such that its first derivative exists and g(x) ≥ 0. If the mean* $E_F[g(X)]$ *exits, then one can define*

$$L_g(y;F) = \frac{\int_0^x g(x)\,dF(x)}{E_F[g(X)]},$$

where y = g(x), f(x) and F(x) are respectively the probability density function (PDF) and the cumulative distribution function (CDF) of the random variable X.

The implicit relation between $L_g(g(x);F)$ and $F(x)$ will be called the concentration curve of the function $g(x)$. It is worth noting that, denoting the elasticity of $g(x)$ as $\eta_g(x)$, the concentration curve of the function $F(x)$ will lie above (below) the egalitarian line if $\eta_g(x)$ is less (greater) than zero for all $x \ge 0$ (see Corollary 1 in Kakwani (1977)). The concentration curve admits the simple implicit representation,

$$L_g(u;F) = \frac{1}{E_F[g(X)]} \int_0^u g\left[F^{-1}(t)\right] dt,$$

which will be used in the next sections.

Analogously to the Lorenz curve, we can derive an index from the concentration curve which, in contrast to the Gini index, can be negative if the area above the egalitarian line is greater than the area below the egalitarian line. According to Kakwani (1977), if $g(x) \geq 0$ for all x, then the concentration index is positive and equal to the Gini index of $g(x)$. For the case $g(x) < 0$ for all x, the concentration curve lies completely above the egalitarian line and consequently, its associated concentration index is negative and equal to minus the Gini index of $g(x)$. Finally, if $g(x)$ is not monotonic, the concentration index is ranged from minus the Gini index of $g(x)$ and the Gini index of $g(x)$.

There have been few attempts to extend the concept of Lorenz curve to higher dimensions. The first approach was proposed by Taguchi (1972a; 1972b; 1988). This definition is not considered in this study given that the curve is not symmetric and its extension to higher dimensions does not look simple. Another proposal was presented by Koshevoy (1995) and investigated thereafter by Koshevoy and Mosler (1996, 1997) who introduced the concepts of Lorenz zonoid and its associated Gini index. Recently, dominance relationships have also been developed (Koshevoy and Mosler, 2007). Even when this is the most popular extension of the Lorenz curve, it is not suitable for this analysis since its parameterization seems to be extremely complex. Instead, we use the definition proposed by Arnold (1983, 1987) for constructing a bivariate Lorenz curve given that its structure is especially suitable to handle with parametric models and it can be easily extended to the multidimensional space, although for simplicity, we consider the bivariate version of the curve.

Let $\mathbf{X} = (X_1, X_2)^T$ be a bivariate random variable with bivariate probability distribution function F_{12} on $\Re_+^2$ having finite second and positive first moments. We denote by F_i, $i = 1, 2$ the marginal CDF corresponding to X_i, $i = 1, 2$ respectively.

Definition 3.3. *The Lorenz surface of F_{12} is the graph of the function,*

$$L(u_1,u_2;F_{12})=\frac{\int_0^{s_1}\int_0^{s_2} x_1 x_2 \; dF_{12}(x_1,x_2)}{\int_0^{\infty}\int_0^{\infty} x_1 x_2 \; dF_{12}(x_1,x_2)}, \tag{3.2}$$

where

$$u_1=\int_0^{s_1} dF_1(x_1), \quad u_2=\int_0^{s_2} dF_2(x_2), \quad 0\le u_1,u_2\le 1.$$

The two-attribute Gini-Arnold index $GA(F_{12})$ is defined as,

$$GA(F_{12})=4\int_0^1\int_0^1 \left[u_1 u_2 - L(u_1,u_2;F_{12})\right] du_1 du_2, \tag{3.3}$$

where the egalitarian surface is given by $L_0(u_1,u_2;F_0)=u_1 u_2$.

As the previous definition of the Gini index has not been explored in detail in the literature, we highlight some nice properties presented by this indicator:

1. The marginal Lorenz curves can be obtained as $L(u_1;F_1)=L(u_1,\infty;F_{12})$ and $L(u_2;F_2)=L(\infty,u_2;F_{12})$.

2. The bivariate Lorenz curve does not depend on changes of scale in the marginals.

3. If F_{12} is a product distribution function (which implies independence between variables), then

$$L_0(u_1,u_2;F_{12}) = L(u_1;F_1)\,L(u_2;F_2),$$

which is just the product of the marginal Lorenz curves.

4. We denote by F_a the one-point distribution at $a \in \Re_+^2$, that is, the egalitarian distribution at *a*. Then, the egalitarian distribution has bivariate Lorenz curve $L(u_1,u_2;F_a)=u_1 u_2$.

5. In the case of a product distribution (independence between variables), the two-attribute Gini-Arnold defined in (3.3) can be written as,

$$1-GA\,(F_{12})=\left[1-G(F_1)\right]\left[1-G(F_2)\right].$$

3.3 The bivariate Sarmanov-Lee Lorenz curve

The Arnold´s Lorenz curve (3.2) can be evaluated implicitly in some relevant bivariate families of distributions. As a previous step, we need to introduce an explicit expression for Arnold's bivariate Lorenz curve since many of our results are based on this version of the curve.

Lemma 3.1. *The bivariate Lorenz curve can be written in the explicit form,*

$$L(u_1,u_2;F_{12})=\frac{1}{E\left[X_1X_2\right]}\int_0^{u_1}\int_0^{u_2}A(x_1,x_2)\,dx_1dx_2,\quad 0\le u_1,\,u_2\le 1, \tag{3.4}$$

where

$$A(x_1,x_2)=\frac{F_1^{-1}(x_1)F_2^{-1}(x_2)f_{12}(F_1^{-1}(x_1)F_2^{-1}(x_2))}{f_1(F_1^{-1}(x_1))f_2(F_2^{-1}(x_2))}. \tag{3.5}$$

Proof: The proof is direct making the change of variable $(u_1,u_2)=(F_1(x_1),F_2(x_2))$ in (3.2).

Note that for either specification of the Arnold Bivariate Lorenz curve (Equations (3.2) and (3.4)), we need to define the structure of dependence between variables given by F_{12}. In this work we propose to use the distribution derived from the Sarmanov-Lee copula which presents several advantages in relation with other models. Its joint PDF and CDF are quite simple and its different probabilistic features (moments, conditional distributions) can be obtained in a explicit form. On the other hand, the covariance structure, in general, is not limited in the sense that it includes correlations ranged from -1 to 1. This model considers the case of independence. Additionally, the Sarmanov-Lee distribution includes several relevant special cases including the classical Farlie-Gumbel-Morgenstern distribution and the variations proposed by Huang and Kotz (1999) and Bairamov and Kotz (2003).

Let $\mathbf{X} = (X_1, X_2)^{\mathrm{T}}$ be a random variable that follows a Sarmanov-Lee distribution with joint PDF,

$$f(x_1, x_2) = f_1(x_1) f_2(x_2) \left\{1 + \omega\phi_1(x_1)\phi_2(x_2)\right\}, \tag{3.6}$$

where $f_1(x_1)$ and $f_2(x_2)$ are the univariate PDF marginals, $\phi_i(t), i = 1, 2$ are bounded non-constant function such that,

$$\int_{-\infty}^{\infty} \phi_i(t) f_i(t) dt = 0, \quad i = 1, 2,$$

and ω is a real number which satisfies the condition $1 + \omega\phi_1(x_1)\phi_2(x_2) \geq 0$ for all x_1, and x_2.

We denote $\mu_i = E[X_i] = \int_{-\infty}^{\infty} t\, f_i(t)\, dt,\ i = 1,\ 2,$ $\sigma_i^2 = \mathrm{var}[X_i] = \int_{-\infty}^{\infty} (t - \mu_i)^2 f_i(t)\, dt, i = 1, 2$ and $v_i = E[X_i\, \phi_i(X_i)] = \int_{-\infty}^{\infty} t\phi_i(t) f_i(t)\ dt,\ i = 1, 2$. Properties of this family have been explored by Lee (1996). Moments and regressions of this family can be easily obtained. The product moment is $E[X_1 X_2] = \mu_1\mu_2 + \omega v_1 v_2$, and the regression of X_2 on X_1 is given by,

$$E\left[X_2 \middle| X_1 = x_1\right] = \mu_2 + \omega v_2\, \phi_1(x_1)$$

Note that the proposed distribution (3.6) has two components: a first component corresponding to the marginal distributions and the second component which defines the structure of dependence, given by the parameter ω and the functions $\phi_i(u)$, $i=1,2$. These two components will be translated to the structure of the associated bivariate Lorenz curve, and the corresponding bivariate Gini index.

The bivariate Sarmanov-Lee Lorenz curve is obtained using the distribution in (3.6) in the explicit version of the Arnold Lorenz curve (3.4).

Theorem 1 *Let* $\mathbf{X}=(X_1, X_2)^T$ *a bivariate Sarmanov-Lee distribution with joint PDF (3.6), with non-negative marginals satisfying* $E[X_1]<\infty, E[X_2]<\infty$ *and* $E[X_1X_2]<\infty$. *Then, the bivariate Lorenz curve is given by,*

$$L_{SL}(u_1,u_2;F_{12})=\pi L(u_1;F_1)L(u_2;F_2)+(1-\pi)L_{g_1}(u_1;F_1)L_{g_2}(u_2;F_2), \qquad (3.7)$$

where

$$\pi=\frac{\mu_1\mu_2}{E[X_1X_2]}=\frac{\mu_1\mu_2}{\mu_1\mu_2+wv_1v_2},$$

and $L(u_i;F_i)$, $i=1,2$ *are the Lorenz curves of the marginal distribution* $X_i, i=1,2$ *respectively, and* $L_{g_i}(u_i;F_i)$, $i=1,2$ *represent the concentration curves of the random variables,* $g_i(X_i)=X_i\phi_i(X_i)$ $i=1,2$, *respectively.*

Proof: The function (3.5) for the Sarmanov-Lee distribution can be written of the form,

$$A_{SL}(x_1,x_2)=F_1^{-1}(x_1)F_2^{-1}(x_2)\left\{1+w\phi_1(F_1^{-1}(x_1))\phi_2(F_2^{-1}(x_2))\right\},$$

and integrating in the domain $(0,u_1)\times(0,u_2)$ we obtain,

$$\mu_1\mu_2 L(u_1;\ F_1)L(u_2;F_2)+wE_{F_1}[g_1(X_1)]E_{F_2}[g_2(X_2)]\,L_{g_1}(u_1;F_1)L_{g_2}(u_2;F_2),$$

and after normalization we obtain (3.7).

The interpretation of Equation (3.7) is quite direct: the bivariate Lorenz curve can be written as a convex linear combination of two components: a first component corresponding to the product of the marginal Lorenz curves (marginal component) and a second component corresponding to the product of the concentration Lorenz curves (dependence component).

As in the unidimensional case, it is possible to derive the Gini index from the expression of the Lorenz curve. The following result provides a convenient write of the two-attribute bivariate Gini defined in (3.3). This expression permits a simple decomposition of the overall equality (1 - G (F_{12})) in two factors: a first factor which represents the equality within variables (associated with the concept of *distribution sensitive inequality* (Kolm, 1977)) and a second factor which represents the equality between variables (related to the so-called *association sensitive inequality* (Atkinson and Bourgignon, 1982)).

Theorem 2 *Let* $\boldsymbol{X}= (X_1, X_2)^T$ *be a bivariate random variable that follows a Sarmanov-Lee distribution with bivariate Lorenz curve* $L(u_1,u_2;F_{12})$. *The two-attribute bivariate Gini index defined in (3.3) is given by,*

$$1-G(F_{12})=\pi[1-G(F_1)]\cdot[1-G(F_2)]+(1-\pi)[1-G_{g_1}(F_1)][1-G_{g_2}(F_2)], \tag{3.8}$$

where $G(F_i)$ i = 1, 2 *are the Gini indices of the marginal Lorenz curves, and* $G_{gi}(F_i)$, i = 1, 2 *represent the concentration indices of the concentration Lorenz curves* $L_{g_i}(u_i,F_i)$, i = 1, 2.[19]

Proof: The proof is direct using expression (3.7) and taking into account that $G(F_{12})=1-4\int_0^1\int_0^1 L(u_1,u_2,F_{12})\,du_1du_2$.

Then the overall equality (OE), given by $1-G(F_{12})$, can be decomposed into two factors,

[19] Note that this index is interpreted as the classical Gini only when the concentration curve is convex, that is when there is a positive association between attributes (then the concentration indices are positive), being the Gini index ranged from 0 to 1 (since $0 \leq OE \leq 1$ and $0 \leq EW \leq 1$ and in consequence $0 \leq EB \leq 1$). Otherwise, concentration indices of each dimension can be negative.

$$OE = EW + EB, \tag{3.9}$$

where

$$OE = 1 - G(F_{12}),$$

$$EW = \pi[1 - G(F_1)][1 - G(F_2)],$$

$$EB = (1-\pi)[1 - G_{g_1}(F_1)][1 - G_{g_2}(F_2)].$$

EW represents the equality within variables and the second factor *EB* represents the equality between variables which includes the structure of dependence of the underlying bivariate income distribution through the functions g_i, i =1, 2. Therefore, *EB* informs about the degree of association between dimensions. It has been repeatedly emphasized that the dependence between dimensions has a significant influence in the assessment of disparities (Decancq and Lugo, 2012; Duclos et al., 2006; 2011; Kovacevic, 2010a; Seth, 2013; Tsui, 1995; 1999; Bourguignon and Chakravarty, 2003).

Let us illustrate the influence of inter-dimensional association in the measurement of multidimensional inequality with the following example. Consider that we are interested in evaluating cross-country inequality based on two indicators of well-being, *e.g.* income and education. Suppose that we are considering two regions *A* and *B*, made up of two countries (C_{1A}, C_{2A}) and (C_{1B}, C_{2B}) respectively, which result in the following distributional matrices:

$$\mathbf{Z}_A = \begin{bmatrix} 70 & 80 \\ 30 & 20 \end{bmatrix} \quad \mathbf{Z}_B = \begin{bmatrix} 70 & 20 \\ 30 & 80 \end{bmatrix},$$

where the *j*th column is the distribution of the *j*th indicator and the *i*th row includes the amount of attributes that the *i*th country has. Note that Z_A is derived from Z_B by switching the amounts of the second variable (education) between the countries considered.

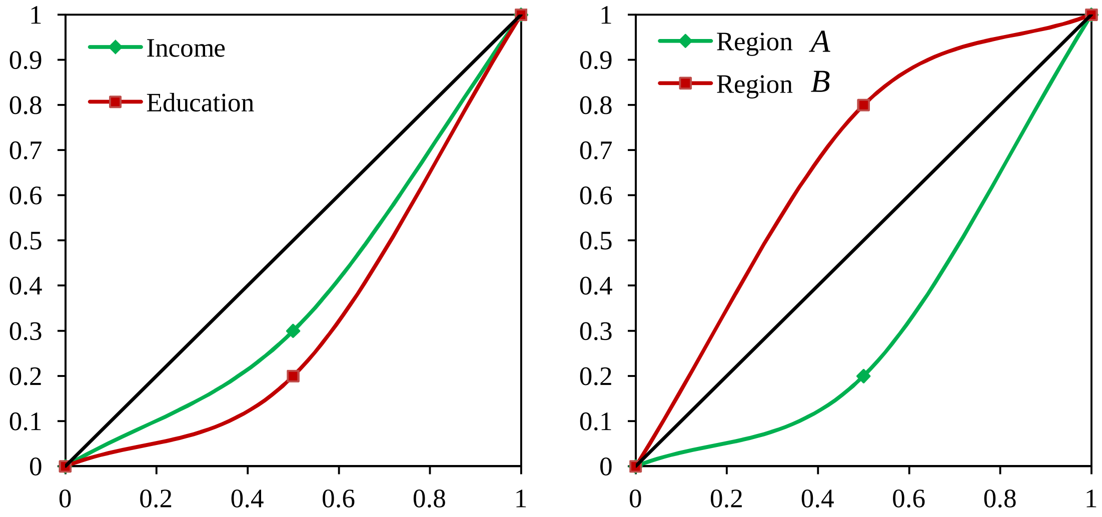

Figure 3.1. Lorenz curves of income and education (left graph) and concentration curves of education of regions Z_A and Z_B (right graph).

The Lorenz curves for both dimensions are plotted in Figure 3.1 (left graph), showing that income is less unequal than education. At this point, it is important to recall the interpretation of these curves. Naturally, in the case of income, the Lorenz curve informs about the percentage of income owned by the poorest k percent of the population ($k \in [0, 100]$). Conversely, the interpretation for the education variable reveals the percentage of education possessed by the least educated k percent of people. It is worth noting that this graph is exactly the same for regions A and B, given that both groups of countries have the same level of inequality in education and income. Therefore, a dimension-by-dimension approach would conclude that these distributions are equally unequal.

However, since distributional matrices Z_A and Z_B are different, a feasible inequality analysis must offer different results for both regions, differences that come from variations in the degree of correlation between dimensions. As supported by Duclos et al. (2011), it is ethically accepted that inequality is higher in the region A since C_{2A} is relatively worse-off in terms of both attributes, whereas in B, C_{2B} has lower level of education and C_{1B} lower level of income. Therefore, not considering the patterns between dimensions would imply to abandon one of the main motivations for measuring multidimensional inequality (Decancq and Lugo, 2012).

The proposed methodology accounts for both types of inequality using the concentration curves of each dimension. Let us illustrate this using the simple example of regions *A* and *B*. Figure 3.1 (right graph) plots the concentration curves of education with respect to the income component. It should be noted that the interpretation of the concentration curve is slightly different from the information provided by the classical Lorenz curve. In this case, concentration curves inform about the percentage of education possessed by the poorest *k* percent of population. Therefore, this curve gives information about the relationship between these variables. In fact, when the concentration curve is concave, it indicates that a negative relationship exists between both attributes, thus resulting in a negative value of the concentration index (Kakwani, 1977). Conversely, convex curves are related to positive concentration indices. Using (3.8), it is directly concluded that the region B is more equal than A, given that the first component is the same for both regions while the second is positive for the region *A* and negative for the region *B*. Therefore, our methodology not only accounts for the two different types of inequality, it also quantifies the influence of each one in the overall multidimensional inequality.

3.4. Deriving the bivariate Lorenz curve for a class of well-being indices

In this section we consider a relevant model based on the previous methodology to study the distribution of well-being as a multidimensional process. Specifically, we take the Human Development Index (HDI) as a theoretical benchmark[20] since this approach allows us to compare and complement our results with previous studies.

[20] The HDI has been highly criticized on the grounds of construction (Grimm et al., 2008; Kelley, 1991), selection of variables (Srinivasan, 1994), arbitrary weighting scheme (McGillivray and White, 1993; Noorbakhsh, 1998), and redundancy with its components (Cahill, 2005; McGillivray, 1991, McGillivray and White, 1993; Ravallion, 1997). These criticisms suggest that the HDI is not an ideal indicator of development. However, it should be emphasized that the conception of human development is complex, abstract and difficult to synthesize. Independently of its limitations, this indicator attracts a great amount of attention from the media and politicians due to its simplicity, transparency and capacity to capture the most important aspects of well-being.

Therefore, in this chapter we assume that well-being at country level focuses on the three dimensions[21] considered in the HDI: income, health and educational attainment. These components, placed on a scale 0 to 1, are transformed indicators of the original variables, namely GNI per capita, life expectancy and the geometric average of expected years of schooling and mean years of schooling. Finally, the HDI is constructed using a geometric mean of the three transformed variables.

Before going any further, it should be emphasized that the bivariate Lorenz curve defined in (3.7) is especially suitable for modeling inequality in the HDI given the construction formula of this indicator, characterized by a multiplicative scheme. Notwithstanding the especial case of the HDI, the bivariate Lorenz curve can be used to measure inequality in other kinds of variables if the marginal distributions are adequately modeled. In this case, given that the indicators considered are ranged from 0 to 1, the beta distribution seems to be the optimal model in this case. Then, we define the bivariate Lorenz curve based on the Sarmanov-Lee distribution considering the beta distribution as model for marginal distributions.

Let $X_i \sim \mathcal{B}e(a_i, b_i)$, $i = 1, 2$ be two classical beta distributions with PDF,

$$f_i(x_i; a_i, b_i) = \frac{x_i^{a_i - 1}(1 - x_i)^{b_i - 1}}{B(a_i, b_i)}, \, 0 \le x_i \le 1, \quad i = 1, 2,$$

where $B(a_i, b_i) = \Gamma(a_i)\Gamma(b_i)/\Gamma(a_i + b_i)$ for $i = 1, 2$. This distribution has been proposed as a model of income distribution by McDonald (1984) and more authors. If we consider the mixing functions $\phi_i(x_i) = x_i - \mu_i$, where $\mu_i = E[X_i] = a_i / a_i + b_i$, $i = 1, 2$, the bivariate Sarmanov-Lee distribution is,

[21] A whole array of indicators have been proposed in the literature to be considered in the measurement of well-being, including education (Morrison and Murtin, 2012), health (Bourguignon and Morrison, 2002; Becker et al., 2005), security (Lawson-Remer et al., 2009; Bilbao-Ubillos, 2013), democracy (Cornwall and Gaventa, 2009; Domínguez et al., 2011), environmental questions (Neumayer, 2001; Briassoulis, 2001) and distributional aspects (Alkire and Foster, 2010; Hicks, 1997; Seth, 2013). Note that the selection of dimensions is not only a technical choice, and it also lead to normative implications.

$$f_{12}(x_1,x_2) = f_1(x_1;a_1,b_1)\,f_2(x_2;a_2,b_2)\left\{1+\omega\left(x_1 - \frac{a_1}{a_1+b_1}\right)\left(x_2 - \frac{a_2}{a_2+b_2}\right)\right\}, \qquad (3.10)$$

where ω satisfies,

$$\frac{-(a_1+b_1)(a_2+b_2)}{\max\{a_1a_2, b_1b_2\}} \le \omega \le \frac{(a_1+b_1)(a_2+b_2)}{\max\{a_1a_2, b_1b_2\}},$$

A good property of this family is that it can be expressed as a linear combination of products of univariate beta densities. The univariate Lorenz curve of the classical beta distribution if given by (Sarabia, 2008),

$$L(u_i, F_i) = G_{Be(a_i+1,b_i)}[G^{-1}_{Be(a_i,b_i)}(u_i)], \quad i = 1,2, \qquad (3.11)$$

where $G_{Be(a,b)} = \left(1/B(a,b)\right)\int_0^z t^{a-1}(1-t)^{b-1}dt$ represents the CDF of the classical beta distribution. In consequence, the concentration curve can be written as,

$$L_{g_i}(u_i,F_i) = \frac{E[X_i^2]G_{Be(a_i+2,b_i)}[G^{-1}_{Be(a_i,b_i)}(u_i)] - E[X_i]^2 L(u_i,F_i)}{var[X_i]}, \; i = 1,2. \qquad (3.12)$$

Note that $v_i = E_{F_i}[X_i\phi_i(X_i)] = var[X_i], i = 1,2$. Finally, combining (3.11) with (3.12) in (3.7), we obtain the bivariate beta Lorenz curve.

This model can be extended easily to the Sarmanov-Lee distribution with generalized beta of the first type (GB1) marginals, with PDF,

$$f_i(x_i;a_i,b_i,p_i) = \frac{p_i x_i^{p_ia_i-1}(1-x_i^{p_i})^{b_i-1}}{B(a_i,b_i)}, \; 0 \le x_i \le 1, \; i = 1,2,$$

And mixing function,

$$\phi(x_i) = x_i - \mu_i = x_i - \frac{\Gamma(a_i+1/p_i)\Gamma(a_i+b_i)}{\Gamma(a_i+b_i+1/p_i)\Gamma(a_i)}, \; i = 1,2.$$

3.4.1. Estimation methods

Before describing the estimation procedure for the statistical tools developed in this chapter, it is necessary to clarify what is actually measured when we refer to multidimensional inequality in well-being. Due to the scarcity of individual data on non-income dimensions for long periods of time and for most countries, we use country-level data, which automatically implies that within country disparities are being ignored. This situation gives us two possible alternatives: we can consider countries as units of observation (unweighted inequality) or we can assume that all the citizens of a given country have the same level of well-being, thus being individuals the subjects of our analysis (population weighted inequality) (Milanovic, 2005). Both approaches are widely used in the literature on income inequality which also points out the advantages and shortcomings of each one.

In this study we focus on unweighted inequality, thus implying that each country counts the same in the global distribution, irrespective of its demographic weight, due to the following reasons. First of all, we are studying inequality in well-being as a multidimensional process, which also considers education and health. Note that the distributions of these attributes are strongly conditioned by public policies which are equally implemented all over the country. Then, the countries can be seen as a set of policies (Decancq et al., 2009) whose effectiveness would contribute developing countries to catch up the advanced nations. Secondly, weighted inequality is extremely sensitive to the performance of the most populous countries such as China and India[22]. Last but not least, one of the main objectives of this study is to compare our results with the classical approach of the economic inequality and with the results presented by previous studies. Consequently, less ambiguous conclusions can be extracted considering unweighted inequality, given that population growth plays a crucial role in the evolution of weighted measures (Firebaugh, 2000).

[22] In the case of income, it is widely accepted that weighted inequality has been reduced over the last two decades. However, if China and India are removed from the sample, an increase in income disparities is concluded (see *e.g.* Milanovic, 2005). In fact, it is also observed that poverty has increased in most countries of Sub-Saharan Africa, thus widening the gap between these nations and the most advanced economies. Therefore, global inequality has increased but outstanding performances of China and India have masked this fact.

Under the theoretical benchmark of the HDI, three different dimensions are considered. Unfortunately, only bidimensional Lorenz curves can be plotted due to obvious limitations. It should be stated, however, that this fact does not restrict our analysis strongly, since compensations between health and education seem to have lack of interest in practice (Muller and Trannoy, 2011). These authors give some arguments to assume that education and health are not dependent on each other, based primarily on political reasons. Generally, policies targeted to health improvements and education programs are nationally implemented by different ministries. Such a division can also be observed at supranational level, having international organizations, such as the World Bank, different departments for health and education. Assuming both attributes independent would imply that an increase in income would compensate low levels of health or a poor performance in education. However, it is practically and politically less attractive that high educational levels would be used as a substitutes of low health standards. On the other hand, it is absolutely valid to assume that better educated people have more possibilities to enjoy a healthy life, so we also consider the possibility that these two dimensions are related[23].

As we stated previously, the theoretical bivariate distribution for modeling two of the components of the HDI is the Sarmanov-Lee distribution with classical beta marginals. We estimate these parameters using the maximum likelihood procedure. First, we describe the estimation of the parameters based on the method of moments which are used as a initial estimates in the maximum likelihood methodology.

Let $\mathbf{X} = (X_1, X_2)^{\mathrm{T}}$ be a bivariate distribution with joint PDF given by Equation (3.10). Let $(x_{11}, x_{21}), \ldots, (x_{1n}; x_{2n})$ a sample of size n from (3.10). For the estimation of the parameters $(a_1; b_1; a_2; b_2; \omega)$, we proceed in two steps:

1. Estimation of the marginal distributions. We define,

$$m_i = \frac{1}{n}\sum_{j=1}^{n} x_{ij},\; s_i^2 = \frac{1}{n}\sum_{j=1}^{n}(x_{ij}-m_i)^2,\; i = 1,2.$$

[23] Including this assumption can lead to incongruent results given that it would imply that international aid to improve health conditions would be targeted to most educated countries, thus increasing inequality levels (Muller and Trannoy, 2011).

And then, the point estimates of the couples (a_i, b_i), i = 1, 2 are,

$$\hat{a}_i = \frac{m_i(m_i - m_i^2 - s_i^2)}{s_i^2},$$

$$\hat{b}_i = \frac{(1-m_i)(m_i - m_i^2 - s_i^2)}{s_i^2},$$

with i = 1, 2.

2. Estimation of the structure of dependence. The estimate of ω parameters is based on the sample relation $\rho = \omega\sigma_1\sigma_2$. Then, if r denotes the sample linear correlation coefficient, and s_i, i = 1, 2, the sample standard deviation of the marginal distributions X_i, i = 1, 2, the point estimate of ω is,

$$\hat{\omega} = \frac{r}{s_1 \cdot s_2}.$$

The previous estimates $\hat{a}_i, \hat{b}_i$, i = 1, 2 and $\hat{\omega}$ are used as initial estimates in the maximum likelihood estimation. If we have a sample of n individuals (x_{1i}, x_{2i}), i = 1,…, n, the maximum likelihood estimation consists in maximizing the log-likelihood function and solving for the parameters a_1, b_1, a_2, b_2 and ω. The likelihood function is given by the following expression:

$$L(a_1, b_1, a_2, b_2, \omega; \underline{x_1}, \underline{x_2}) = \prod_{i=1}^{n} f_{12}(x_{1i}, x_{2i}; a_1, b_1, a_2, b_2, \omega).$$

Using the maximum likelihood estimates of the parameters a_i and b_i, i = 1, 2, the Gini index of the marginal distributions can be obtained by the formula,

$$G(Be(a_i, b_i)) = \frac{\Gamma(a_i + b_i)\Gamma\left(a_i + \frac{1}{2}\right)\Gamma\left(b_i + \frac{1}{2}\right)}{\Gamma\left(a_i + b_i + \frac{1}{2}\right)\Gamma(a_i + 1)\Gamma(b_i)\sqrt{\pi}}, \qquad (3.13)$$

and the concentration indices can be computed using the following expression:

$$G(F_{12}) = 1 - 2\int_0^1 L_{g_i}(u_i, F_i)\, dF_i\,, \tag{3.14}$$

where $L_{g_i}(u_i, F_i)$ is defined in (3.12).

Finally the parameter π, which determines the weight of the inequality within dimensions in the overall multidimensional inequality, is given by,

$$\pi = \frac{\left(\dfrac{a_1}{a_1+b_1}\right)\left(\dfrac{a_2}{a_2+b_2}\right)}{\left(\dfrac{a_1}{a_1+b_1}\right)\left(\dfrac{a_2}{a_2+b_2}\right) + \omega\left(\dfrac{a_1 b_1}{(a_1+b_1)^2(1+a_1+b_1)}\right)\left(\dfrac{a_1 b_1}{(a_1+b_1)^2(1+a_1+b_1)}\right)}. \tag{3.15}$$

The computation of the bivariate Gini index in (3.8) is straightforward using Equations (3.13), (3.14) and (3.15).

3.5. Multidimensional inequality in well-being

In this section we present the results of applying the concepts developed in this chapter to well-being data for the last three decades. We use the most recent available data from International Human Development Indicators (UNDP, 2012) on the HDI and its three components for the period 1980-2010 with five years intervals. Income is represented by Gross National Income per capita measured in PPP 2005 US dollars, to make incomes comparable across countries and over time. The second component is measured by life expectancy at birth, which is considered an indicator of the health level. The education index comprises two variables, expected years of schooling and mean years of schooling, which are aggregated using with the geometric mean. This first educational variable indicates the number of years that a child of school entrance age can expect to receive if prevailing patterns of age-specific enrolment rates persist throughout the child's life (UNDP, 2012). The second represents average number of years of education received by people aged 25 and older, converted from education attainment levels using official durations of each level (Barro and Lee, 2010).

Table 3.1. Marginal Gini indices and bivariate Gini indices for the Sarmanov-Lee distribution with classical beta marginals

	Unidimensional Gini			Bivariate Gini		
	Education	Health	Income	Education/Income	Health/Income	Education/Health
1980	0.2651	0.1296	0.2003	0.3581	0.2634	0.3245
1985	0.2410	0.1216	0.1929	0.3315	0.2548	0.2979
1990	0.2260	0.1221	0.1936	0.3160	0.2549	0.2830
1995	0.2129	0.1252	0.1953	0.3014	0.2569	0.2693
2000	0.1993	0.1240	0.1929	0.2882	0.2527	0.2533
2005	0.1818	0.1192	0.1855	0.2718	0.2428	0.2325
2010	0.1676	0.1100	0.1776	0.2571	0.2307	0.2142
1980-1990	-14.76	-5.78	-3.36	-11.75	-3.25	-12.81
1990-2000	-11.82	1.52	-0.35	-8.80	-0.84	-10.49
2000-2010	-18.92	-12.69	-8.58	-12.11	-9.53	-18.23
1980-2010	-36.80	-15.12	-11.30	-28.21	-12.41	-33.99

Originally, we had non-available data for 26 countries for one or more years before 1995. Consequently, our sample comprised only 105 countries, covering less than the 75 percent of global population. In order to offer comparable results across periods and to not restrict the sample considerably, missing values have been estimated. The estimation is based on two complementary methodologies which jointly offer feasible and consistent results according to the sample: piecewise cubic Hermite interpolating polynomial (PCHI) and the average rate of change, which is used when PCHI offers unfeasible estimations or out of range results. After this procedure, our dataset includes 132 countries[24] whose indicators of income, health and education are available for eight points of time. Consequently, the sample covers over 90 percent of the world population during the whole period. Notwithstanding this large coverage, many African and Eastern European countries are not included due to the scarcity of data. Given that practically all absentees are developing countries, our estimates can be biased downward. Therefore, the conclusions derived from our results should be cautiously interpreted.

Using the methodology described in previous sections, we have estimated the parameters of the Sarmanov-Lee distribution considering the beta distribution for

[24] For a description of the countries included see Appendix 4.

modeling marginal distributions[25]. As stated before, since we are considering the HDI as a benchmark, three dimensions of well-being are included in the analysis, thus leading to three different bidimensional distributions: income with education, income with health and, finally, education with health. As a result, we calculate three bivariate Gini indices (Table 3.2) which give us summarized information about how global inequality in well-being has evolved over de study period.

Before analyzing the evolution of multidimensional inequality, let us look at inequality patterns within each dimension independently. In line with previous studies (McGillivray and Markova, 2010; Decancq, 2011), it is observed that the unidimensional Gini index of education decreases continuously during the entire period, pointing out a reduction by almost 37 percent over the last three decades. This period of convergence[26] is mainly driven by the increase in the mean years of schooling which have been doubled in the last 40 years thanks to the efforts in education performed in developing countries, especially in Asia (World Bank, 2006; Morrison and Murtin, 2012).

For the health indicator we observe a reduction of disparities over the first ten years. Conversely, the nineties are characterized by a slightly increase in inequality in health derived from the expansion of AIDS in Sub-Saharan Africa (Neumayer, 2003; Becker et al., 2005), effect that was partially offset by the decrease in infant mortality (Deaton, 2004). The differences in health levels fell sharply during the rest of the period mainly due to the expansion of life expectancy in East and South Asia and the North of Africa (Goesling and Firebaugh, 2004). Taking the study period as a whole, a process of convergence in health levels is concluded, which has lead to a decrease in inequality by 15 percent.

Income differences across countries have received by far more attention than the other dimensions. In line with previous studies (see e.g. Pritchett, 1997; Milanovic, 2005; World Bank, 2001), our results suggest an increase in economic inequality during the

[25] Parameter estimates are presented in Table A.4.1.
[26] Note that we use the concepts of convergence and reduction of disparities indistinctly because we are estimating unweighted inequality which measures differences in national levels of well-being across countries.

period 1985-1995. The raise in income disparities is derived from the failure of Africa in the eradication of poverty, which has offset the outstanding economic performance of Asian territories, thus widening (in the aggregate) the gap between poor and wealthy countries. In fact, Asian countries converged rapidly to developed economies during the last 30 years (see *e.g*. Sala-i-Martin, 2006), but it was not enough to eclipse the divergence process characterized by most African nations. Notwithstanding this trend, thanks to the decrease in disparities during the course of the last decade of the study period, income inequality across countries has been reduced by 7 percent over the last 30 years.

Once we have investigated the evolution of inequality in each of the indicators considered in this study, it is worth focusing on the bivariate Gini index. Taking the period as a whole, a decreasing inequality pattern is observed for education and income components jointly. This result indicates that the reduction of inequality in education has offset the increase in income disparities during the nineties given that a decrease of 8 percent is reported for this decade. As a consequence of the continuous decrease in the bidimensional inequality of this distribution, a decrease by slightly over 28 percent is observed in the last 30 years. Note that the same dynamic is observed when considering the joint distribution of health and education. Bidimensional inequality in this case presents the highest decrease over the past three decades with a reduction of the bivariate Gini index by 34 percent. Our estimates reveal that the increase in health inequality over the period 1985-1995 is completely eclipsed by the decrease in educational disparities across countries and consequently bivariate Gini index falls during the past three decades. In contrast, bidimensional inequality in health and income trends upward from 1985 to 1995. Note that such an ascending pattern is due to the process of divergence experienced by both indicators during that period. This trend is reversed during the last fifteen years given that health and education reduced their inequality levels. Taking the study period as a whole, multidimensional inequality in the bivariate distribution of income and health presents the lowest reduction of disparities, which is about 12 percent for the last three decades.

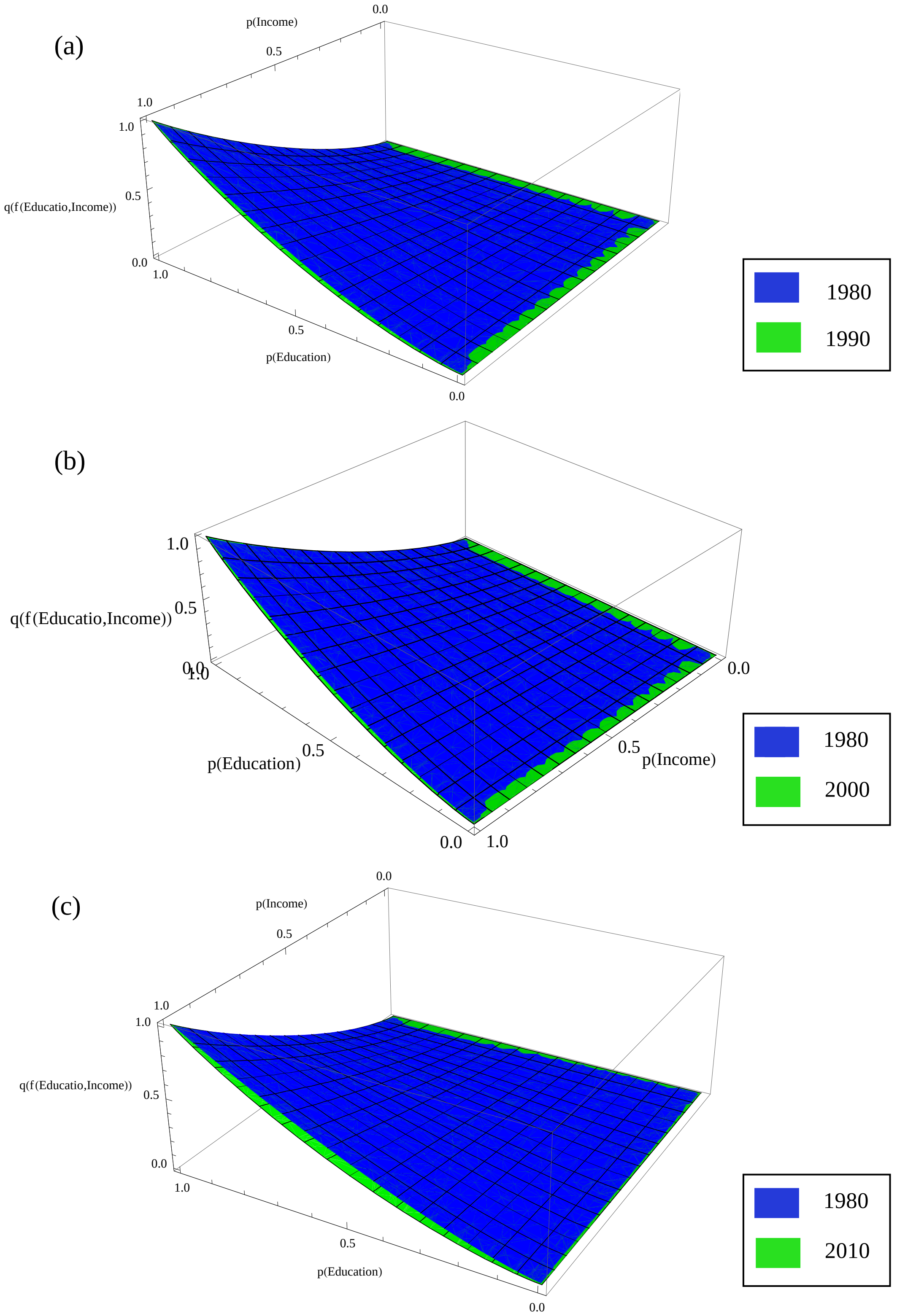

Figure 3.2. Evolution of multidimensional inequality in education and income over the periods: 1980-1990 (a), 1980-2000 (b) and 1980-2010 (c)

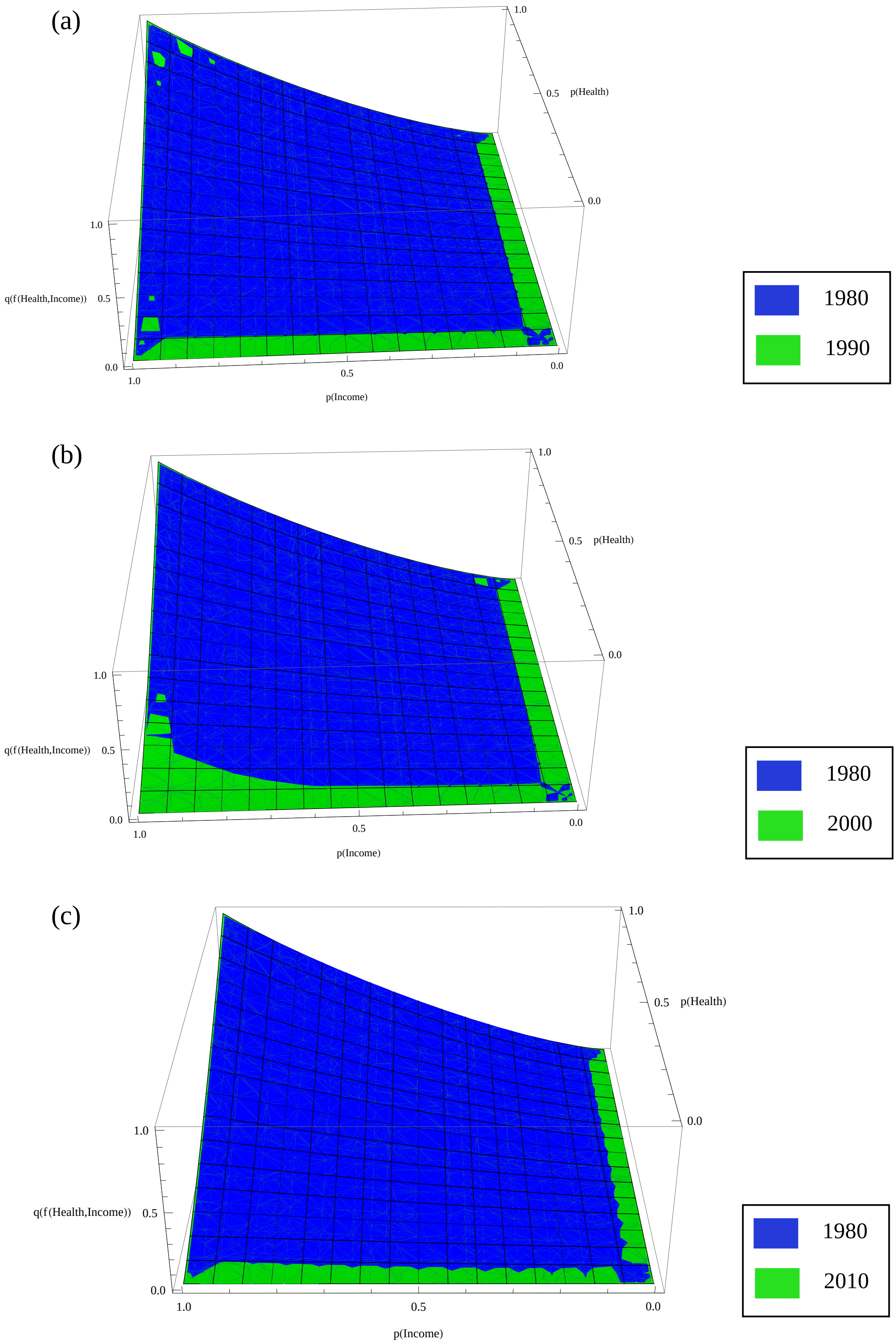

Figure 3.3. Evolution of multidimensional inequality in health and income over the periods: 1980-1990 (a), 1980-2000 (b) and 1980-2010 (c)

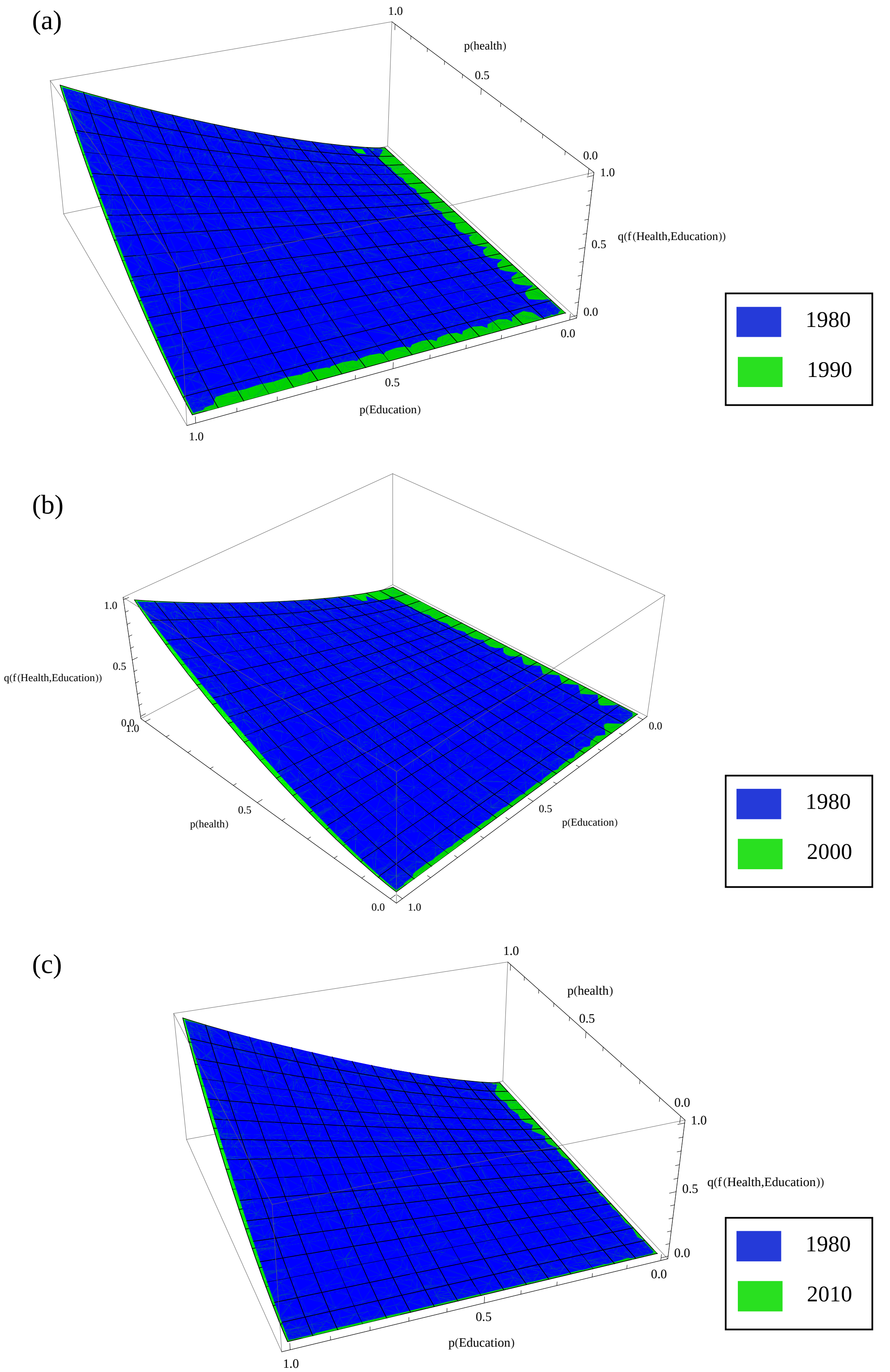

Figure 3.4. Evolution of multidimensional inequality in health and education over the periods: 1980-1990 (a), 1980-2000 (b) and 1980-2010 (c)

Table 3.2. Decomposition of equality using the Sarmanov-Lee distribution with beta marginals

	Education/Income			Health/Income			Education/Health		
	Overall	Within	Between	Overall	Within	Between	Overall	Within	Between
1980	0.6419	0.5757	0.0662	0.7366	0.6908	0.0457	0.6755	0.6340	0.0415
1985	0.6685	0.6015	0.0670	0.7452	0.7046	0.0406	0.7021	0.6617	0.0404
1990	0.6840	0.6127	0.0712	0.7451	0.7034	0.0417	0.7170	0.6743	0.0427
1995	0.6986	0.6211	0.0776	0.7431	0.6990	0.0441	0.7307	0.6828	0.0479
2000	0.7118	0.6344	0.0774	0.7473	0.7019	0.0454	0.7467	0.6954	0.0514
2005	0.7282	0.6560	0.0722	0.7572	0.7125	0.0447	0.7675	0.7147	0.0528
2010	0.7429	0.6756	0.0673	0.7693	0.7276	0.0417	0.7858	0.7357	0.0501
1980-1990	6.55	6.43	7.64	1.16	1.82	-8.81	6.15	6.36	2.96
1990-2000	4.07	3.54	8.62	0.29	-0.22	8.84	4.14	3.12	20.23
2000-2010	4.37	6.49	-13.01	2.94	3.66	-8.16	5.23	5.80	-2.55
1980-2010	15.74	17.35	1.71	4.44	5.32	-8.85	16.33	16.05	20.62

Our estimates suggest that, at the beginning of the period, the greatest levels of inequality are found when considering education and income together, presenting the Gini index a value close to 0.36. Bivariate inequality in education and health presented the second highest value 0.32. The joint distribution of income and health is the least unequal among the relationships considered whose disparities can be quantified in 0.26. The different inequality patterns observed above have altered the position of each distribution in terms of inequality. In 2010, the join distribution of education and health is seen as the most equal (whose bivariate Gini index reached the value 0.21), followed by health and income with a Gini index of 0.23, and, finally, the bivariate distribution of education and health remains as the most unequal with inequality levels close to 0.25.

Bivariate Gini index can be decomposed in two components, using Equations (3.8) and (3.9). Overall equality (1- $G(F_{12})$) is decomposed in terms of equality within each dimension and equality between dimensions, thus allowing us to analyze the evolution of each component in terms of global disparities. From Table 3.3, it is pointed out that the evolution of global equality in well-being is mainly determined by the equality within each component of the HDI, while the factor associated to differences between dimensions represents a residual proportion, which is lower than 10 percent in all cases. Note that equality within-dimensions increased steadily over time for all of the distributions considered, thus reinforcing its position as a dominant factor in overall equality. In contrast, equality between variables presents different evolutions depending on the indicators studied.

In the case of the joint distribution of education and income, this component rose rapidly over the course of the first 20 years of the study period. However, as a consequence of the decrease experienced during the last decade (by 13 percent), our results point out an increase in equality between these indicators by less than 2 percent. For income and health, instead, we observe a sharp decrease in this component during the eighties and a recovery period of this kind of equality over the course of the nineties, whereas the last decade is characterized by a deterioration of equality between variables. As a result, this component has been reduced by slightly over 8 percent, thus representing an even minor proportion of overall equality at the end of the study period. The equality between health and education improved slightly over the eighties (increasing by almost 3 percent), whereas the next ten years were characterized as a period of strong increase in this component, by slightly over 20 percent. In contrast, the last decade presented a small reduction of 2.5 percent of equality between these two indicators. As a result of the aforementioned trends in each decade, this factor has increased by 20 percent over the past 30 years.

The main conclusion suggested by the previous results is that multidimensional inequality in well-being has decreased over time. Having reached this point it is important to recall that bivariate Gini indices only provide summarized information about the evolution of well-being distribution. Bidimensional Lorenz curves are necessary to study whether the previous conclusion can be extrapolated to the whole distribution or it is just a general result derived from the aggregation procedure inherent to all inequality measures[27].

[27] This result can also be investigated using stochastic dominance relationships in multidimensional environments (see Duclos et al., 2011). However when no dominance is achieved, this methodology in unable to provide information about with parts of the distribution improves or worsen in terms of inequality.

Figure 3.2 shows bidimensional Lorenz curves for the bidimensional distribution of education and income in three periods of time: 1980-1990, 1980-2000 and 1980-2010. In line with previous results, the decrease in inequality over the past decades is also observed, since the curve in 2010 lies above the curve in 1980 almost completely. However, it is not possible to conclude that well-being distribution in 2010 Lorenz dominates[28] well-being distribution in 1980 given that the Lorenz curve in 2010 lies below its analogous in 1980 for countries with low level of income and high educational standards. Therefore, the poorest and the least educated countries currently have more unequal distribution than 30 years ago in terms of well-being, whereas the rest of nations enjoy comparatively lower levels of well-being inequality than in 1980. Note that for the other two periods of time presented in Figure 3.2, it is more evident that for poor and less educated people the Lorenz curve in 1980 lies above the Lorenz curve in 1990 and 2000 respectively.

For the join distribution of income and health (Figure 3.3), the absence of Lorenz dominance is even more patent. We observe that the distribution in 1980 is more equal at the bottom quantile and in some points at the top of the distribution, for the richest and healthiest countries. For the comparison of the years 1980 and 2000, a greater proportion of the area of the Lorenz curve in 1980 lies above the curve in 2000, including wealthy nations with low levels of education. For the whole period, we see that the Lorenz dominance of the distribution in 1980 is relegated to the bottom of the distribution which includes poor and least educated countries.

The bivariate Lorenz curves for the joint distribution of health and education are presented in Figure 3.4. Our estimates suggest that the curve in 1980 dominates the Lorenz curve in 1990 at the bottom of the distribution, but the dominance relationship becomes weaker over time. In fact, for the 30 years of the study period, our estimates point out that the Lorenz curve of 2010 lies above the curve in 1980 almost completely.

[28] Lorenz dominance relationships are concluded when the Lorenz curve of a particular distribution lies completely above the Lorenz curve of another distribution.

3.6 Conclusions

In this chapter, using the definition proposed by Arnold (1983), we have obtained closed expressions for the bivariate Lorenz curve, considering a convenient model for the underlying bidimensional distribution. We have studied a relevant type of models based on the class of bivariate distributions with given marginals described by Sarmanov and Lee (Lee, 1996; Sarmanov, 1966). The derived expression of the bivariate Lorenz curve can be easily interpreted as a convex linear combination of products of classical and concentrated Lorenz curves. Consequently, the closed expression for the bivariate Gini index (Arnold, 1987) is made up of two terms, each one including the marginal Gini indices of the variables involved and the concentration indices. This indicator can be decomposed in two factors, corresponding to the equality within and between variables.

The methodology developed in this chapter has been used to study the evolution of global multidimensional inequality in well-being. In particular, we take the HDI as a theoretical benchmark, thus focusing on three dimensions of quality of life, namely income, health and education. Following the construction of the HDI, the original variables are normalized in the scale 0 to 1. Therefore the beta distribution seems to be especially suitable to model the marginal distributions in this case.

Our results point out that bidimensional inequality has been reduced in all of the relationships considered. However, inequality measures only offer summarized information of the evolution well-being differences across countries, thus some internal dynamics can be masked. In particular, it has been concluded that, in spite of the decrease in the bivariate Gini index, the poorest, least educated and least healthy countries have a more unequal distribution than 30 years ago. The most relevant fact that should be noted is that these patterns cannot be concluded using inequality measures, even the multidimensional ones. Therefore, our analysis emphasizes that the extension of the Lorenz curves to multidimensional environments is essential to analyze the internal dynamics of well-being distribution and to offer a complete panorama of the evolution of disparities in well-being.

Chapter 4

International convergence in well-being indicators: a non-parametric approach

4.1. Introduction

The study of convergence has risen to prominence among academics since the presentation of the classical works of Solow (1956, 1957) and Swan (1956). Several papers have tried to determine if there is a long-run tendency towards equalization, a question that lies in the heart of the convergence debate.

According to the Solow-Swan neoclassical growth model, if countries only differ in their level of capital, poor countries tend to grow faster than developed nations due to the assumption of diminishing returns of capital. This theory is the so-called absolute β-convergence, which assumes that all economies in the world converge to the same steady state. Much of the existing literature on convergence hypothesis (see *e.g.* de la Fuente, 1997; Islam, 2003; Sala-i-Martin, 1996) supports that there has been a process of divergence among world economies in the last decades. Therefore, it is concluded that the currently rich nations are expected to be even wealthier in the future, hence leaving developing countries behind. Note however that economic systems usually differ in technology, population growth and human capital. As a consequence, differences in the structural parameters would result in different steady states. This concept, called in the classical literature conditional β-convergence, has been tested by numerous studies (see *e.g*. Barro and Sala-i-Martin, 1990; 1992; Sala-i-Martin, 1996), which point out that, when taking the structural characteristics into account, poor countries converge to their own steady states faster than the advanced economies[29].

The concept of σ-convergence has also been widely studied given its close relationship with the process of β-convergence. It is assumed that there is σ-convergence if the dispersion of per capita income decreases over time. Then, β-convergence is a necessary condition for σ-convergence, but not sufficient (Sala-i-Martin, 1996). Previous studies conclude the existence of σ-divergence across world economies for the second half of the last century (Decancq et al., 2009; Milanovic, 2005; Pritchett, 1997; World Bank, 2006), revealing that international inequality across countries tends to increase over time.

Conventionally, the specification of absolute β-convergence focuses on testing a common linear trend between the growth rate of per capita income and the initial level of output. This regression is augmented with structural variables for testing conditional β-convergence. A negative sign of the coefficient on initial per capita income is interpreted as a support for convergence across countries. However, many authors have questioned the assumption of linearity (Durlauf et al., 2001; Fiaschi and Lavezzi, 2007; Huang, 2005), concluding the existence of multiple growth regimes associated with different levels of development.

Traditionally, income variables have played a main role in the measurement of quality of life. However, there is a discontent with the hegemony of per capita GDP as an indicator of well-being since there are other relevant dimensions which are imperfectly captured by purely economic variables. There is by now nearly consensus that development is a multidimensional concept, which, in addition to income, should also consider social indicators. This line of argumentation has received an increasing

[29] This specification of convergence has been criticized for the so-called Galton’s fallacy problem and its inability to reflect the existence of multiple poles that might lead to multiple stable steady state equilibrium (See, *e.g*. Quah, 1993; 1996; Bliss, 1999).

amount of attention from academics over the last decades, thus resulting in many attempts to synthesize different aspects of well-being in a composite index, which offers a more comprehensive perspective of such a process than per capita income alone.

In 1979, David Morris from the Overseas Development Council designed the Physical Quality Life Index (Morris, 1979), constructed by a weighted average of infant mortality, literacy rate and life expectancy at age one. Becker et al. (2005) developed an indicator which combined income and longevity for measuring inequality in well-being . More recently, there have been many proposals to construct a composite index centered on the notion that development entails more than just economic aspects (see *e.g.* Alkire and Foster, 2010; Bilbao-Ubillos, 2013; Edgier and Tatlidil, 2006; Fakuda-Parr et al., 2009; Grimm et al., 2008; Morrison and Murtin, 2012).

This line of thinking has induced academics to test the convergence hypothesis in other dimensions such as health and education. A theoretical framework of convergence in life expectancy is developed in Mayer-Foulkes (2003), concluding a convergence clubs pattern, whereas global convergence is found to be weak. This result is confirmed in Sab and Smith (2001), who also point out the existence of strong absolute and conditional convergence in education. Mazumdar (2003) tests the existence of convergence in five dimensions of well-being, including calorie intake, life expectancy, infant mortality, per capita GDP and adult literacy rate, concluding divergence in all variables except for income among the advanced economies.

A natural extension of these works is to test the hypothesis of convergence in an aggregate index of quality of life, considering jointly social factors and economic indicators. Note that this approach makes it possible to draw general conclusions regarding the evolution of cross-country trends in quality of life. There have been various attempts to test whether a catching-up process in human well-being has taken place in the last decades (see *e.g.* Konya and Guisan, 2008; Mayer-Foulkes, 2010; Noorbakhsh, 2006), concluding that living standards have converged slowly over the last 30 years. Nevertheless, some authors have questioned the linearity of this process.

Mayer-Foulkes (2010), using series of superposed transitions, concludes that complex relations of divergence and convergence exist in the components of the HDI. In fact, nonlinear parametric models, such as the quadratic specification, have been also proposed as a possible approximation of this phenomenon (Mazumdar, 2002; 2003). Note, however, that the parametric approach requires making *a priori* assumptions about the evolution of the convergence speed, thus the model might present misspecification bias. We opt for a semiparametric specification which lets the data describe by themselves the intensity and direction of the convergence/divergence process.

Through the more flexible methodology of partially linear models (PLM), this chapter aims to provide a reappraisal of the convergence process in terms of quality of life, using the Human Development Index (HDI) as an indicator of this phenomenon, for the period 1980-2011. Having reached this point, it should be emphasized that considering the hypothesis of convergence in a composite indicator presents some limitations that should be taken into account (Mazumdar, 2003). In fact, these are the same criticisms that are attached to any multidimensional indicator of well-being, namely the arbitrarily of the weights and the lack of meaningfulness of the resulting indicator. Therefore, we also adopt a dimension-by-dimension approach to obtain more detailed conclusions regarding convergence in quality of life.

The rest of the chapter is organized as follows. Section 4.2 describes the characteristics of the HDI as an indicator of well-being. Section 4.3 relates the convergence hypothesis to the non-income variables. A detailed explanation of the data used and the methodology applied is presented in Section 4.4. Section 4.5 explores the hypothesis of σ-convergence and presents the evolution of global inequality in well-being over the last three decades. β-convergence is tested in Section 4.6 using non-parametric techniques. Finally, Section 4.7 includes some conclusions and discusses possible policy implications.

4.2. Measuring development: beyond income

Since it was launched in 1990, the HDI attracts a large amount of attention from the media, academics and policymakers. This indicator was designed following the Sen's capability approach (Sen, 1988; 1989; 1999) which considers development as a process of enhancing individuals' choices. This new paradigm of development was presented in the first Human Development Report which stated:

> "*Human development is a process of enlarging people's choices. In principle, these choices can be infinite and change over time. But at all levels of development, the three essential ones are for people to lead a long and healthy life, to acquire knowledge and to have access to resources needed for a decent standard of living*." (UNDP, 1990; p.10).

To materialize this eminently subjective concept into a single number, three dimensions were proposed, which were considered essential to measure the complex reality of human development. Therefore, the HDI is made up of three intermediate indices, using country-level data on income, health and education, which reflect achievements in each dimension respect to the level of subsistence and the historical maximum value.

Since 2010, the HDI of the country *i* in year *t* is constructed using a geometric mean of the three individual indices as follows:

$$HDI_{it} = \left(I_{it}^{health} \, I_{it}^{Education} \cdot I_{it}^{Income}\right)^{1/3} \; .$$

The health index (I_{it}^{health}) is measured by life expectancy at birth (LE), which is considered an indicator of longevity. This indicator is standardized according to the following expression:

$$I_{it}^{health} = \frac{LE_{it} - LE_{\min}}{LE_{\max} - LE_{\min}} \; ,$$

where the minimum is the so-called level of subsistence fixed at 20 years, and the upper bound is the maximum value observed between 1980 and 2011, that is 85 which corresponds to Japan in 2011. It should be noted that life expectancy only measures years of life, but no insights about the quality of these years are made. Notwithstanding its limitations, life expectancy is the sole variable that has not been changed since the HDI was launched, due to the scarcity of data on health indicators for long temporal periods (Klugman et al., 2011).

The education index ($I_{it}^{Education}$) comprises two variables, expected years of schooling (EYS) and mean years of schooling (MYS), which are computed with the geometric mean, given by:

$$I_{it}^{Education} = \left[\left(\frac{MYS_{it} - MYS_{\min}}{MYS_{\max} - MYS_{\min}} \right) \cdot \left(\frac{EYS_{it} - EYS_{\min}}{EYS_{\max} - EYS_{\min}} \right) \right]^{1/2}.$$

MYS and EYS have lower bounds of zero given that societies would survive without education. The maximum corresponds to Czech Republic in 2005 with 13.1 expected years of schooling, whereas MYS variable has a fixed maximum of 18 years, which is achieved by several developed countries. These variables have been introduced in 2010 substituting adult literacy rate and the combined gross enrolment ratio. These indicators were considered uninformative since no discrimination across countries was provided, especially in developed nations whose literacy rates are superior to 95 percent.

Income is represented by per capita Gross National Income (GNI) measured in PPP 2005 US dollars, to make incomes comparable across countries and over time. It should be noted that income is regarded as the mean to acquire goods and services, concept which is different to how much is produced in a particular economy. Thus, per capita GDP has been replaced by per capita GNI given that such a variable represents the economic reality of countries more accurately[30] in terms of the

[30] As is exemplified by UNDP (2010), the GNI of Timor Leste is several times its GDP due to international aid.

capability approach, due to the consideration of international aid and foreign remittances. The intermediate index of income (I_{it}^{Income}) is computed as follows:

$$I_{it}^{Income} = \frac{\ln(GNI_{it}) - \ln(GNI_{\min})}{\ln(GNI_{\max}) - \ln(GNI_{\min})},$$

where the maximum value is 107721 (per capita GNI for Qatar in 2011), whereas the minimum value is fixed at the level of subsistence which is 100 US$. The logarithmic transformation was introduced in 2001 with the objective to reflect that income is conceived as a mean to purchase goods and services, thus the concavity of the logarithmic function characterizes impact of diminishing returns of income on well-being.

In spite of its popularity, the HDI has been highly criticized on the grounds of construction (Grimm et al., 2008; Kelley, 1991), selection of variables (Srinivasan, 1994)[31], arbitrary weighting scheme (McGillivray and White, 1993; Noorbakhsh, 1998), and redundancy with its components (Cahill, 2005; McGillivray, 1991; Ravallion, 1997)[32].

Some authors argue that the HDI omits important aspects of well-being that should be incorporated in the index. Among them, we emphasize democracy (Domínguez et al., 2011), social cohesion (Bilbao-Ubillos, 2011), personal safety (Bilbao-Ubillos, 2013) and environment (Briassoulis, 2001; Neumayer, 2001; Sagar and Najam, 1998). Distributional aspects have also been proposed for their consideration in the construction of the index (Alkire and Foster, 2010; Hicks, 1997; Seth, 2009; 2013) since inequality in the different dimensions of well-being has a deep impact on the progress of a particular country. Conversely, some authors have suggested removing the income component from the HDI (Anand and Sen, 2000).

Concerning the construction of the HDI, two main criticisms need to be addressed. On the other hand, an equal weighting scheme seems to be arbitrary, hence not based on

[31] For a review of the proposed dimensions that should be taken into account see Alkire (2002).
[32] A review of the criticisms focused on the limitations of the HDI can be found in Kovacevic (2010b).

social choice nor normative arguments. Notwithstanding this subjective format, statistical methods (principal components analysis) have been applied to determine the weights supported by the data, concluding that the simple average is empirically justified (Ogwang and Abdou, 2003).

The traditional arithmetic mean is considered problematic since the components of the index are regarded as perfect substitutes, thus implying that the marginal rate of substitution is constant. This axiom can lead to incongruent results, in the sense that the maximization of the HDI in a society may result in corner solutions, promoting one dimension and disregarding others (Klugman et al., 2011). The formula introduced in 2010 marks a conceptual change concerning the relationship between the three dimensions given that some degree of complementarity is introduced.

Several studies point out that there is a high rank correlation between the HDI and its underlying components, thus reflecting a problem of redundancy in the information provided by the composite index. This result implies that "*assessing inter-country development levels on any one of these variables yields similar results to those that the index itself yields*" (McGillivray 1991, pp. 1462). Therefore, the HDI would not provide us with complementary information than the traditional indicator of development, *i.e*. per capita GDP, offers. Note that the previous statement would lead to the loss of the relevance of this study. Since there is apparently no difference between income and human development, the conclusions reached by previous studies on the convergence hypothesis would apply. However, it has been evidenced that the distributions of income, health, education and the HDI differ substantially from each other (McGillivray and Markova, 2010; McGillivray and Pillarisetti, 2004; Pillarisetti, 1997), also pointing out different evolutions over time. This result emphasizes the point that the consideration of a growth-centered approach or a more comprehensive definition of human development would strongly affect the assessment of progress. Consequently, our conclusions about convergence might be also altered.

The criticisms exposed before suggest that the HDI is not an ideal indicator of well-being. However, the evaluation of quality of life is complex, abstract and difficult to synthesize. Independently of its limitations, the HDI seems to be the most adequate alternative to perform cross-country analyses of well-being since it is constructed using homogeneous data for a wider period of time and for more countries than other related indices.

4.3. Convergence in well-being

The concept of convergence in well-being has been essentially studied using inequality measures of composite indicators (see *e.g.* Decancq et al., 2009; McGillivray and Markova, 2010; McGillivray and Pillarisetti, 2004). There is a common result which indicates that well-being levels are converging over time but at slow rate:

> ‘‘*For most of the past 40 years human capabilities have been gradually converging. From a low base, developing countries as a group have been catching up with rich countries in such areas as life expectancy, child mortality, and literacy. A worrying aspect of human development today is that overall state of converging is slowing and for a large group of countries divergence is becoming the order of the day*.” (UNDP, 2005, pp. 25).

Since the concept of β-convergence was derived from the Solow model, its theoretical framework is especially suitable for income. The principal mechanism behind the convergence hypothesis is the assumption of diminishing returns of capital. Accordingly, the vast majority of the papers that test convergence in living standards focuses on income variables whereas social aspects of development are assumed to play little role. However, there have been few attempts to test the convergence hypothesis in a more comprehensive indicator than per capita GDP (Mazumdar, 2002; Noorbakhsh, 2006; Konya and Guisan, 2008; Konya, 2011; Mayer-Foulkes, 2010), thus extrapolating the concept of diminishing returns to the non-income dimensions of the HDI.

According to Noorbakhsh (2006), the concept of diminishing returns can be “equally applicable” to the educational variables and health indicators of the HDI but with some peculiarities. Diminishing returns were associated with the mobility of capital in the pure economic model. In contrast, for non-income aspects, they are linked to the assumption that investment returns in education and health diminish with the level of investment. In a country with low levels of primary education, relatively less investment is necessary to increase the mean years of schooling than in a developed nation, since tertiary education is the most expensive type of education. Therefore, investment returns to expand educational standards will be higher in countries with low levels of education. Moreover, given the nature of the educational indicators considered in the HDI, which are basically quantitative variables that do not account for the quality of education, they have upper limits that make plausible the existence of a convergence process across countries.

Similarly, for the health dimension, it is supported that investment returns in health are higher in countries with low life expectancy, since less amount of investment is needed to improve health levels in countries with high rates of mortality. In fact, according to the last report of Millennium Development Goals (MDG), a large proportion of the deaths of children under five could be saved through low-cost prevention and treatment measures (United Nations, 2012). Moreover this type of medical research is easily exported, whereas advanced medical technology is more difficult to be implemented in developing countries mainly due to the lack of suitable personnel which also increases the amount of investment (Mazumdar, 2000).

4.4. Data and methodology

We use the most recent available data from International Human Development Indicators (UNDP, 2012) on the HDI and its three components for 132 countries over the period 1980-2011 with different data frequency. For the period 1980 to 2005 we have 5-year intervals and from 2005 to 2011 the data have annual frequency.

Originally, our data comprised only 105 countries, covering less than the 75 percent of global population. We had non-available data for 26 countries in one or more years before 1995. In order to offer comparable results across periods and to not restricting the sample considerably, missing values have been estimated. The estimation is based on two complementary methodologies which jointly provide feasible and consistent results according to the sample: piecewise cubic Hermite interpolating polynomial (PCHI) and the average rate of change, which is used when PCHI offers unfeasible estimates or out of range results. After this procedure, our dataset covers over 90 percent of the world population during the whole period, including 132 countries whose indicators of income, health and education are available for thirteen points of time.

Our analysis begins with the study of the whole distribution, considering countries as units of observations. This examination will provide relevant information on the evolution of inequality and the formation of clusters of countries, which would evidence the existence of the so-called convergence clubs. Note that traditional summary statistics cannot inform us about these dynamics since they provide partial findings that only focus on the dispersion of the distribution. To offer a broader picture of the distributional dynamics of well-being, it is necessary to estimate the density of the cross-country distribution of the HDI ($f(x)$). As is usual in the literature, non-parametric estimation techniques are considered, thus avoiding the need to decide *ex-ante* the functional form of $f(x)$, letting the data to state by themselves the shape of the distribution.

Figure 4.1 presents kernel density estimates of the HDI distribution for each decade, computed using a Gaussian kernel with optimal bandwidth (Silverman, 1986). The horizontal axis represents the country level of HDI and the vertical axis refers to the associated density. The evolution of the distribution of the HDI offers optimistic insights in terms of development given that it has completely moved to the right over the last 30 years. However, the intensity of the increase in well-being levels differs across countries in the sense that least developed countries have converged among them whereas medium developed countries have converged towards highly developed nations. This distributional behavior has lead to a twin-peaks distribution (Quah, 1993; 1996), which is colloquially interpreted as if medium developed countries were vanishing (Sala-i-Martin, 2000). As a result, a bimodal distribution comes up, mainly due to the efforts of medium developed countries such as China and India, in catching up the advanced economies. On the other hand, countries on the left tail of the distribution are concentrated around a new pole at the end of the period, which basically comprises Sub-Saharan African countries.

It should be noted that the upper mode includes many more countries than the lower mode, which means that, in general terms, national levels of quality of life are converging, inequality is decreasing and global well-being is expanding. In sum, the increase in well-being levels over the last three decades has led to a more equal distribution of the HDI, however the polarization has increased in the sense that two well-defined worlds have come out (Noorbakhsh, 2006).

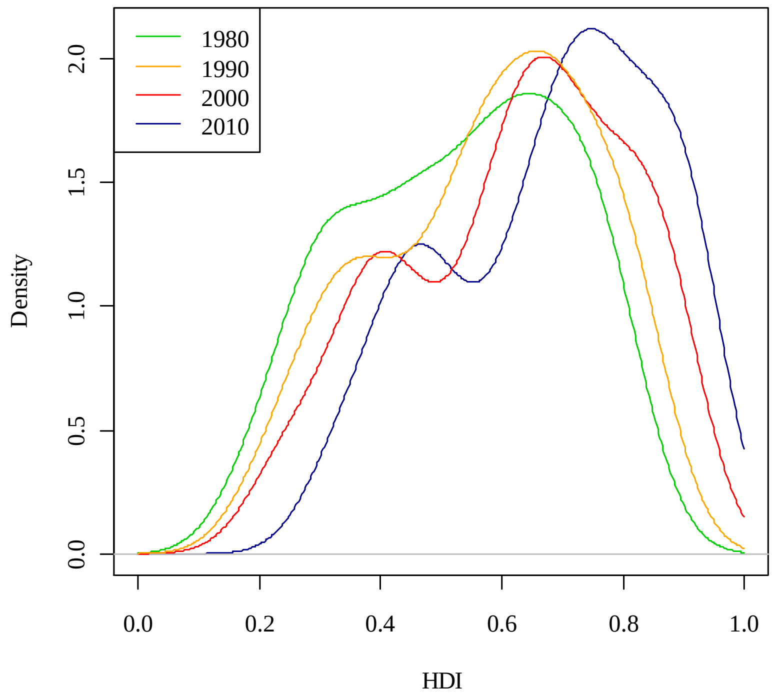

Figure 4.1. Global distribution of the HDI in 1980, 1990, 2000 and 2010

Even when some results can be extracted from the previous analysis, only tentative conclusions can be achieved regarding the existence of σ-convergence, which assumes that the cross-sectional dispersion of a variable tends to decrease over time (Barro, 1991; Barro and Sala-i-Martin, 1990). To quantify the changes in the dispersion of well-being distribution over the last three decades, we have computed four inequality measures. Therefore, we analyze the so-called σ-convergence, not only using the classical indicator (*i.e.* the variance) but also considering the Gini, Theil´s Entropy and the Atkinson indices[33].

All measures indicate the amount of dispersion of well-being distribution across countries[34], however, different weighting schemes are applied for each part of the distribution depending on the measure considered. The Theil index is a special case of the generalized entropy measures when the sensitivity parameter is set to 1 (Cowell, 2011). Such a parameter determines the weight assigned to the upper tail, which in the case of the Theil index, indicates that the same weight is attached to all countries independently of its level of development. The Atkinson index is interpreted as the proportion of total income that would be required to achieve an equal level of welfare. This inequality measure also includes a parameter which is called inequality aversion parameter, since it adjusts the index to be more sensitive to changes in the lower tail (Atkinson, 1970). The expressions of the Gini, the Theil and the Atkinson indices are, respectively, the following:

$$G^{(t)} = \frac{1}{2n^2\mu(Y^{(t)})}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|Y_i^{(t)} - Y_j^{(t)}\right| \ , \tag{4.1}$$

$$T^{(t)} = \frac{1}{n}\sum_{i=1}^{n}\frac{Y_i^{(t)}}{\mu(Y^{(t)})}\log\left(\frac{Y_i^{(t)}}{\mu(Y^{(t)})}\right) , \tag{4.2}$$

and

[33] The inclusion of other measures to study the dispersion of the distribution, responds to the problems presented by the variance, which is "*unsatisfactory in that were we simply to double everyone's incomes (and thereby double mean income and leave the shape of the distribution essentially unchanged)*" (Cowell, 2011; 27).

[34] Note that each measure has different properties and attaches different weights to each part of the distribution. Consequently, results based on different inequality measures can vary substantially.

$$A_{\varepsilon}^{(t)} = 1 - \left[\frac{1}{n} \sum_{i=1}^{n} \left(\frac{Y_i^{(t)}}{\mu(Y^{(t)})} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}, \quad \forall \varepsilon \neq 1 \tag{4.3}$$

where $x_i^{(t)}$ denotes HDI or one of its intermediate indices for the country i at time t, μ is the arithmetic mean of the indicator under study, n is the number of countries, and finally, ε is the inequality aversion parameter of the Atkinson index, which has been set to 2 to analyze the evolution of inequality levels when high aversion to inequality is assumed.

Consequently, our analysis provides a broad picture of the evolution of inequality over the last 30 years which allows us to determine whether distances between countries have been reduced in terms of quality of life. At this point, it should be stated that inequality measures reflect the dispersion of a particular indicator of well-being. However, this methodology does not consider the bimodality that we observed in Figure 4.1. To analyze this feature of the global distribution of the HDI, we apply the polarization measures developed by Esteban and Ray (1994). These authors consider that polarization informs about the degree of concentration of the population in question around different clusters. Using this concept, there would be maximum polarization when there are two poles of an equal size, each one located at opposite ends of the distribution.

According to Esteban and Ray (1994), the degree of polarization of a specific distribution around different poles is given by the following expression:

$$P^{ER}(f, \alpha) = \sum_{i=1}^{m} \sum_{j=1}^{m} p_i^{1+\alpha} p_j \left| \mu_i^{(t)} - \mu_j^{(t)} \right|, \tag{4.4}$$

where $\mu_i^{(t)}$ is the average of the variable under study for the group i, i = 1, ..., m, at year t and p_i is the share of the number of countries in the group i[35]. The parameter α

[35] In the original paper of Esteban and Ray (1994), p_i was the population share of the group. However, as we focus on unweighted inequality measures, we compute unweighted polarization measures to make both analyses comparable.

ranges from 1 to 1.6 and reflects the degree of sensitivity of this measure to polarization.

Note that the computation of the previous measure requires defining *ex-ante m* mutually exclusive groups which would represent each of the clusters. In this study we analyze the case of bipolarization given the pattern that we observed from density estimates of well-being distribution (Figure 4.1). Following Davies and Shorrocks (1989), we define the optimal two-group partition using the average of the variable under study.

As stated before, a necessary condition for σ-convergence is the existence of β-convergence patterns, thus implying that developing countries increase their levels of HDI relatively faster than the advanced nations. The hypothesis of absolute β-convergence is evidenced when there is a negative relationship between the value of a variable at the beginning of the period and its growth rate, which conventionally is tested using the following model:

$$\dot{y}_i = \alpha + \beta y_{i0} + \varepsilon_i \ , \tag{4.5}$$

where y_{i0} is the logarithm of the HDI or any intermediate index which are denoted as Y_{it}, $\dot{y}_i = (1/T)\ln(Y_{it}/Y_{i0})$ is the growth rate of Y_{it} and, finally, ε_{it} is the unexplained residual. Positive values of the β parameter imply divergence, whereas negative values support the existence of a catching-up process between developing and developed countries. Equation (4.5) assumes that all countries of the sample converge to the same steady state. However, nations have different structural features which lead to a multiple steady state equilibrium (Sala-i-Martin, 1996). This assumption is related to the so-called conditional convergence hypothesis, traditionally specified as an augmented regression of Equation (4.5):

$$\dot{y}_i = \alpha + \beta y_{i0} + \boldsymbol{\omega}_i' \delta + \varepsilon_i , \tag{4.6}$$

where the matrix $\boldsymbol{\omega}_i$ contains the structural variables which are constant in the steady state. In this study, a set of regional dummy variables[36] are considered to represent regional features that have influence on the growth rate of the indicators included in the analysis[37].

A number of studies, however, have challenged the assumption of linearity for testing income convergence. Using nonlinear specifications, it has been concluded that the catching-up process is not adequately represented by a linear trend, thus classifying countries into different groups which exhibit a variety of convergence patterns (Azomahou et al., 2011; Durlauf, 2001; Durlauf et al., 2001; Liu and Stengos, 1999). In this line, a generalization of the process of convergence in well-being is considered in Mazumdar (2002), who includes quadratic and logarithmic terms to represent nonlinearities in the convergence speed. Having reached this point, it is important to recall that parametric specifications require making *a priori* assumptions about the functional form of the relationship under study. Therefore, we consider a more flexible model which allows the data to describe by themselves the direction of the convergence or divergence process. Following the notation in Equations (4.5) and (4.6), we specify a semiparametric partially linear regression[38] for testing absolute and conditional β-convergence, given respectively by the following expressions:

$$\dot{y}_i = f(y_{i0}) + \eta_i, \tag{4.7}$$

$$\dot{y}_i = f(y_{i0}) + \boldsymbol{\omega}_i{}'\delta + \eta_i, \tag{4.8}$$

[36] This set of dummies is considered as a proxy of structural variables given that no theory has been developed for convergence in human development. Note that this is a common practice in the convergence literature when conditional variables are not available (Azomahou et al., 2011; Dobson and Ramlogan, 2002; de la Fuente, 2002). The only study that includes a set of conditional variables in the analysis of well-being convergence is Noorbakhsh (2006), concluding that only few of them are significant. It could be possible to use the same set of variables in this study, however this would imply to restrict our sample considerably since, as stated by the author, "*variables selected to represent the structural conditions would be widely different for the high human development countries*" (Noorbakhsh, 2006, pp. 8).

[37] The regions included are the Western Europe, North America and Oceania (WENAO), East Asia, the Arab States, Europe and Central Asia, Latin America and the Caribbean, South Asia and Sub-Saharan Africa. A description of the countries included in each region is provided in Appendix 5. To avoid problems of perfect collinearity, we omit the WENAO dummy which is considered as a base variable.

[38] For a detailed explanation of the econometric technique, see Wand (2005) and Ruppert et al. (2003).

where y_{i0} denotes Y_{i0} expressed in natural logarithms, η_i is the error term identically and independently distributed with mean 0 and variance σ_η^2, and $f(y_{i0})$ is an unknown unidimensional smooth function $f(y_0) = E[\dot{y}|y_0]$ represented by a linear combination of polynomial functions, regression parameters and radial basis functions[39] which need to be chosen to be numerically stable. In this study, the smooth function in Equations (4.7) and (4.8) is expressed as a radial basis function of degree three:

$$f(y_0) = \beta_0 + \beta_1 y_{i0} + \sum_{k=1}^{K} u_k \left| y_{i,0} - k_k \right|^3, \tag{4.9}$$

where β_i, for i = 0, 1 are the so-called fixed effects. The unknown vector of parameters $\mathbf{u}=(u_1, u_2,...,u_K)'$ follows a Normal distribution with mean 0 and variance $\sigma_u^2\Sigma'$, being K the number of bases, and k_k are fixed knots[40]. Note that if u_k = 0 for all k, then the semiparametric model used in this study turns out to be the linear specification of β-convergence, since the last term disappears.

The estimation is based on penalized spline smoothing, which minimizes the following expression:

$$\min_{\beta,u} \left\| y - \mathbf{H}\theta \right\|^2 + \lambda^3 \theta' \mathbf{D}\theta,$$

where $\theta = [\beta, u]$ is the parameter vector, $\mathbf{H}$ is a matrix that contains the polynomial basis functions and the k radial basis functions, $\lambda^3\theta'\mathbf{D}\theta$ is called roughness penalty since it penalizes fits that are too rough (Ruppert et al., 2003). The first parameter $\lambda>0$, estimated by restricted maximum likelihood, determines the amount of smoothing, thus controlling the trade-off between roughness and goodness of fit. Finally, $\mathbf{D}$ is a block identity penalty matrix whose first two elements are zero given that the fixed effects (*i.e.* intercept and linear term) are not penalized.

[39] Other options would be B-splines, natural cubic splines or truncated polynomials. All of these alternatives would provide very similar results (Ruppert et al., 2003).

[40] According to Ruppert (2002), overfitted or underfitted estimates are likely to be obtained depending on the number of knots specified. In this work, the knots are calculated by default as $q_k = (k + 1/K + 2), \forall k = 1,...,K$, where K = max [n/4, 20].

Alternatively, if we substitute Equation (4.9) in (4.8), penalized spline regression can be seen as a Linear Mixed Model (LMM) given by the following expression,

$$y = \mathbf{X}\beta + \mathbf{Z}u + \boldsymbol{\varepsilon}, \tag{4.10}$$

where:

$$E\begin{bmatrix} u \\ \varepsilon \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \quad Cov\begin{bmatrix} u \\ \varepsilon \end{bmatrix} = \begin{bmatrix} \mathbf{G} & 0 \\ 0 & \mathbf{R} \end{bmatrix},$$

$$\mathbf{X} = \begin{bmatrix} 1 & \cdots & 0 & 1 & y_{0,1} \\ \vdots & & \vdots & \vdots & \vdots \\ 1 & \ddots & 0 & 1 & y_{0,i} \\ 0 & & 0 & 1 & \vdots \\ \vdots & & \vdots & \vdots & \vdots \\ 0 & \cdots & 1 & 1 & y_{0,n} \end{bmatrix}; \quad y = \begin{bmatrix} \dot{y}_1 \\ \vdots \\ \dot{y}_i \\ \vdots \\ \dot{y}_n \end{bmatrix}; \quad \beta = \begin{bmatrix} \omega_1 \\ \vdots \\ \omega_6 \\ \beta_0 \\ \beta_1 \end{bmatrix},$$

$$\mathbf{Z} = \begin{bmatrix} (y_{0,1} - q_1)^3 & \cdots & (y_{0,1} - q_k)^3 \\ \vdots & & \vdots \\ (y_{0,i} - q_1)^3 & \ddots & (y_{0,i} - q_k)^3 \\ \vdots & & \vdots \\ (y_{0,n} - q_1)^3 & \cdots & (y_{0,n} - q_k)^3 \end{bmatrix}; \quad u = \begin{bmatrix} u_1 \\ \vdots \\ u_k \end{bmatrix},$$

$\mathbf{G} = \sigma_u^2 \boldsymbol{\Sigma}$ and $\mathbf{R} = \sigma_\varepsilon^2 \mathbf{I}_n$ are positive definite covariance matrices.

As demonstrated by Ruppert et al. (2003), the estimated best linear unbiased predictions (EBLUP) of β and u are:

$$EBLUP(\beta) \equiv \hat{\beta} = (\mathbf{X}'\hat{\mathbf{V}}^{-1}\mathbf{X})\mathbf{X}'\hat{\mathbf{V}}^{-1}y,$$

$$EBLUP(u) \equiv \hat{u} = \hat{\mathbf{G}}\mathbf{Z}'\hat{\mathbf{V}}^{-1}(y - \mathbf{X}\hat{\beta}),$$

where $\mathbf{V}$ = $ZGZ'+R$ = $I_n + \lambda\mathbf{Z}\Sigma\mathbf{Z}'$ (Crainiceanu and Ruppert, 2004). The smoothing parameter is expressed as $\lambda = \sigma_u^2 / \sigma_\varepsilon^2$, thus the ratio of the variance components determines the amount of shrinkage in this case.

Matrices $\hat{\mathbf{V}}$ and $\hat{\mathbf{G}}$ are estimated using REML estimates of their parameters. Following this representation of the penalized regression (Equation (4.10)) with parameters θ, λ and σ_ε^2, the log-likelihood is expressed as follows:

$$L(\beta, \sigma_\varepsilon^2, \lambda) = -\left[n\log(\sigma_\varepsilon^2) + \log\{\det(\mathbf{V})\} + \frac{(y - \mathbf{X}\beta)'\mathbf{V}^{-1}(y - \mathbf{X}\beta)}{\sigma_\varepsilon^2} \right].$$

Thus, the restricted log-likelihood function is:

$$REL(\beta, \sigma_\varepsilon^2, \lambda) = L(\beta, \sigma_\varepsilon^2, \lambda) - (p+1)\log(\sigma_\varepsilon^2) - \log\left\{\det(X'\mathbf{V}^{-1}X)\right\}.$$

The maximization of this function over θ, λ, σ_ε^2 provides the REML estimators.

We also compute a test to analyze the adequacy of the semiparametric models with respect to the linear specifications in Equation (4.5) and (4.6) (Crainiceanu and Ruppert, 2004). Assuming that $\mathbf{u}$ is identically and independently distributed with mean 0 and variance $\mathbf{G} = \sigma_u^2 \mathbf{\Sigma}$,[41] testing the null hypothesis $u_1 = u_2 = \ldots = u_k$ is equivalent to:

$$H_0 : \sigma_u^2 = 0,$$
$$H_a : \sigma_u^2 > 0.$$

Note that if the null hypothesis is not rejected, convergence in human development is correctly represented by the conventional linear model. Otherwise, a more flexible semiparametric approximation is required.

We use the restricted log-likelihood ratio test (RLRT) expressed as follows:

$$RLRT_n = \sup_{H_a \cup H_0} REL(\boldsymbol{\theta}, \sigma_\varepsilon^2, \lambda) - \sup_{H_0} REL(\boldsymbol{\theta}, \sigma_\varepsilon^2, \lambda),$$

[41] Note that the standard choice of $\mathbf{\Sigma}$ is the identity matrix (Craniceanu and Ruppert, 2004).

where REL is the restricted maximum likelihood for the non-restricted model (PLM) and the restricted specification (linear model) respectively.

The computation of RLRT[42] is relatively simple, however the derivation of its distribution under the null has to be bootstrapped since the observations of the dependent variable are not independent under the alternative. Therefore, the asymptotic probabilistic theory does not hold.

4.5. Well-being inequality and sigma-convergence

In this section we study whether a process of σ-convergence in the HDI and each of its intermediate indices took place in the last three decades. This concept of convergence assumes that dispersion from the cross-country mean tends to decrease over time (Barro, 1991; Barro and Sala-i-Martin, 1992). In the empirical literature, the variance of the logarithm of the variable under consideration is the most commonly used measure of dispersion. We also have considered three additional measures of inequality: the Gini (Equation (4.1)), the Theil (Equation (4.2)) and the Atkinson (Equation (4.3)) indices, whose evolution over the last three decades is presented in Figure 4.2. To facilitate the comparison of results, inequality has been normalized to be 100 in 1980.

Table 4.1. Rate of σ-convergence

	1980-1990	1990-2000	2000-2011	1980-2011
HDI	-0.0723	-0.0270	-0.1197	-0.2054
Education Index	-0.1379	-0.1030	-0.1483	-0.3414
Health Index	-0.0451	0.0325	-0.1227	-0.1351
Income Index	-0.0040	0.0201	-0.0853	-0.0706

[42] For a detailed explanation of the procedure for testing the null hypothesis of non-significance of the variance component in linear mixed models with one variance component see Crainiceanu and Ruppert (2004). In that paper, the finite and asymptotic distributions of the RLRT are derived to provide consistent results.

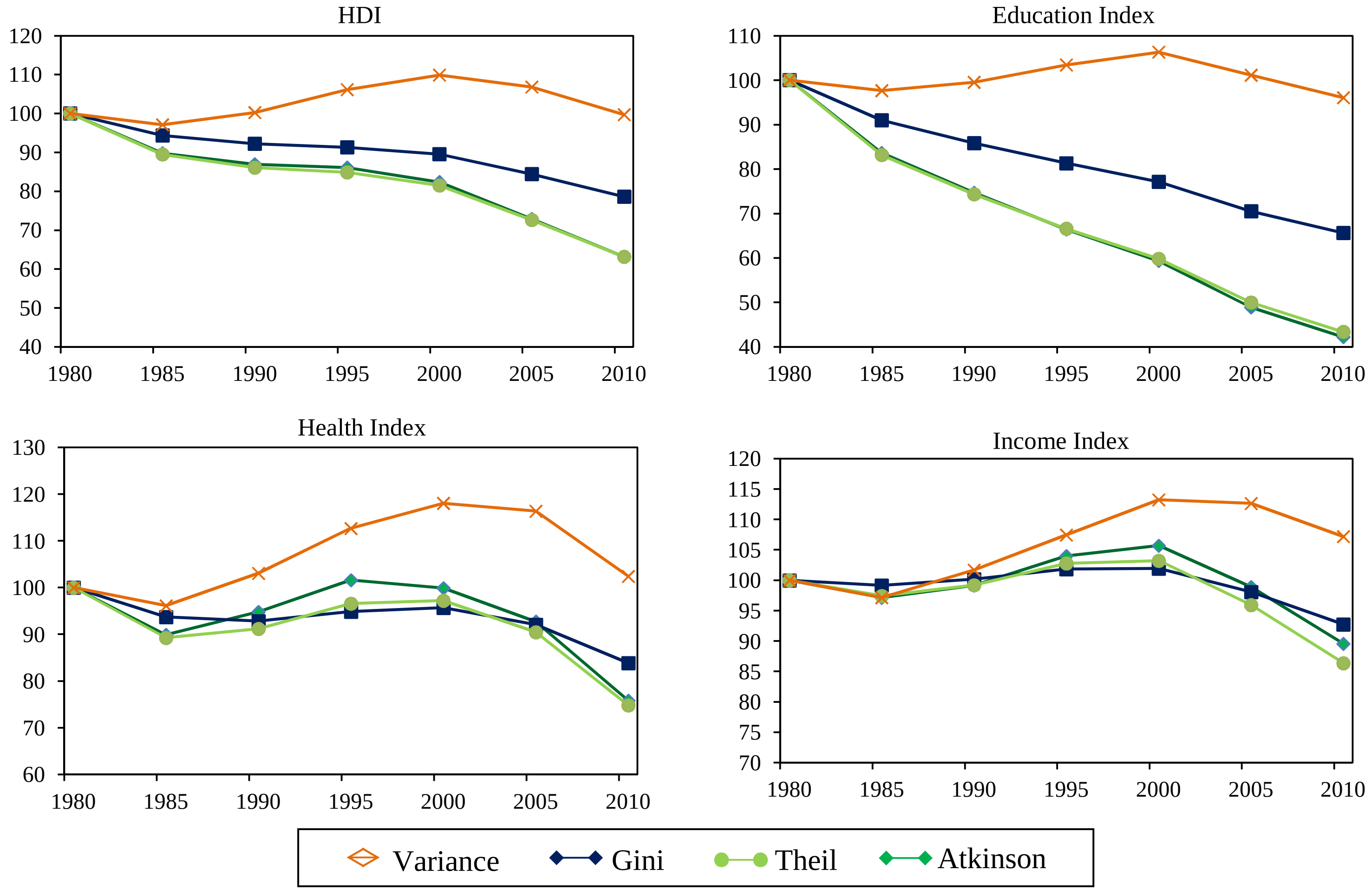

Figure 4.2. Inequality in the HDI and its components (1980 = 100)

In line with previous studies, our results reveal the presence of a global convergence process in living quality of life the study period (Decancq et al., 2009; Martínez, 2012; McGillivray and Markova, 2010). It is observed that inequality in human development decreases about 20 percent according to the Gini index and about 40 percent for the Theil and Atkinson indices. Convergence patterns are also observed for each component of the HDI but the intensity of this process varies across dimensions. The dispersion of the educational indicator decreases continuously during the entire period, thus experiencing the greatest fall of inequality, ranged from 35 to 60 percent depending on the inequality measure analyzed. This fall in education inequality is mainly driven by outstanding evolution of Asian countries (Morrison and Murtin, 2012; World Bank, 2006).

In line with previous investigations, the fall of health inequality has been remarkably lower, ranging from 15 to 30 percent over the last three decades (McGillivray and Markova, 2010). A process of divergence is observed in this dimension during the nineties as a consequence of the rapid extension of AIDS in Sub-Saharan Africa (Becker et al., 2005; Neumayer, 2003). Conversely, a much more egalitarian distribution is observed for the second half of the study period, mainly due to the enhancement of life expectancy in East and South Asia and in the North of Africa (Goesling and Firebaugh, 2004).

Table 4.2. Bipolarization of the HDI and its components

	HDI		Income		Health		Education	
	$\alpha = 1$	$\alpha = 1.6$	$\alpha = 1$	$\alpha = 1.6$	$\alpha = 1$	$\alpha = 1.6$	$\alpha = 1$	$\alpha = 1.6$
1980	0.1468	0.0961	0.1526	0.1006	0.1379	0.0910	0.1750	0.1154
1985	0.1482	0.0976	0.1529	0.1009	0.1338	0.0882	0.1724	0.1137
1990	0.1499	0.0988	0.1587	0.1047	0.1335	0.0877	0.1739	0.1147
1995	0.1527	0.1006	0.1625	0.1072	0.1394	0.0913	0.1755	0.1157
2000	0.1561	0.1027	0.1661	0.1096	0.1460	0.0955	0.1768	0.1166
2005	0.1547	0.1018	0.1657	0.1093	0.1439	0.0943	0.1725	0.1137
2011	0.1493	0.0982	0.1617	0.1067	0.1324	0.0869	0.1678	0.1106

The study of economic disparities has generated an increasing number of papers, which generally point out that cross-country inequality increased over the second half of the last century (see *e.g*. Milanovic, 2005; Pritchett, 1997; World Bank, 2001). The failure of Africa in the eradication of poverty increased income disparities in spite of the success of Asia which rapidly converged to developed countries in the last 30 years. However, in the last decade, this tendency has reversed, and consequently income inequality across countries has been reduced by about 10 percent over study period.

Note however that the patterns described previously do not apply for the variance because, as an absolute indicator, it does not take into consideration the mean of the distribution. It should be stated that, for variables that show sharply positive or negative trends over time, the coefficient of variation would provide more realistic information of the convergence or divergence process (Kenny, 2005). In fact, a number of papers consider relative measures of well-being inequality to study σ-convergence (Marchante et al., 2006; Ferrara and Nisticó, 2013; Konya and Guisan, 2008; Noorbakhsh, 2006), which is also convenient in this case since the mean of the HDI has increased considerably over the last decades, from 0.433 in 1980 to 0.621 in 2012 (UNDP, 2013).

We have calculated percentage of change of coefficient of variation which is called the rate of σ-convergence (O'Leary, 2001)[43]. Therefore, negative values indicate convergence in the sense of sigma, whereas positive trends point out divergence patterns. Table 4.1 shows the growth rate of the coefficient of variation for the countries included in the sample, calculated for each dimension of the HDI and the index itself over the whole period and within each decade.

As for other inequality measures, once the mean is taken into account, countries converge in the sense of sigma, which implies that the dispersion tends to decrease over the time. However, different patterns are observed across dimensions. Whereas a continuous decrease is observed for the dispersion of the education indicator, income

[43] This is a common measure in the study of σ-convergence used in several studies. For the specific case of well-being convergence it has been used in Marchante et al. (2006) and Ferrara and Nisticó (2013).

inequality and health disparities are characterized by some fluctuations. Focusing on the evolution of inequality in the composite index, it is concluded that, as in the case of education, a smooth linear process of convergence took place over the last three decades. These dynamics point out that the uneven behaviors of different aspects of development are hidden when studying convergence in composite indices. Notwithstanding this fact, it is also important to remark that some common trends are observed. In fact, convergence in the considered indicators of quality of life has taken place mainly during the last decade in all cases.

As we concluded from Figure 4.1, even when a more equal distribution of well-being is observed at the end of the study period, two well-defined clusters of countries are distinguished. This pattern is related to the so-called convergence clubs which cannot be measured using the traditional approach of inequality. Therefore, we apply the methodology proposed by Esteban and Ray (1994) to study the evolution of bipolarization in well-being over the last 30 years (Table 4.2). This measure includes a sensitivity parameter α (see Equation (4.4)) that has been set to 1 and 1.6 in this study, following Ezcurra and Pascual (2005).

Our results suggest that polarization has been reduced around 4 percent for the case of non-income variables. In contrast, the economic component has increased its polarization by slightly over than 6 percent. These patterns confirm again that the distributions of social dimensions differ substantially from those of economic indicators. It should be worth noting that the previous results reveal that inequality and polarization assess completely different phenomena. For instance, inequality in education was substantially reduced over the last three decades while the change in polarization levels has been hardly appreciable. More notable is the case of the income component whose inequality was slightly reduced over the study period but an increase in polarization is observed. As a consequence, consistently with the conclusions obtained from kernel density estimates, the polarization in the distribution of the HDI has increased by 2 percent over the last 30 years even when the traditional approach of inequality suggests a more equal distribution of well-being.

4.6. Beta-convergence in well-being

Table 4.3 presents the estimation results of absolute convergence according to Equation (4.7) using as a dependent variable the growth rate of the HDI and its intermediate indices. For comparative purposes, we also present the conventional linear estimation of β-convergence (Equation (4.5)). It is observed that all dimensions show statistically significant negative coefficients on y_{i0}, thus suggesting a negative relationship between the growth rate of the considered indicators and their value in 1980.

It should be, however, noted that the speed of convergence differs across dimensions. The highest speed is observed for the education dimension, followed by health and finally, income has seen the lowest reduction in the gap between developed and developing countries. Consequently, even when little advances have been achieved in income levels, significant improvements in non-income dimensions and human well-being have been accomplished. This result highlights again the relevance of considering non-income dimensions in the study of the convergence hypothesis, since their distributional patterns differ substantially from income. On the other hand, it should be also noted that, in line with previous studies, the magnitude of the coefficient is relatively low in all cases, thus indicating a weak absolute convergence process in living standards over the last 30 years (Mazumdar, 2002; 2003; Noorbakhsh, 2006; Konya and Guisan, 2008).

According to the results of the RLRT test, the null hypothesis of linearity is rejected for the income and education indices (see the last row in Table 4.3) given that the bootstrapped p-values are practically equal to zero. As a result, we might conclude that the convergence process has been nonlinear for both indicators. This conclusion would imply that, using parametric models, the convergence speed is overestimated or underestimated for some levels of income and education. These dynamics are observed from Figure 4.3 which shows the estimated function $f(y_{i0})$ with the corresponding 95 confidence interval for testing absolute convergence. The parametric counterpart (y_{i0}) is also plotted for comparative purposes.

Table 4.3. Parametric and semiparametric estimations. Absolute convergence

Variable	HDI	Education Index	Health Index	Income Index
Parametric specification				
y_{i0} [a]	-0.0196***	-0.0402**	-0.0147***	-0.0126***
	(0.0017)	(0.0037)	(0.0023)	(0. 0038)
Constant	0.0191***	0.0334***	0.0154***	0.0119***
	(0.0011)	(0.0020)	(0.0018)	(0.0025)
Adjusted R^2	0.5325	0.6860	0.2732	0.1313
Semiparametric specification				
$f(y_{i0})$	Figure 4.3a	Figure 4.3b	Figure 4.3c	Figure 4.3d
Smoothing parameter	564	2.084	2.497	1.014
RLRT test[b]	0.0000 (1.0000)	19.7747 (0.0000)	0.2115 (0.1926)	19.0775 (0.0000)

The dependent variables are the average growth rate of variables in columns.
*** significance at 1 percent level, ** significance at 5 percent level, * significance at 10 percent level. Standard errors in parenthesis.
(a) Bootstrapped standard errors (based on 999 simulations).
(b) Bootstrapped *p*-value in parenthesis (based on 10000 simulations).

Education shows a decreasing and convex pattern which suggests that the speed of convergence is underestimated for less educated countries given that the parametric estimate lies below the confidence bands of the PLM model. The income dimension presents a high convergence speed for low and medium developed countries, whereas a stagnation phase is observed for highly developed economies, which turns into a convergence process for the most advanced nations. Therefore, an important part of the estimated linear trend lies outside the nonparametric confidence interval, thus indicating that the conventional specification to test β-convergence would mask nonlinearities which are actually captured by the semiparametric model. On the other hand, semiparametric estimates reveal that the speed of convergence of the health index and the HDI follow linear trends which are statistically equal to the conventional convergence models since the parametric estimations lie inside the confidence bands of the PLM estimates in both cases.

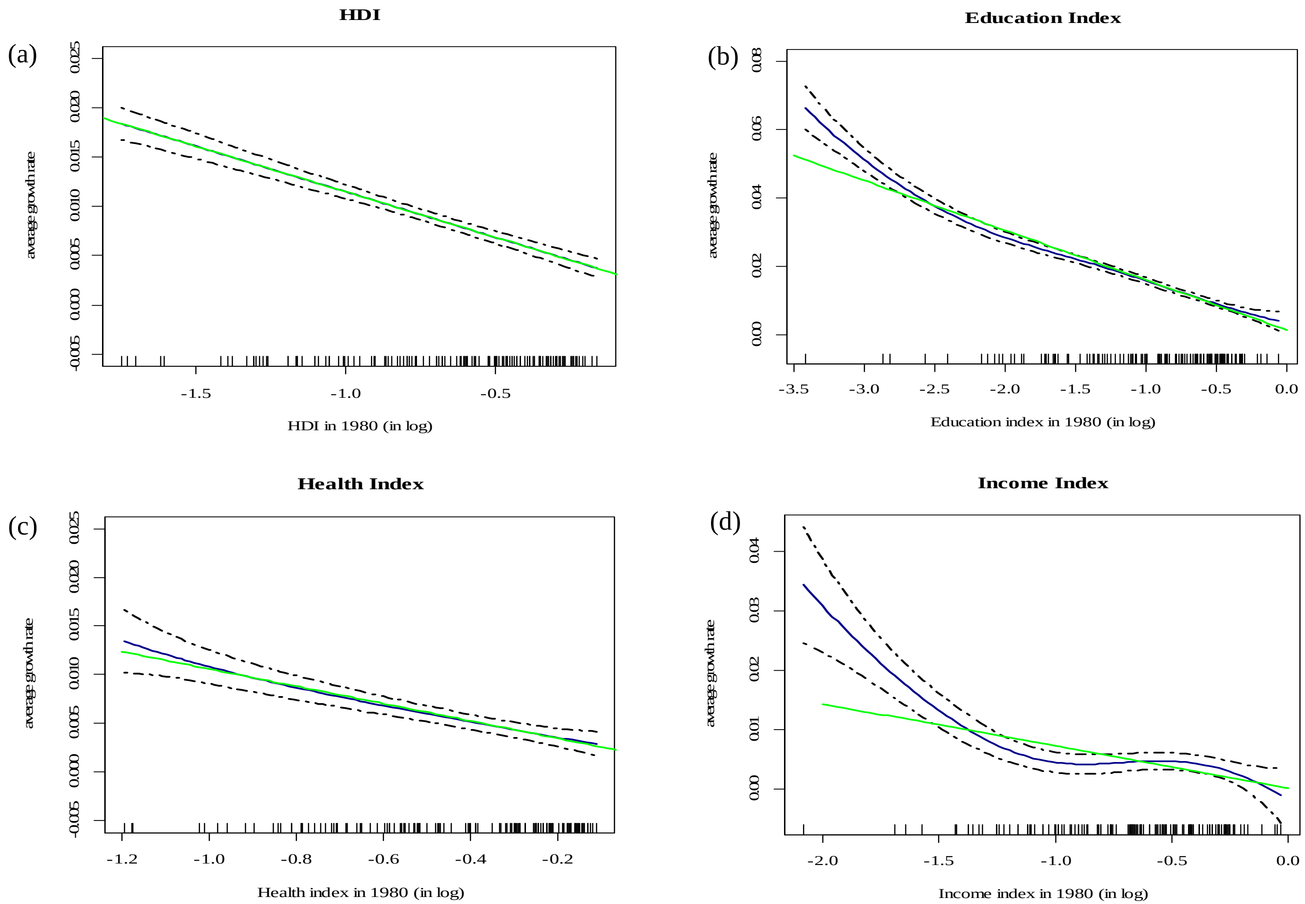

Figure 4.3. Nonparametric estimation of $f(y_{i0})$ according to Equation (4.7). In each case y_{i0} represents the natural logarithm of the HDI or its intermediate indices. The solid blue line corresponds to the estimate of $f(y_{i0})$ and the dashed curves delimit the 95 percent confidence bands. The solid green line represents the classical linear estimation of β-convergence

Table 4.4. Parametric and semiparametric estimations. Conditional convergence

Variable	HDI		Education index		Health index		Income index	
	OLS[a]	PLM	OLS[a]	PLM	OLS[a]	PLM	OLS[a]	PLM
$f(y_{i0})$	-0.0302*** (0.0024)	Figure 4.4a	-0.0511*** (0.0064)	Figure 4.4b	-0.0323*** (0.0035)	Figure 4.4c	-0.0220*** (0.0045)	Figure 4.4d
Intercept	0.0282*** (0.0018)	0.0036 (0.0027)	0.0434*** (0.0043)	0.0057 (0.0037)	0.0311*** (0.0029)	0.0006 (0.0006)	-0.0205*** (0.0033)	0.0039 (0.0033)
Arab States	-0.0023*** (0.0007)	-0.0016 (0.0010)	-0.0037* (0.0020)	-0.0010 (0.0015)	-0.0018** (0.0007)	-0.0009 (0.0009)	-0.0034*** (0.0009)	-0.0025 (0.0016)
East Asia	-0.0023** (0.0009)	-0.0013 (0.0011)	-0.0080*** (0.0019)	-0.0034** (0.0014)	-0.0027*** (0.0008)	-0.0017* (0.0009)	-0.0008 (0.0020)	0.0013 (0.0018)
Europe and Central Asia	-0.0025* (0.0009)	-0.0020** (0.0009)	-0.0033*** (0.0011)	-0.0013 (0.0012)	-0.0031*** (0.0006)	-0.0025*** (0.0009)	-0.0015 (0.0011)	-0.0002 (0.0016)
Latin America	-0.0037*** (0.0006)	-0.0029*** (0.0009)	-0.0073*** (0.0016)	-0.0030** (0.0012)	-0.0021*** (0.0006)	-0.0011 (0.0008)	-0.0047*** (0.0010)	-0.0033** (0.0015)
South Asia	-0.0022 (0.0013)	-0.0015 (0.0014)	-0.0054 (0.0037)	-0.0010 (0.0019)	-0.0031* (0.0016)	-0.0024* (0.0013)	-0.0029 (0.0025)	-0.0009 (0.0023)
Sub-Saharan Africa	-0.0079*** (0.0011)	-0.0073*** (0.0011)	-0.0089*** (0.0027)	-0.0043*** (0.0015)	-0.0105*** (0.0017)	-0.0099*** (0.0011)	-0.0093*** (0.0020)	-0.0080*** (0.0017)
Smoothing parameter		1.048		1.545		477.8		1.284
Adjusted R^2	0.7026		0. 7362		0.6133		0.3399	
RLRT test[b]	12.3821 (0.0001)		12.0611 (0.0001)		0.0000 (1.0000)		14.0182 (0.0001)	

The dependent variables are the growth rate of variables in columns.

*** significance at 1 percent level, ** significance at 5 percent level, * significance at 10 percent level. Standard errors in parenthesis.

(a) Bootstrapped standard errors (based on 999 simulations)

(b) Bootstrapped *p*-value in parenthesis (based on 10000 simulations)

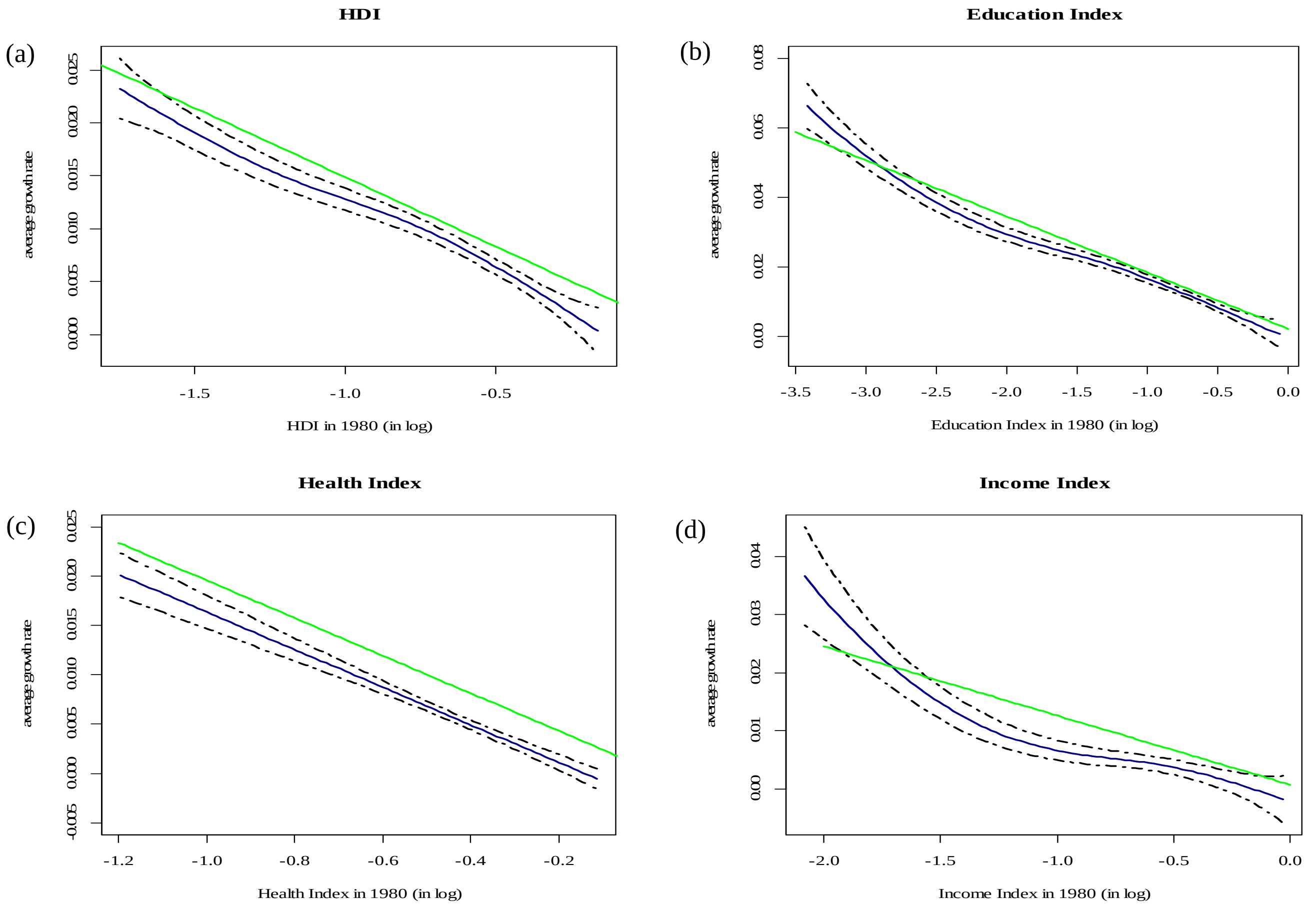

Figure 4.4. Nonparametric estimation of $f\,(y_{i0})$ according to Equation (4.8). In each case y_{i0} represents the natural logarithm of the HDI or its intermediate indices. The solid blue line corresponds to the estimate of $f\,(y_{i0})$ and the dashed curves delimit the 95 percent confidence bands. The solid green line represents the classical linear estimation of β-convergence

We have augmented Equations (4.5) and (4.7) with regional dummies which capture the existence of specific characteristics in each region, thus allowing for the existence of different steady states. Estimated results for conditional convergence (Equations (4.6) and (4.8)) are presented in Table 4.4. To avoid problems of prefect collinearity, we have omitted the dummy variable of the Western Europe, North America and Oceania (WENAO). From the parametric models, it is observed that the estimates of the speed of convergence increase substantially when regional dummy variables are included. This raise is particularly evident in the case of health and education, whose speed of convergence under the conditional framework is almost double the rate of absolute convergence.

Dummy variables are significant when looking at the OLS results and their negative sign indicates that the steady state level of well-being in these regions is lower than that of WENAO, which includes most of the advanced economies. Therefore, a significant heterogeneity in regional steady states is observed, thus suggesting the existence of a conditional convergence process.

To analyze how conditional convergence speed evolves with the level of development, nonparametric estimates for each dimension are presented in Figure 4.4. Conditional convergence patterns tend to be similar to the absolute ones but with higher slopes. As concluded by the RLRT test, the convergence process of the HDI seems to be nonlinear in this case, presenting lower convergence speed for medium developed countries. In fact, according to the results of the RLRT test, the health component is the sole dimension that is adequately represented by the conventional linear specification. Notwithstanding this fact, it should be noted that the parametric trend lies outside the confidence bands of PLM estimates although both lines have the same slope. This result would imply that even when the convergence speed is the same in both cases, the estimated steady states differ due to different estimates of the structural parameters.

4.7. Conclusions

In this chapter, we re-examine the hypothesis of β-convergence in well-being across different economies during the period 1980-2011. The HDI is used as an indicator of such a process, which considers education and health as essential as income in the measurement of quality of life. Specifically, we analyze the concepts of σ-convergence, which assumes that dispersion in living standards tends to decrease over time; and β-convergence, which implies a negative relationship between the initial level of a particular indicator and its growth rate. Conversely to previous studies which estimate parametric models based on a linear trend, we opt for a flexible semiparametric approach. This specification does not require making *a priori* assumptions about the model specification thus letting the data state by themselves how the convergence rate evolves as the level of human well-being increases. For comparative purposes, the parametric model is also estimated. Regional dummies are included to capture differences in the steady-state, which is associated with the concept of conditional β-convergence.

As a preliminary analysis, nonparametric kernel density estimates have been used to analyze graphically the evolution of well-being distribution. Our results suggest that HDI distribution has completely moved to the right over the last 30 years. A bimodal distribution is observed at the end of the period, which indicates the existence of convergence clubs. To quantify the variation in global inequality, we have computed four inequality measures for the HDI and its three components. Our results point out that the gap between developed and developing countries has been substantially reduced in a wide range of indicators of quality of life. The educational dimension shows the greatest reduction, around 60 percent, followed by health whose inequality levels fell about 30 percent, and finally, the income dimension only reduced its inequality by 10 percent over the study period. These trends have resulted in a much more egalitarian distribution of human well-being than 30 years ago. Given that the concentration of the population around different clusters cannot be measured using the traditional approach of inequality, we have applied the polarization measures developed by Esteban and Ray (1994). This analysis points out that bipolarization of well-being has increased slightly over the last three decades. However this result cannot be extrapolated to each dimension considered in the HDI separately. In fact, non-income dimensions reduced their polarization by about 4 percent, whereas the economic component increased it by slightly over 6 percent.

Regarding β-convergence, our results at least suggest weak absolute convergence in living standards over the last 30 years. This finding is robust to the introduction of regional dummies which leads to higher rates of convergence speed. PLM models reveal that whereas the absolute convergence process in human well-being is adequately represented by a linear trend, the conditional convergence process shows nonlinear patterns. Our results point out that, even when little advances have been achieved in income levels, significant improvements in non-income dimensions and human well-being have been accomplished. This conclusion highlights the relevance of considering non-income dimensions in the study convergence hypothesis, since their distributional patterns differ substantially from economic variables.

This study reveals that some degree of equalization in well-being levels took place in the last decades. However, the convergence process is rather slow and hence the action of international organizations is essential to achieve faster rates of convergence. International cooperation in social policies also plays a crucial role in increasing well-being levels in developing countries, thus moving on the direction of MDG. Given that less developed countries have a scarcity of technological and capital resources (UNDP, 2003; 2005; World Health Organization, 2003) more efforts from donor countries and international aid agencies are needed (Noorbakhsh, 2006). In fact, the fulfillment of a global partnership seems to be the key in achieving the eight targets of MDG by 2015 (United Nations, 2012). The role of the national governments is also important especially in expanding schooling rates. Their efforts to improve primary care are also determinant to enhance the quality of life (Kenny, 2005), thus encouraging a catching-up process between developing countries and leader economies.

Conclusiones

Resultados

La presente tesis doctoral aborda el estudio de la desigualdad en el bienestar desde cuatro perspectivas diferentes, correspondientes a cada uno de los capítulos que conforman este trabajo. La primera parte se basa en el estudio clásico de desigualdad de renta en la que se consideran variables puramente económicas. En los últimos años se ha señalado por parte de numerosos académicos la necesidad de considerar variables no monetarias, que representan aspectos igualmente relevantes del bienestar y que serían recogidos de forma imperfecta por el PIB (Sen, 1985; Streeten, 1994; Stiglitz et al., 2009).

En este contexto, ha habido numerosos intentos de sintetizar la compleja realidad que caracteriza el bienestar en un único indicador. De entre las diferentes propuestas, la que ha recibido mayor atención por parte de los medios, la comunidad científica y el ámbito político es el IDH. Bajo el marco normativo de este indicador, factores como la salud o la educación se postulan como aspectos fundamentales para medir los niveles de calidad de vida. Siguiendo este enfoque, la segunda parte este estudio evalúa la desigualdad del bienestar bajo una perspectiva multidimensional, utilizando para ello el nuevo paradigma del desarrollo sobre el que se asienta el IDH.

Numerosos estudios han analizado la desigualdad de ingresos ya sea a nivel global o desde una perspectiva regional o nacional. En consecuencia, se han propuesto una gran variedad de modelos paramétricos para estimar la distribución de la renta subyacente, a partir de la cual se calculan las correspondientes medidas de desigualdad. Recientemente, se ha presentado una nueva propuesta denominada *distribución Gaussiana modificada*, que permite ajustar de forma satisfactoria datos de ingreso a nivel individual para una muestra suficientemente extensa (Guo y Gao, 2012). Pese a las ventajas que presenta esta distribución respecto a la clásica distribución Gaussiana, sus propiedades estadísticas no han sido estudiadas en la literatura. En el Capítulo 1 se obtienen las propiedades probabilísticas de esta familia así como diversas medidas de desigualdad. Se presentan además dos métodos de estimación alternativos que permiten obtener los parámetros de esta distribución de forma consistente.

La relación que guarda esta distribución con otras familias pone de manifiesto algunos resultados interesantes. Se ha demostrado que la distribución Gaussiana modificada puede expresarse en términos de la distribución chi-cuadrado teniendo en cuenta que los grados de libertad deben ser multiplicados por el número de observaciones de la muestra. Asimismo, se ha investigado la relación entre la distribución Gaussiana modificada y la stretched exponential, modelo ampliamente utilizado en ciencias sociales. Este análisis revela que la distribución Gaussiana modificada presenta colas más pesadas que la stretched exponential y por tanto, es un modelo más adecuado para modelizar datos con esta característica como es el caso de las distribuciones de renta. Por último, se demuestra que el modelo estudiado en este capítulo se puede expresar como una distribución tipo Weibull si incluimos un parámetro de localización y se imponen ciertas restricciones sobre el parámetro de escala.

A modo de ilustración se han ajustado datos referentes a la renta individual en España para los años 1994, 1996 y 1999. A partir de dichas estimaciones es posible calcular las diferentes medidas de desigualdad previamente obtenidas. Los resultados referentes a los índices de desigualdad ponen de manifiesto que ha habido pocos avances en relación a las diferencias de renta entre españoles durante el periodo de estudio. Por otro lado, señalar que la curva de Lorenz en 1999 domina por completo a la curva de 1994, lo que indica que la distribución de renta en España es más equitativa al final del periodo de estudio.

En ocasiones, la información de la variable renta en términos de microdatos no está disponible. Asimismo, esta información puede no ser homogénea, lo que hace difícil las comparaciones entre países. Alternativamente, podemos utilizar varios de los estadísticos descriptivos de las encuestas primarias que en la mayoría de los casos son libremente accesibles. En este contexto, obtener distribuciones de renta a partir de información parcial resulta crucial a la hora de estudiar patrones de desigualdad y pobreza. El Capítulo 2 está dedicado a la estimación de las distribuciones de renta para 127 países durante la década de los noventa utilizando información limitada, en concreto, el ingreso medio y el valor del índice de Gini. En este estudio se propone como modelo paramétrico las llamadas *distribuciones de Lamé* que representan dos versiones curvadas de las distribuciones de Singh-Maddala y de Dagum. La principal característica de esta familia es que incluye modelos parsimoniosos, capaces de ajustar satisfactoriamente distribuciones de renta con sólo dos parámetros y cuyas curvas de Lorenz vienen caracterizadas por un único parámetro. A partir de las estimaciones nacionales y haciendo uso de los pesos poblacionales de cada país se obtienen las distribuciones de renta en siete regiones así como a nivel mundial, calculándose para cada una de ellas diferentes medidas de desigualdad y pobreza.

Los resultados obtenidos muestran una disminución de los ratios de pobreza a nivel global independientemente de la línea de pobreza considerada. No obstante, esta tendencia puede ocultar dinámicas regionales desiguales. De hecho, mientras que las regiones asiáticas han experimentado avances destacables durante la década de los noventa, con reducciones de los ratios de pobreza extrema superiores al 60 por ciento, África Subsahariana no ha conseguido erradicar la pobreza de forma significativa, mostrando ratios cercanos al 36 por ciento a lo largo de todo el periodo. Por otro lado, la transición a la economía de mercado ha afectado sustancialmente a la situación económica de los países del este de Europa, que muestra los mayores incrementos relativos en los ratios de pobreza, los cuales se han duplicado en tan solo 10 años.

Respecto a la evolución de las diferencias en los niveles de renta a nivel mundial, se observa una caída significativa de las mismas a lo largo del periodo de estudio. Los resultados obtenidos sugieren que esta tendencia tiene su origen en la reducción de la desigualdad entre países, gracias al crecimiento económico alcanzado por algunas de las naciones más pobladas, como China o India, que han sido capaces de compensar la caída de los ingresos en numerosos países africanos. Por otro lado, las desigualdades internas de los países se han incrementado de forma notable, si bien no lo suficiente como para compensar la reducción de las diferencias entre países. A nivel regional se observan, en términos generales, tendencias opuestas a las globales, siendo Asia del este y el Pacífico el único territorio que ha disminuido sus niveles de desigualdad.

El capítulo anterior estudia la evolución de la desigualdad de renta a nivel mundial. Sin embargo, en la evaluación del bienestar deben contemplarse otros aspectos como son la educación o la salud, cuyas distribuciones no siguen necesariamente los mismos patrones que el ingreso (Bourguignon y Morrison, 2002). En el Capítulo 3 se ha investigado la evolución de la distribución de los niveles nacionales de bienestar a nivel global durante los últimos treinta años. Para ello se ha desarrollado una nueva herramienta metodológica basada en la extensión del concepto de curva de Lorenz al espacio multidimensional. Para modelizar la distribución subyacente se considera la distribución de Sarmanov-Lee dado que presenta una estructura de correlación flexible con marginales dadas. Se ha obtenido además la expresión del índice de Gini asociado a dicha curva que se descompone en términos de la equidad interna de las variables, lo que se asocia al concepto de *distributive sensitive inequality* (Kolm, 1977), y el grado de asociación entre las dimensiones, lo que se corresponde con la denominada *association sensitive inequality* (Atkinson y Bourguignon, 1982).

La metodología anterior se emplea para modelizar las curvas de Lorenz bivariadas para los componentes del IDH. Por tanto, se estudia la desigualdad de las distribuciones conjuntas de ingreso y salud, educación y salud y educación e ingreso. Las estimaciones sugieren que todas la variables incluidas en el IDH han reducido sus niveles de desigualdad durante las últimas tres décadas. Como consecuencia, la desigualdad bidimensional también muestra tendencias decrecientes que parecen ser más acentuadas cuando se consideran conjuntamente variables de educación y salud. De hecho, estas dos dimensiones son las que muestran una mayor reducción de las disparidades de forma independiente, debido a los avances alcanzados por los países asiáticos. La distribución bidimensional que presenta una menor caída de las disparidades es la de salud e ingreso. Las estimaciones del índice de Gini para esta distribución muestran diferentes tendencias en la primera y segunda mitad del periodo de estudio. Mientras que las disparidades presentan una tendencia positiva de 1985 a 1995, la segunda parte del periodo se caracteriza por una caída significativa de la desigualdad.

Señalar que el índice de Gini bivariado proporciona información sobre la evolución de las distribuciones en términos agregados. La estimación de las curvas de Lorenz para las tres distribuciones bidimensionales indica que el resultado anterior debe extrapolarse con cautela a todas las partes de la distribución. De hecho, se observa que los países con menores niveles de renta, los que tienen estándares educativos más bajos y reducidos niveles de salud, presentan una situación más desigual al final del periodo de estudio. Esta tendencia pone de manifiesto la importancia de utilizar medidas de desigualdad que reflejen la evolución de la distribución en su conjunto y permitan establecer conclusiones para las diferentes partes de la misma.

La reducción de la desigualdad en los niveles nacionales de bienestar, se asocia en la literatura con la llamada convergencia σ. Este resultado implica la existencia de convergencia β, dado que se trata de una condición necesaria de la anterior. Sin embargo, esta dependencia no implica que el ratio de convergencia tenga que ser necesariamente lineal. El cuarto capítulo investiga la hipótesis de convergencia β en el bienestar utilizando un modelo semiparamétrico que permite introducir no linealidades en el proceso de convergencia.

A partir de las estimaciones obtenidas es posible concluir que la brecha entre los países desarrollados y en desarrollo se ha estrechado para cada uno de los índices intermedios del IDH individualmente, así como para el índice compuesto. Destacar que el proceso de convergencia ha sido especialmente lento a lo largo de las últimas tres décadas. Los bajos ratios de convergencia podrían estar ocultando no linealidades que se compensasen a nivel agregado. Esta cuestión se aborda mediante la utilización de contrastes de especificación que revelan que el proceso de convergencia en el bienestar ha sido lineal bajo el modelo incondicional. Por otro lado, bajo la hipótesis de convergencia condicional, que introduce en el modelo factores regionales que afectarían al estado estacionario de los países, este proceso presenta no linealidades que serían ignoradas por las especificaciones lineales clásicas.

Asimismo, las estimaciones realizadas revelan que las dimensiones no monetarias han evolucionado de forma positiva, mientras que las variables de renta apenas han convergido. Este resultado señala la importancia de considerar variables no económicas en el estudio de la convergencia en los niveles de bienestar, dado que las diferentes dimensiones presentan patrones distributivos distintos.

Futuras líneas de investigación

A partir de los diferentes enfoques y metodologías utilizadas, los cuatro capítulos de la presente tesis doctoral ofrecen una visión global de la evolución de las disparidades en el bienestar durante las últimas décadas. Si bien se ha dado respuesta a algunas preguntas importantes en el campo de la evaluación de la desigualdad, a lo largo del trabajo surgen ciertas cuestiones que pueden abordarse en futuras investigaciones.

Respecto a la distribución estudiada en el Capítulo 1, es posible desarrollar un modelo más general que incluya como caso particular la distribución Gaussiana modificada, y que permita trabajar con tamaños muestrales más pequeños. Esta nueva propuesta viene definida por la siguiente función de densidad:

$$f(x;\mu,\sigma,\alpha,\beta) = \frac{K}{\sigma}\left(\frac{x-\mu}{\sigma}\right)^{\alpha} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^{\beta}\right\}, \qquad x \geq \mu$$

y $f(x, \mu, \sigma, \alpha, \beta) = 0$ si $x < \mu$, siendo $\beta, \alpha, \sigma > 0$ y $\mu \in \Re$. El modelo anterior incluye como casos particulares las distribuciones gamma, gamma generalizada, Weibull y

Gaussiana modificada. A partir del modelo planteado es posible contrastar diversos submodelos mediante el test de razón de verosimilitudes.

En el Capítulo 2, se ha demostrado que el modelo empleado ofrece resultados satisfactorios en términos de bondad de ajuste. Sin embargo, el estudio de la distribución de la renta mundial está limitado a la década de los noventa debido a la escasa disponibilidad de datos en años posteriores. Asimismo, se ha estudiado la dinámica distributiva del ingreso en tres momentos de tiempo, 1990, 1995 y 2000, por lo que, el análisis a partir de datos quinquenales no mostraría las tendencias a corto plazo. La ampliación de la base de datos *World Income Inequality Database* (UNU-WIDER, 2008) contribuirá a extender este análisis durante un periodo temporal más largo, que incluya la primera década del siglo XXI. Además, se plantea la posibilidad de estimar la distribución de la renta anualmente, lo que permitirá investigar de forma más detallada las dinámicas distributivas del bienestar durante los años noventa.

En los dos últimos capítulos de este trabajo se analiza el bienestar desde una perspectiva multidimensional, utilizando el marco normativo propuesto por el IDH. Destacar que este indicador ha sido ampliamente criticado desde sus inicios en lo relativo a su construcción (Grimm et al., 2008; Kelley, 1991), a las variables utilizadas (Srinivasan, 1994), a las dimensiones consideradas (Alkire, 2002), a la redundancia con sus componentes (Cahill, 2005; McGillivray, 1991, McGillivray y White, 1993; Ravallion, 1997) y a la arbitrariedad de los pesos asignados a cada una de las dimensiones (McGillivray y White, 1993; Noorbakhsh, 1998). Si bien el IDH es uno de los indicadores más populares para realizar comparaciones en los niveles de calidad de vida a nivel nacional, la consideración de otros índices podría revelar resultados complementarios sobre la evolución de la desigualdad en los niveles de bienestar. En concreto, se propone emplear el denominado *full income* desarrollado por Becker et al. (2005), que contempla cuestiones monetarias y de salud, para estudiar la evolución de la desigualdad en el bienestar utilizando las herramientas desarrolladas en este estudio. Asimismo, se propone extender la curva de Lorenz bidimensional a más de dos dimensiones, modelizada a partir de la distribución de Sarmanov-Lee. A partir de dicha curva es posible obtener un índice de Gini *n*-dimensional descomponible en la desigualdad interna de las dimensiones y el grado de asociación entre ellas.

En el primer y segundo capítulo se ha estudiado la desigualdad de renta entre personas. Las estimaciones realizadas consideran la situación económica de cada uno de los individuos del análisis y por tanto los resultados que se obtienen a partir de dicho estudio presentan la evolución completa de la desigualdad de ingreso. Sin embargo, la disponibilidad de datos individuales de variables como la salud o la educación es relativamente escasa, especialmente para países en desarrollo durante largos periodos temporales. Dadas las limitaciones presentes en los datos, en los dos últimos capítulos los sujetos del análisis son los países en lugar de los individuos. Por tanto, cuando las variables no monetarias entran en juego, la desigualdad se mide en términos nacionales, de modo que los indicadores obtenidos informan sobre las diferencias entre los niveles de bienestar medios de los países incluidos en el análisis. A partir de la información disponible en la encuesta *Demographic and Health Survey* (DHS), será posible estudiar la evolución de la desigualdad del bienestar a nivel global contemplando tanto las disparidades internas de cada uno de los países como las diferencias entre ellos, ofreciendo así un análisis detallado y exhaustivo de la evolución de la distribución de los niveles de calidad de vida.

Conclusions

Main results

This doctoral thesis studies inequality in well-being using four different approaches, corresponding to each of the chapters of this work. The first part focuses on the classical study of income inequality, considering only purely economic variables. However, over the last decades there has been a growing consensus that other non-income variables represent aspects as relevant as wealth in the measurement of well-being, which would be imperfectly represented by the GDP (Sen, 1985; Streeten, 1994; Stigitz et al., 2009).

In this context, there have been many attempts to synthesize different aspects of well-being in a composite index which offers a more comprehensive perspective of such a process than per capita income alone. Among the different proposals, the HDI is the indicator that receives most attention from the media, policy-makers and academics. Under the framework provided by this indicator, factors such as health and education are postulated as essential aspects when evaluating the levels of quality of life in a particular country. Following this approach, the second part of this study assesses inequality in well-being using a multidimensional perspective, under the new paradigm of development.

Several studies have analyzed income inequality from both global and regional perspectives. Consequently, a large number of parametric models have been proposed to estimate the underlying income distribution from which to calculate different inequality measures. Recently, a new proposal called *modified Gaussian distribution* has been presented. It has been demonstrated that this model fits data on individual incomes for large samples satisfactorily (Guo and Gao, 2012). In spite of the advantages of this model whith respect to the classical Gaussian distribution, its statistical properties have not been studied in the literature. In Chapter 1, we obtain the probabilistic properties of this family as well as several inequality measures. We also describe two alternative estimation methods, which provide feasible estimates of the parameters of this distribution.

The relationship between this model and other families reveals some interesting findings. Regarding its relationship with the chi-square distribution, it has been demonstrated that the modified Gaussian distribution can be expressed in terms of this model, multiplying the degrees of freedom by the number of observations of the sample. The connection of this distribution with the stretched exponential, which is a widely used model in the field of social sciences, has been also investigated. This analysis reveals that the modified Gaussian distribution presents fatter tails than the stretched exponential. This is, therefore, a more adequate model to fit data with this characteristic as in the case of income distributions. Finally, this model can be expressed in terms of the Weibull distribution if a location parameter is included and some restrictions are imposed on the scale parameter.

To illustrate all the results derived from the previous analysis, we have fitted data on individual incomes in Spain for the years 1994, 1996 and 1999. Using these estimates, it is possible to compute the inequality measures developed in this chapter. The computed inequality measures point out that little advances have been achieved in terms of inequality in Spain during the study period. However, the Lorenz curve in 1999 dominates the curve completely in 1994, which implies that income distribution in Spain is more equal at the end of the study period.

On the other hand, it should be mentioned that individual data are generally hard to find. Moreover, this information could be heterogeneous in some occasions, which makes it difficult to perform cross-country comparisons. Alternatively, we could use the information provided by summary statistics of the primary surveys, which are available in most cases. In this context, the estimation of income distribution from such pieces of information is crucial when studying inequality and poverty patterns. Chapter 2 focuses on the estimation of national income distributions for 127 countries during the nineties, using the mean income and the value of the Gini index. In particular, we consider the so-called *Lamé family of distributions*, which are curved versions of the Singh-Maddala and Dagum distributions. The main characteristic of these distributions is that they represent parsimonious models which fit income data with just two parameters and whose Lorenz curves are characterized by only one parameter. Using national estimates and population weights for each country, regional and global income distributions are computed and several inequality and poverty measures are calculated.

Our results show a decrease in global poverty rates irrespective of the line considered given that income distribution in 2000 stochastically dominates income distribution in 1990 up to the last poverty line considered in this chapter. However, this global trend could hide uneven regional dynamics. In fact, while the Asian regions have seen notable advances during the course of the nineties, with reductions in extreme poverty of over 60 percent, Sub-Saharan Africa has failed in the eradication of poverty, presenting poverty rates close to 36 percent over the whole period. On the other hand, the transition to a market economy has substantially affected the economic situation of the Eastern European countries, showing the highest growth in poverty rates, which doubled in just one decade.

Regarding the evolution of income disparities at global level, we observe a fall of global inequality over the study period. Our results suggest that this trend is mainly due to the reduction of disparities between countries, derived from the outstanding economic growth of some of the most populous countries in the world, such as China and India, which have eclipsed the fall of income in most African countries. On the other hand, internal inequalities of the countries have increased notably over the nineties but not enough to offset the decrease in the differences across countries.

Conversely, ascending patterns are observed at regional level, except in the case of East Asia and the Pacific, which is the sole territory that has reduced its inequality levels.

Even when income has been the preferred indicator to measure disparities in well-being, other equally relevant factors should be included in the definition of this phenomenon such as health or education, whose distributions do not necessarily follow the same patterns of economic indicators (Bourguignon and Morrison, 2002). In Chapter 3, we have investigated the evolution of the national levels of well-being over the last 30 years. To achieve this goal, we have developed a new tool based on the extension of the Lorenz curve to the multidimensional space. The Sarmanov- Lee distribution is considered to model the underlying bivariate distribution since it has a flexible correlation structure with given marginals. The expression of the Gini index associated with that curve has also been obtained. This indicator can be decomposed in terms of equality within variables, which is associated with the concept of *distribution sensitive inequality* (Kolm, 1977), and the degree of association between dimensions (which corresponds to the so-called *association sensitive inequality* (Atkinson and Bourguignon, 1982)).

The described methodology has been used to model bivariate Lorenz curves for the components of the HDI. Therefore, we study inequality levels in the joint distributions of income and education, health and education and health and income. Our estimations suggest that all the variables included in the HDI have reduced their inequality levels in the last three decades. As a consequence, bidimensional inequality also shows decreasing patterns, which seem to be sharper when variables of health and education are jointly considered. In fact, separately, these two dimensions have seen the greatest reduction in their internal disparities, mainly driven by the advances achieved by Asian countries. The bidimensional distribution of income and health presents the lowest fall of disparities mainly due to the increase in inequality from 1985 to 1995. Despite the fact that this trend seems to be compensated in the second half of the study period, it should be noted that the ascending pattern observed in this period has slowed the fall of inequalities in national levels of income and health.

It should be recalled that the bivariate Gini index only provides summarized information about the evolution of well-being distribution. The estimation of the Lorenz curves for the three bidimensional distributions reveals that the previous result cannot be applied to all countries. In fact, it is observed that countries with low levels of income, poor educational standards and reduced levels of health present a more unequal situation at the end of the study period. This trend emphasizes the importance of using inequality measures that reflect the evolution of the whole distribution, thus providing conclusions for its different parts.

The reduction of inequality in well-being is associated with the concept of σ convergence in economics literature. This result would imply the existence of the so-called β convergence as a necessary condition. However, the dependence between these two concepts does not imply that the convergence ratio is necessarily linear. Chapter 4 investigates the hypothesis of β convergence in quality of life using a semiparametric model that allows us to introduce nonlinearities in the process of convergence.

According to our estimates, it is possible to conclude that the gap between developed and developing countries has narrowed for the HDI and each of its intermediate indices. Nevertheless, a slow process of convergence can be observed over the last three decades. It could be possible that the low rates of convergence are hiding nonlinearities in this process. To study this hypothesis, we use specification tests which reveal that the process of convergence in well-being has been linear in the unconditional framework. On the other hand, under the hypothesis of conditional convergence, this process presents nonlinearities that would be ignored by the classical lineal specifications.

It should also be remarked that our estimates reveal that non-income aspects have improved considerably, while the economic dimension has slightly converged. This result highlights the relevance of considering non-economic variables in the study of convergence in well-being, since different dimensions follow distributional patterns that vary significantly.

Future research

Using different approaches and methodologies, the four chapters offer an overview of the evolution of inequality in well-being during the past decades. Whereas many important issues in the field of evaluation of disparities have been addressed, different questions arise during this work. We present here four important ones which could lead to future research.

Regarding the distribution studied in Chapter 1, it is possible to develop a more general model that includes as a particular case the *modified Gaussian distribution*, allowing us to deal with smaller sample sizes. This new proposal is given by the following density function:

$$f(x;\mu,\sigma,\alpha,\beta) = \frac{K}{\sigma}\left(\frac{x-\mu}{\sigma}\right)^{\alpha} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^{\beta}\right\}, \qquad x \geq \mu$$

and $f(x, \mu, \sigma, \alpha, \beta) = 0$ if $x < \mu$, being $\beta, \alpha, \sigma > 0$ and $\mu \in \Re$. The previous model includes as special cases the distributions gamma, generalized gamma, Weibull and modified Gaussian. Using the general distribution, it is possible to test different sub-models using the likelihood ratio test.

In Chapter 2, it has been shown that the model considered provides satisfactory results in terms of goodness of fit. However, the study of world income distribution focuses on the nineties due to the restricted availability of data in subsequent years. Moreover, the study of the distributional dynamics of income is limited to three points of time, 1990, 1995 and 2000. It is possible that the analysis performed using 5-year data would hide short-term dynamics. The future improvement and extension of the *World Income Inequality Database* (UNU-WIDER, 2008) will contribute to the expansion of this analysis over a longer period of time, also including the first decade of the XXIst century. Moreover, annual income distributions could be estimated, allowing us to investigate the distributional dynamics of wealth during the nineties in more detail.

In response to the growing discontent with the use of solely economic variables to assess well-being levels, in the last two chapters, this work analyzes well-being from a multidimensional perspective using the normative framework proposed by the HDI. However, it should be noted that this indicator has been highly criticized on the grounds of construction (Grimm et al., 2008; Kelley, 1991), selection of variables (Srinivasan, 1994), arbitrary weighting scheme (McGillivray and White, 1993; Noorbakhsh, 1998), and redundancy with its components (Cahill, 2005; McGillivray, 1991; Ravallion, 1997). Even when the HDI is one of the most popular indicators for making comparisons of levels of quality of life, the consideration of other composite indices could reveal complementary results about the evolution of inequality in well-being. In particular, we propose to use the so-called *full income* developed by Becker et al. (2005), which includes income aspects and health variables, to study the evolution of inequality in well-being using the tools developed in this study. On the other hand, we propose to extend the bidimensional Lorenz curve to higher dimensions using the Sarmanov-Lee distribution. We would also obtain its associated *n*-dimensional Gini index, which could be decomposed in the inequality within-variables and the degree of association among dimensions.

In addition, note that Chapters 1 and 2 investigate inequality between individuals, which implies that we are considering the particular economic background of each person included in the analysis. As a consequence, the results derived from that study would offer complete conclusions about the evolution of income inequality. However, the availability of individual data on non-income variables such as health and education is relatively limited, especially in developing countries over long periods of time. Due to the restrictions of the data, the subjects of the analysis in the last two chapters are countries instead of individuals. Therefore, when non-income dimensions are included in the analysis, inequality measures inform about the differences in well-being levels across the countries considered in the study. Using data from the *Demographic and Health Survey* (DHS), it could be possible to examine the global evolution of inequality in well-being, considering both the internal disparities of each country and the differences between them, thus offering a detailed and comprehensive analysis of well-being distribution.

References

Adams, R.H. & Page, J. (2003). Poverty, inequality and growth in selected Middle East and North Africa countries, 1980–2000. *World Development*, 31(12), 2027–2048.

Aigbokhan, B.E. (2008). Growth, inequality and poverty in Nigeria. Prepared for United Nations Economic Commission for Africa (UNECA). Addis Ababa, Ethiopia.

Alkire, S. (2002). Dimensions of Human Development. *World Development*, 30, 181–205.

Alkire, S. & Foster, J. (2010). Designing the Inequality-Adjusted Human Development Index (IHDI). Human Development Research Paper, 2010/28, UNDP.

Anand, S. & Segal, P. (2008). What do we know about global income inequality?. *Journal of Economic Literature*, 46, 57–94.

Anand, S. & Sen, A. (2000). The Income Component of the Human Development Index. *Journal of Human Development*, 1, 83–106.

Arnold, B.C. (1983). *Pareto Distributions*. Maryland: International Co-operative Publishing House.

Arnold, B.C. (1987). *Majorization and the Lorenz Curve.* Lecture Notes in Statistics 43, New York: Springer Verlag.

Arnold, B.C., Castillo, E. & Sarabia, J.M. (2006). Families of multivariate distributions involving the Rosenblatt transformation, *Journal of the American Statistical Association*, 101, 1652–1662.

Atchison, J. & Brown, J.A.C. (1957). *The lognormal Distribution*. Cambridge: Cambridge University Press.

Atkinson, A.B. (1970). On the Measurement of Inequality. *Journal of Economic Theory*, 2, 244–263.

Atkinson, A.B. (2003). Multidimensional deprivation: contrasting social welfare and counting approaches. *Journal of Economic Inequality*, 1, 51–65.

Atkinson, A.B. & Bourguignon, F. (1982). The comparison of multi-dimensioned distributions of economic status. *Review of Economic Studies*, 49, 181–201.

Azomahou, T.T., Nguyen-Van, P. & Pham, T.K.C. (2011). Testing convergence of European regions: A semiparametric approach. *Economic Modelling*, 28, 1202–1210.

Baer, W. (1995). *The Brazilian economy: growth and development*. 4th Ed., New York: Praeger.

Bairamov, I & Kotz, S. (2003). On a new family of positive quadrant dependent bivariate distributions. *International Mathematical Journal*, 3, 1247–1254.

Barro, R.J. (1991). Economic growth in a cross section of countries. *The Quarterly Journal of Economics*, 106, 407–443.

Barro, R. & Lee, J.-W. (2010). A New Data Set of Educational Attainment in the World, 1950-2010. NBER Working Paper 15902, National Bureau of Economic Research, Massachusetts.

Barro, R.J. & Sala-i-Martin, X. (1990). Economic growth and convergence across the United States. NBER working paper No.3419.

Barro, R. & Sala-i-Martin, X. (1992). Convergence. *Journal of Political Economy*, 100, 407–433.

Becker, S.G., Philipson, T.J. & Soares, R.R. (2005). Quantity and Quality of Life and the Evolution of World Inequality. *American Economic Review*, 95, 277–291.

Bhalla, S. (2002). *Imagine There is No Country*. Washington, DC: Institute for International Economics.

Bilbao-Ubillos, J. (2011). The limits of Human Development Index: The complementary role of economic and social cohesion, development strategies and sustainability. *Sustainable Development*, doi: 10.1002/sd.525.

Bilbao-Ubillos, J. (2013). Another Approach to Measuring Human Development: The Composite Dynamic Human Development Index. *Social Indicators Research*, doi: 10.1007/s11205-012-0015-y

Bliss, C. (1999). Galton's Fallacy and economic convergence. *Oxford Economic Papers*, 51, 4–14.

Bosworth, B. & Collins, S.M. (2008). United States-China trade: where are the exports?. *Journal of Chinese Economic and Business Studies*, 6(1), 1–21.

Bourguignon, F. & Chakravarty, S.R. (2003). The measurement of multidimensional poverty. *The Journal of Economic Inequality*, 1, 25–49.

Bourguignon, F. & Morrison, C. (2002). Inequality among World Citizens. *American Economic Review*, 92, 727–744.

Briassoulis, H. (2001). Sustainable development and its indicators: Through a (Planner's) Glass darkly. *Journal of Environmental Planning and Management*, 44, 409–427.

Cahill, M.B. (2005). Is the Human Development Index Redundant?. *Eastern Economic Journal*, 31, 12–19.

Castilho, M., Menéndez, M. & Sztulman, A. (2012). Trade liberalization, inequality, and poverty in Brazilian states. *World Development*, 40, 821-835.

Castillo, E., Hadi, A.S., Balakrishnan, N. & Sarabia, J.M. (2005). *Extreme Value and Related Models with Applications in Engineering and Science*, New York: John Wiley.

Chen, S. & Ravallion, M. (2004). How Did the World's Poorest Fare since the Early 1980s?. *The World Bank's Research Observer*, 19, 141–170.

Chen, S. & Ravallion, M. (2010). The developing world is poorer than we thought, but no less successful in the fight against poverty. *Quarterly Journal of Economics*, 125, 1577–1625.

Chotikapanich, D. & Griffiths, W.E. (2008). Estimating Income Distributions Using a Mixture of Gamma Densities," Department of Economics - Working Papers Series 1034, The University of Melbourne.

Chotikapanich, D., Grifiths, W., Rao, D.S.P. & Valencia, V. (2009). Global Income Distribution and Inequality: 1993 and 2000. University of Melbourne Research Paper No. 1062.

Chotikapanich, D. & Rao, D.S.P. (1998). Inequality in Asia 1975-1990: A Decomposition Analysis. *The Asia Pacific Journal of Economics and Business*, 2, 63–78.

Chotikapanich, D., Rao, D. S. P. & Tang, K. K. (2007). Estimating income inequality in China using grouped data and the generalized beta distribution. *Review of Income and Wealth*, 53, 127–147.

Chotikapanich, D., Valenzuela, M.R. & Rao, D.S.P. (1997). Global and Regional Inequality in the Distribution of Income: Estimation with Limited Incomplete Data. *Empirical Economics*, 20, 533–546.

Clementi, F., Di Matteo, T., Gallegati, M. & Kaniadakis, G. (2008). The k-generalized distribution: a new descriptive model for the size distribution of incomes. *Physica A*, 287, 3201–3208.

Cornwall, A. & Gaventa, J. (2009). From users and choosers to makers and shapers: Repositioning participation in social policy. *IDS Bulletin*, 31, 50–62.

Cowell, F.A. (2011). *Measuring Inequality*. Fifth ed., New York: Oxford University Press.

Crainiceanu, M.C. & Ruppert, D. (2004). Likelihood ratio test in linear mixed models with one variance component. *Journal of the Royal Statistical Society: Series B*, 66, 165–185.

Dagum, C. (1977). A new model a personal income distribution: Specification and estimation. *Economie Appliquée,* 30, 413–437.

Davies, J.B., Green, D.A. & Paarsch, H.J. (1998). Economic statistics and social welfare comparisons. A review. In Dekker, M. (Ed.): *Handbook of Applied Economic Statistics,* New York: CRC Press, pp. 1–38.

Davies, J.B. & Shorrocks, A.F. (1989). Optimal grouping of income and wealth data. *Journal of Econometrics,* 42, 97–108.

De la Fuente, A. (1997). The empirics of growth and convergence: a selective review. *Journal of Economic Dynamics and Control*, 21, 23–73.

De la Fuente, A. (2002). On the sources of convergence: A close look at the Spanish regions. *European Economic Review,* 46, 569–599.

Deaton, A. (2004). Health in Age of Globalization. Brookings Trade Forum 2004, Brookings Institution, Washington D.C.

Deaton, A. (2010). Price indexes, inequality, and the measurement of world poverty. *American Economic Review*, 100, 1–34.

Decancq, K. (2011). Global inequality: A multidimensional perspective. Available at SSRN: http://dx.doi.org/10.2139/ssrn.1833253.

Decancq, K., Decoster, A. & Schokkaert, E. (2009). Evolution of World Inequality in Well-being. *World Development*, 37, 11–25.

Decancq, K. & Lugo M.A. (2012). Inequality of Wellbeing: A Multidimensional Approach. *Economica*, 79, 721–746.

Deininger, K. & Squire, L. (1996). New Data Set Measuring Income Inequality. *World Bank Economic Review*, 10, 565–591.

Di Matteo, T., Aste, T. & Hyde, S.T. (2004). Exchanges in complex networks: income and wealth distributions, in: F. Mallamace, H.E. Stanley (Eds.), *The Physics of Complex Systems,* Amsterdam: IOS Press, pp. 43.

Dobson, S. & Ramlogan, C. (2002). Economic growth and convergence in Latin America. *Journal of Development Studies*, 38, 83–104.

Domínguez, R., Guijarro, M. & Trueba, C. (2011). Recuperando la dimensión política del desarrollo humano. *Sistema*, 220, 11–31.

Donaldson, D. & Weymark, J.A. (1980). A single-parameter generalization of the Gini index of inequality. *Journal of Economic Theory*, 22, 67–86.

Duclos, J.-C., Sahn, D. & Younger, S. (2006). Robust multidimensional poverty comparisons. *The Economic Journal*, 116, 943–968.

Duclos, J.-C., Sahn, D. & Younger, S. (2011). Partial multidimensional inequality orderings. *Journal of Public Economics*, 95, 225–238.

Durlauf, S.N. (2001). Manifesto for a growth econometrics. *Journal of Econometrics*, 100, 65–69.

Durlauf, S.N., Kourtellos, A. & Minkin, A. (2001). The local Solow growth model. *European Economic Review*, 45, 928–940.

Edgier, V.S. & Tatlidil, H. (2006). Energy as an indicator of human development: A statistical approach. *The Journal of Energy and Development*, 31, 213–232.

Esteban, J.M. & Ray, D. (1994) On the measurement of polarization. *Econometrica*, 62, 819–51.

Ezcurra, R. & Pascual, P. (2005). Is there convergence in income inequality levels among the European regions?. *Applied Economics Letters*, 12, 763–767.

Fakuda-Parr, S., Lawson-Remer, T. & Randolph, S. (2009). An Index of Economic and Social Rights Fulfillment: Concept and Methodology. *Journal of Human Rights*, 38, 195–221.

Ferrara, A.R. & Nisticò, R. (2013). Well-Being Indicators and Convergence across Italian Regions. *Applied Research in Quality of Life*, 8, 15–44.

Fiaschi, D. & Lavezzi, A.M. (2007). Nonlinear economic growth: Some theory and cross-country evidence. *Journal of Development Economics*, 84, 271–290.

Figueira, F.C., Moura Jr., N.J. & Ribeiro, M.B. (2011). The Gompertz–Pareto income distribution. *Physica A*, 390, 689–698.

Firebaugh, G. (2000). The Trend in Between-Nation Income Inequality. *Annual Review of Sociology*, 26, 323–339.

Gastwirth, J.L. (1971). A general definition of the Lorenz curve. *Econometrica*, 39, 1037–1039.

Goesling, B. & Firebaugh, F. (2004). The trend in international health inequality. *Population and Development Review*, 30, 131–146.

Grimm, M, Harttgen, K., Klasen, S. & Misselhorn, M. (2008). A Human Development Index by Income Groups. *World Development*, 36, 2527–2746.

Grüen, C. & Klasen, S. (2008). Growth, inequality, and welfare: comparisons across space and time. *Oxford Economic Papers*, 60, 212–236.

Guo, Q. & Gao, L. (2012). Distribution of individual incomes in China between 1992 and 2009. *Physica A*, 391, 5139–5145.

Henle, J.M., Horton, N.J. & Jakus, S.J. (2008). Modelling Inequality with a Single Parameter. In: D. Chotikapanich (Ed.), *Modeling Income Distributions and Lorenz Curves*. Series: Economic Studies in Inequality, Social Exclusion and Well-Being, 5, New York: Springer Verlag, pp. 255–269.

Heston, A., Summers. R. & Aten B. (2012). Penn World Table Version 7.1. Center for Inter-national Comparisons of Production, Income and Prices at the University of Pennsylvania.

Hicks, D.A. (1997). The Inequality-Adjusted Human Development Index: A Constructive Proposal. *World Development*, 25, 1283–1298.

Hobin, B. & Franses, P.H. (2001). Are living standards converging? *Structural Change and Economics Dynamics*, 12, 171–200.

Holzmann, H., Vollmer, S. & Weisbrod, J. (2007). Income distribution dynamics and pro-poor growth in the world from 1970 to 2003 (No. 161). Discussion papers//Ibero America Institute for Economic Research.

Huang, H.C. (2005). Diverging evidence of convergence hypothesis. *Journal of Macroeconomics*, 27, 233–255.

Huang, J.S. & Kotz, S. (1999). Modifications of the Farlie-Gumbel-Morgenstern distributions. A tough hill to climb. *Metrika*, 49, 135–145.

Islam, N. (2003). What have we learnt from the convergence debate?. *Journal of Economic Surveys*, 17, 309–362.

Johnson, N.L., Kotz, S. & Balakrishnan, N. (1970). *Continuous univariate distributions*, vol. 1. New York: Wiley

Johnson, N.L., Kotz, S. & Balakrishnan, N. (1995). *Continuous univariate distributions*, vol. 2. New York: Wiley.

Kakwani, N.C. (1977). Applications of Lorenz Curves in Economic Analysis. *Econometrica*, 45, 719–728.

Kakwani, N.C. (1984). Welfare ranking in income distribution. *Advances in Econometrics*, 3, 191–213.

Kaniadakis, G. (2009). Maximum entropy principle and power-law tailed distributions. *The European Physical Journal B*, 70, 3–13.

Kelley, A. (1991). The Human Development Index: "Handle with Care". *Population and Development Review*, 17, 315–324.

Kenny, C. (2005). Why are we worried about income? Nearly everything that matters is converging. *World Development*, 33, 1–19.

Kleiber, C. & Kotz, S. (2003). *Statistical Size Distributions in Economics and Actuarial Sciences*, Hoboken, NJ: John Wiley.

Klugman, J., Rodríguez, F. & Choi, H.-J. (2011). The HDI 2010: New Controversies, Old Critiques. *Journal of Economic Inequality*, 9, 249–288.

Kolm, S.C. (1977). Multidimensional Equalitarianisms. *Quarterly Journal of Economics*, 91, 1–13.

Konya, L. (2011). New panel data evidence of human development convergence from 1975 to 2005. *Global Business and Economics Review*, 13, 57–70.

Konya, L. & Guisan, M.C. (2008). What does the human development index tell us about convergence? *Applied Econometrics and International Development*, 8, 19–40.

Koshevoy, G. (1995). Multivariate Lorenz majorization. *Social Choice and Welfare*, 12, 93–102.

Koshevoy, G. & Mosler, K. (1996). The Lorenz zonoid of a multivariate distribution. *Journal of the American Statistical Association*, 91, 873–882.

Koshevoy, G. & Mosler, K. (1997). Multivariate Gini indices. *Journal of Multivariate Analysis*, 60, 252–276.

Koshevoy, G. & Mosler, K. (2007). Multivariate Lorenz dominance based on zonoids. *AStA Advances in Statistical Analysis*, 91(1), 57–76.

Kovacevic, M. (2010a). Measurement of Inequality in Human Development-A Review. Human Development Research Paper No. 35. UNDP.

Kovacevic, M. (2010b). Review of HDI Critiques and Potential Improvements. Human Development Research Paper No. 33. UNDP.

Laherre, J. & Sornette, D. (1999). Stretched exponential distributions in nature and economy: fat tails with characteristic scales. *European Physical Journal B*, 2, 525–539.

Lawson-Remer, T., Fukuda-Parr, S. & Randolph, S. (2009). An index of economic and social rights fulfillment: Concept and methodology. *Journal of Human Rights*, 8, 195–221.

Lee, M-L.T. (1996). Properties of the Sarmanov Family of Bivariate Distributions, *Communications in Statistics, Theory and Methods*, 25, 1207–1222.

LIS (2000). Luxembourg Income Study. http://www.lisproject.org/

Liu, Z. & Stengos, T. (1999). Non-linearities in cross-country growth regressions: a semiparametric approach. *Journal of Applied Econometrics*, 14, 527–538.

Londoño, J.L. & Székely, M. (2000). Persistent Poverty and Excess Inequality: Latin America, 1970-1995. *Journal of Applied Economics*, 1, 93–134.

Maasoumi, E. (1986). The measurement and decomposition of multi-dimensional inequality. *Econometrica*, 54, 991–997.

Marchante, A.J., Ortega, B. & Sánchez, J. (2006). The Evolution of Well-Being in Spain (1980–2001): a regional analysis. *Social Indicators Research*, 76, 283–316

Marshall, A.W., Olkin, I. & Arnold, B.C. (2011). *Inequalities: Theory of Majorization and Its Applications*. Second ed., New York: Springer.

Martínez, R. (2012). Inequality and the new human development index. *Applied Economics Letters*, 19, 533–535.

Mauro, J.C. & Smedskjaer, M.M. (2012). Unified physics of stretched exponential relaxation and Weibull fracture statistics. *Physica A*, 391, 6121–6127.

Mayer-Foulkes, D. (2003). Convergence Clubs in Cross-Country Life Expectancy Dynamics. In: van der Hoeven & R., Shorrocks, A. (Eds.), *Perspectives on Growth and Poverty*. Tokyo: United Nations University Press, pp. 144–171.

Mayer-Foulkes, D. (2010). Divergences and Convergences in Human Development. Human Development Research Paper No. 2010/20. UNDP.

Mazumdar, K. (2000). Inter-country inequality in social indicators of development. *Social Indicators Research*, 49, 335–345.

Mazumdar, K. (2002). A Note on Cross-Country Divergence in Standard of Living. *Applied Economics Letters*, 9, 87–90.

Mazumdar, K. (2003). Do Standards of Living Converge? A cross-country Study. *Social Indicators Research*, 64, 29–50.

McDonald, J.B. (1984). Some generalized functions for the size distribution of income. *Econometrica*, 52, 647–663.

McDonald, J.B. & Xu, Y.J. (1995). A generalization of the beta distribution with applications. *Journal of Econometrics*, 66, 133–152.

McGillivray, M. (1991). The Human Development Index: Yet Another Redundant Composite Development Indicator? *World Development*, 19, 1461–1468.

McGillivray, M. & Markova, N. (2010). Global Inequality in Well-being Dimensions. *Journal of Development Studies*, 46, 371–378.

McGillivray, M. & Pillarisetti, J.R. (2004). International Inequality in Well-Being. *Journal of International Development*, 16, 563–574.

McGillivray, M. & Shorrocks, A. (2005). Inequality and multidimensional well-being. *Review of Income and Wealth*, 51(2), 193–199.

McGillivray, M. & White, H. (1993). Measuring development? The UNDP's Human Development Index. *Journal of International Development*, 5, 183–192.

Milanovic, B. (2002). True World Income Distribution, 1988 and 1993: First Calculation Based on Household Surveys Alone. *Economic Journal* 112, 51–92.

Milanovic, B. (2005). *Worlds Apart: Measuring International and Global Inequality*. Princeton and Oxford: Princeton University Press.

Minoiu, C. (2007). Poverty Analysis based on kernel density estimates from grouped data. Mimeo.

Morris, M.D. (1979). *Measuring the Conditions of the World's Poor: The Physical Quality of Life Index*. New York: Pergamon.

Morrison, C. & Murtin, F. (2012). The Kuznets Curve of Human Capital Inequality: 1870-2012. *Journal of Economic Inequality*, 11, 283–301.

Moura Jr., N.J. & Ribeiro, M.B. (2009). Evidence for the Gompertz curve in the income distribution of Brazil 1978–2005. *The European Physical Journal B*, 69, 101–120.

Muliere, P. & Scarsini, M. (1989). A note on stochastic dominance and inequality measures. *Journal of Economic Theory*, 49, 314–323.

Muller, C. & Trannoy, A. (2011). A dominance approach to the appraisal of the distribution of well-being across countries. *Journal of Public Economics,* 95, 239–246.

Nagaraj, R. (2000). Indian economy since 1980: Virtuous growth or polarisation?. *Economic and Political Weekly*, 2831–2839.

Neumayer, E. (2001). The Human Development Index and Sustainability - A Constructive Proposal. *Ecological Economics*, 39, 101–114.

Neumayer, E. (2003). Beyond income: convergence in living standards, big time. *Structural Change and Economic Dynamics*, 14, 275–296.

Noorbakhsh, F. (1998). The Human Development Index: Some Technical Issues and Alternative Indices. *Journal of International Development*, 10, 589–605.

Noorbakhsh, F. (2006). International convergence or higher inequality in human development? Evidence for 1975-2002. WIDER Research Paper No. 2006/15.

O'Leary, E. (2001). Convergence of Living Standards among Irish Regions: The Roles of Productivity, Profit Outflows and Demography, 1960–1996. *Regional Studies*, 35, 197–205.

Ogwang, T. & Abdou, A. (2003). The Choice of Principal Variables for Computing Human Development Indicators. *Social Indicators Research*, 64, 139–152.

Oliver-Alonso, J., Ramos, X. & Raymond-Bara, J.L. (2001). Recent trends in Spanish income distribution: a robust picture of falling income inequality. In: FUNCAS Working Paper 166, Madrid.

Pareto, V. (1897). The new theories of economics. *The Journal of Political Economy*, 5, 485–502.

Patil, G.P. (2002). Weighted distributions. In: A.H. El-Shaarawi, W.W. Piegorsch (Eds.), *Encyclopedia of Environmetrics*, 4, New York: Wiley, pp. 2369–2377.

Patil, G.P., Rao, C.R. & Zelen, M. (1988). Weighted distributions. In: S. Kotz, N.L. Johnson (Eds.), *Encyclopedia of Environmetrics*, 9, Wiley: New York, pp. 565–571.

Piketty, T. & Saez, E. (2003). Income inequality in the United States, 1913–1998. *The Quarterly Journal of Economics,* 118(1), 1–41.

Pillarisetti, J.R. (1997). An empirical note on inequality in the world development indicators. *Applied Economic Letters*, 4, 145–147.

Pinkovskiy, M. & Sala-i-Martin, X. (2009). Parametric estimations of the World Income Distribution. NBER Working Paper No. 15433.

Pritchett, L. (1997). Divergences, Big Time. *Journal of Economic perspectives*, 11, 3–17.

Quah, D. (1993) Galton's fallacy and tests of the convergence hypothesis. *Scandinavian Journal of Economics*, 95, 427–443

Quah, D. (1996) Regional convergence clusters across Europe. *European Economic Review*, 40, 1951–1958.

Ravallion, M. (1997). Good and Bad Growth: The Human Development Reports. *World Development*, 25, 631–638.

Ruppert, D. (2002). Selecting the number of knots for penalized splines. *Journal of Computational and Graphical Statistics*, 11, 735–757.

Ruppert, D., Wand, M.P. & Carroll, R.J. (2003). *Semiparametric Regression*. London: Cambridge University Press.

Ryu, H.K. & Slottje, D.J. (1996). Two flexible functional form approaches for approximating the Lorenz curve. *Journal of Econometrics*, 72, 251–274.

Sab, R. & Smith, S.C. (2001). Human capital convergence: International evidence (Vol. 1). IMF Working Paper. International Monetary Fund.

Sagar, A.D. & Najam, A. (1998). The Human Development Index: A Critical Review. *Ecological Economics*, 25, 249–264.

Sala-i-Marin, X. (1996). The Classical Approach to Convergence Analysis. *The Economic Journal*, 106, 1019–1033.

Sala-i-Martin, X. (2000). Apuntes de crecimiento económico. Barcelona: Antoni Bosch Editor.

Sala-i-Martin, X. (2006). The World Distribution of Income: Falling Poverty and... Convergence, Period. *Quarterly of Journal Economics*, 121, 351–397.

Salem, A.B. & Mount, T.D. (1974). A convenient descriptive model of income distribution: the gamma density. *Econometrica*, 42, 1115–1127.

Sarabia, J.M. (2008). Parametric Lorenz curves: models and applications. In: D. Chotikapanich (Ed.), *Modeling Income Distributions and Lorenz Curves*. Series: Economic Studies in Inequality, Social Exclusion and Well-Being, 4, New York: Springer-Verlag, pp. 167–190.

Sarabia, J.M., Castillo, E., Pascual, M. & Sarabia, M. (2007). Bivariate income distribution with lognormal conditionals. *Journal of Economic Inequality*, 5, 371–383.

Sarabia, J.M., Jordá, V. & Trueba, C. (2013). The Lamé class of Lorenz curves. *Communications in Statistics, Theory and Methods*, DOI: 10.1080/03610926.2013.775306.

Sarabia, J.M. & Prieto, F. (2009). The Pareto-positive stable distribution: a new descriptive model for city size data. *Physica A*, 388, 4179–4191.

Sarmanov, O.V. (1966). Generalized Normal Correlation and Two-Dimensional Frechet Classes, *Doklady (Soviet Mathematics)*, 168, 596–599.

Schinckus, C. (2009). Economic uncertainty and econophysics. *Physica A*, 388, 4415–4423.

Schinckus, C. (2010). Econophysics and economics: sister disciplines?. *American Journal of Physics*, 78, 325–327.

Sen, A. (1985). *Commodities and Capabilities*. North-Holland, Amsterdam.

Sen, A. (1988). The Concept of Development, in Chenery H. and Srinivasan, T.N. (Eds.), *Handbook of Development Economics*. Amsterdam: Elsevier, pp. 9–26.

Sen, A. (1989). Development as Capabilities Expansion. *Journal of Development Planning*, 19, 41–58.

Sen, A. (1999). *Development as Freedom*. Oxford: Oxford University Press.

Seth, S. (2009). Inequality, Interactions, and Human Development. *Journal of Human Development and Capabilities*, 10, 375–396.

Seth, S. (2013). A class of distribution and association sensitive multidimensional welfare indices. *Journal of Economic Inequality*. DOI10.1007/s10888-011-9210-3

Shorrocks, A.F. (1983). Ranking income distributions. *Economica*, 54, 3–17.

Silverman, B.W. (1986): *Density Estimation for Statistics and Data Analysis*, London: Chapman and Hall.

Singh, S.K. & Maddala, G.S. (1976). A function for the size distribution of incomes. *Econometrica*, 44, 963–970.

Slottje, D.J. (1987). Relative Price Changes and Inequality in the Size Distribution of Various Components. *Journal of Business and Economics Statistics*, 5, 19–26.

Slottje, D.J. (1990). Using grouped data for constructing inequality indices, parametric vs. non parametric methods. *Economics Letters*, 32, 193–197.

Solow, R.M. (1956). A Contribution to the Theory of Economic Growth. *Quarterly Journal of Economics*, 70, 65–94.

Solow, R.M. (1957). Technical Change and the Aggregate Production Function. *Review of Economics and Statistics*, 39, 312–320.

Solt, F. (2009). Standardizing the world income inequality database. *Social Science Quarterly*, 90(2), 231–242.

Sornette, D. (2004). *Critical Phenomena in Natural Sciences, Chaos, Fractals, Self-organization and Disorder: Concepts and Tools,* second ed., Heidelberg: Springer.

Sornette, D. (2009). Probability distributions in complex systems. arXiv:0707.2194v1.

Srinivasan, T.N. (1994). Human development: a new paradigm or a reinvention of the wheel?. *American Economic Review*, 84, 238–243.

Stiglitz, J.E., Sen, A. & Fitoussi, J.P. (2009). Report by the commission on the measurement of economic performance and social progress. Commission on the Measurement of Economic Performance and Social Progress.

Streeten, P. (1994). Human development: means and ends. *The American Economic Review*, 84(2), 232–237.

Swan, T.W. (1956). Economic Growth and Capital Accumulation, *Economic Record*, 32, 334–361.

Taguchi, T. (1972a). On the two-dimensional concentration surface and extensions of concentration coefficient and Pareto distribution to the two-dimensional case-I. *Annals of the Institute of Statistical Mathematics*, 24, 355–382.

Taguchi, T. (1972b). On the two-dimensional concentration surface and extensions of concentration coefficient and Pareto distribution to the two-dimensional case-II. *Annals of the Institute of Statistical Mathematics*, 24, 599–619.

Taguchi, T. (1988). On the structure of multivariate concentration—some relationships among the concentration surface and two variate mean difference and regressions. *Computational Statistics & Data Analysis*, 6(4), 307–334.

Tsui, K.Y. (1995). Multidimensional generalizations of the relative and absolute inequality indices: the Atkinson-Kolm-Sen approach. *Journal of Economic Theory*, 67, 251–265.

Tsui, K.Y. (1999). Multidimensional inequality and multidimensional generalized entropy measures: An axiomatic derivation. *Social Choice and Welfare*, 16, 145–157.

Tsui, K.Y. (2002). Multidimensional poverty indices. *Social Choice and Welfare*, 19, 69–93.

TransMonee (1999). Monitoring Social Conditions and Public Policy in Central and Eastern Europe. UNICEF, available at: http://eurochild.gla.ac.uk/Documents/monee/.

UNDP (1990). *Concept and Measurement of Human Development*. New York: United Nations Development Program.

UNDP (2003). *Human Development Report. Millennium Development Goals: A Compact Among Nations to End Human Poverty*. New York: United Nations Development Program.

UNDP (2005). *Human Development Report. International cooperation at a crossroads: Aid, trade and security in an unequal world*. New York: United Nations Development Program.

UNDP (2010). *Human Development Report. The Real Wealth of Nations: Pathways to Human Development*. New York: United Nations Development Program.

UNDP (2012). International Human Development Indicators. Retrieved from http://hdr.undp.org/en/statistics/, last accessed November 10, 2012.

UNDP (2013). *Human Development Report. The Rise of the South: Human Progress in a Diverse World*. New York: United Nations Development Program.

United Nations (2012). *The Millennium Development Goals Report*. New York: United Nations.

UNU-WIDER (2008). World Income Inequality Database, Version 2.0c.

Wand, M.P. (2005). SemiPar: an R package for semiparametric regression. Working Paper.

World Bank (2001). *World Development Report 2000/2001: Attacking poverty*. Oxford and New York: Oxford University Press.

World Bank (2006). *World Development Report 2005/2006: Equity and Development*. Oxford and New York: Oxford University Press.

World Health Organization (WHO) (2003). *The World Health Report: Shaping the Future*. Geneva: World Health Organization.

Yitzhaki, S. (1983). On an extension of the Gini inequality index. *International Economic Review*, 24, 617–628.

Appendix

Appendix 1.

Regions and countries included in Chapter 2

Western Europe, North America, and Oceania: Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Korea (republic of), Luxembourg, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States.

Middle East and North Africa: Algeria, Iran, Egypt, Israel, Jordan, Morocco, Tunisia, Yemen.

East Asia and the Pacific: China, Hong Kong, Indonesia, Lao People's Democratic Republic, Malaysia, Philippines, Singapore, Taiwan, Thailand, Viet Nam.

Eastern Europe and Central Asia: Armenia, Azerbaijan, Belarus, Bosnia and Herzegovina, Bulgaria, Croatia, Cyprus, Estonia, Georgia, Hungary, Kazakhstan, Kyrgyzstan, Latvia, Lithuania, Macedonia, Moldova (Republic of), Poland, Romania, Russian Federation, Serbia, Slovak Republic, Slovenia, Tajikistan, Turkey, Turkmenistan, Ukraine, Uzbekistan.

Latin America and the Caribbean: Argentina, Bahamas, Barbados, Bolivia (Plurinational State of), Brazil, Chile, Colombia, Costa Rica, Cuba, Dominican Republic, Ecuador, El Salvador, Guatemala, Guyana, Haiti, Honduras, Jamaica, Mexico, Nicaragua, Panama, Paraguay, Peru, Puerto Rico, Trinidad and Tobago, Uruguay, Venezuela

South Asia: Bangladesh, India, Nepal, Pakistan, Sri Lanka.

Sub-Saharan Africa: Botswana, Burundi, Cameroon, Cape Verde, Central African Republic, Cote d'Ivoire, Ethiopia, Gabon, Gambia, Ghana, Guinea, Guinea-Bissau, Kenya, Lesotho, Madagascar, Malawi, Mali, Mauritania, Mauritius, Mozambique, Namibia, Niger, Nigeria, Rwanda, Senegal, Sierra Leone, South Africa, Tanzania, Uganda, Zambia.

Appendix 2.

Observed income shares for selected countries included in Chapter 2

Table A2.1. Observed income shares in 1990

	d1	d2	d3	d4	d5	d6	d7	d8	d9	d10
Argentina	2.3	3.4	4.0	5.1	6.3	7.7	9.1	11.4	15.5	35.3
Brazil	0.8	1.6	2.3	3.1	4.2	5.5	7.4	10.5	16.6	47.9
Burundi	3.4	4.5	5.5	6.6	7.6	8.7	10.1	12.0	15.0	26.6
Central African Republic	0.7	1.3	2.0	2.9	4.0	5.5	7.6	10.9	17.3	47.7
Chile	1.3	2.4	3.1	4.0	4.9	6.1	7.7	10.2	15.3	45.1
Costa Rica	1.4	3.0	4.3	5.4	6.6	8.0	9.9	12.4	16.4	32.7
Czech Republic	5.5	6.6	7.3	8.0	8.7	9.6	10.8	12.0	13.7	17.9
El Salvador	1.5	3.0	4.2	5.2	6.5	8.0	9.7	12.0	16.4	33.6
Finland	4.9	6.5	7.4	8.2	9.0	9.9	10.8	11.8	13.4	18.1
Gambia	0.3	0.8	1.4	2.0	2.9	4.0	5.8	9.2	15.7	58.0
Germany	3.3	5.2	6.1	7.1	8.0	9.2	10.5	12.2	14.8	23.5
Guinea	0.8	2.1	3.2	4.4	5.9	7.5	9.6	12.3	16.9	37.2
Honduras	0.8	1.8	2.6	3.6	4.7	6.2	8.2	11.3	17.1	43.9
Indonesia	2.9	4.2	5.0	5.9	6.8	8.0	9.3	11.3	14.7	31.8
Jamaica	0.9	2.2	3.2	4.3	5.8	7.3	9.4	12.3	17.5	37.1
Netherlands	3.4	5.9	6.8	7.6	8.6	9.6	10.8	12.3	14.4	20.7
Niger	3.0	4.5	5.4	6.3	7.3	8.4	9.7	11.5	14.5	29.4
Norway	3.5	5.5	6.7	7.8	8.9	10.0	11.1	12.4	14.2	19.8
Pakistan	3.5	4.7	5.4	6.2	6.9	7.8	9.0	10.7	13.9	32.1
Paraguay	2.2	3.7	4.7	5.8	6.8	8.1	9.9	12.7	16.6	29.5
Poland	3.8	5.1	6.2	7.2	8.2	9.3	10.5	12.1	14.7	22.8
Portugal	3.4	4.6	5.9	7.0	8.0	9.2	10.4	12.3	15.0	24.2
Romania	4.5	6.1	7.1	7.9	8.7	9.5	10.6	12.0	14.0	19.6
Senegal	0.6	1.4	2.2	3.0	3.9	5.1	6.5	9.1	14.4	53.8
Spain	3.1	4.8	5.9	7.0	8.1	9.3	10.6	12.4	15.1	23.7
Tunisia	2.3	3.6	4.7	5.8	7.0	8.3	10.0	12.2	15.6	30.7
United Kingdom	2.9	4.5	5.5	6.6	7.7	9.0	10.5	12.3	15.0	26.0
Venezuela	1.6	3.1	4.2	5.3	6.6	8.0	9.7	12.2	16.4	32.9
Yemen, Republic of	2.4	3.7	4.8	6.0	7.1	8.3	9.8	11.8	15.2	30.9

Source: UNU-WIDER World Income Inequality Database version 2.0c

Table A2.2. Observed income shares in 1995

	d1	d2	d3	d4	d5	d6	d7	d8	d9	d10
Algeria	2.9	4.1	5.2	6.3	7.5	8.8	10.3	12.3	15.7	26.9
Argentina	1.7	2.7	3.9	4.9	6.1	7.4	9.0	11.3	15.4	37.3
Belarus	4.3	5.9	6.9	7.7	8.5	9.4	10.5	11.9	14.0	20.9
Belgium	3.0	5.0	6.0	7.0	8.0	9.0	11.0	12.0	14.0	23.0
Brazil	0.9	1.7	2.5	3.4	4.4	5.7	7.5	10.4	16.3	47.3
Bulgaria	3.4	5.3	6.4	7.4	8.4	9.4	10.6	12.1	14.4	22.5
China	3.4	4.9	5.8	6.7	7.6	8.6	9.8	11.5	14.4	27.4
Colombia	1.0	2.2	3.0	3.9	4.9	6.0	7.7	10.0	14.4	47.0
Costa Rica	1.3	2.7	3.8	4.9	6.1	7.5	9.4	12.1	16.5	35.7
Cote d`Ivoire	3.1	4.1	5.1	6.1	7.2	8.4	9.9	12.0	15.5	28.8
Czech Republic	4.9	6.2	7.2	7.8	8.7	9.7	10.8	12.0	13.8	18.7
Dominican Republic	1.5	2.5	3.4	4.3	5.4	6.7	8.5	11.1	16.2	40.7
Ecuador	1.6	2.9	3.7	4.6	5.6	6.7	8.3	10.8	15.7	40.3
El Salvador	1.0	2.4	3.5	4.6	5.8	7.2	9.0	11.6	16.4	38.7
Estonia	2.1	4.1	5.4	6.7	7.8	9.0	10.5	12.5	16.0	25.9
Ethiopia	3.0	4.2	5.0	5.8	6.7	7.7	9.0	10.8	13.9	33.8
Finland	4.8	6.4	7.3	8.0	8.8	9.6	10.5	11.7	13.4	19.6
France	3.0	5.0	6.0	7.0	8.0	9.0	11.0	12.0	15.0	23.0
Germany	3.0	5.0	7.0	8.0	8.0	10.0	11.0	12.0	14.0	22.0
Greece	2.0	4.0	5.0	7.0	8.0	9.0	11.0	13.0	16.0	25.0
Honduras	1.0	2.0	2.8	3.8	4.9	6.3	8.0	10.8	16.2	44.2
Hungary	4.1	5.8	6.9	7.9	8.8	9.7	10.7	11.9	13.8	20.5
Ireland	3.0	5.0	5.0	6.0	7.0	9.0	10.0	12.0	15.0	26.0
Israel	0.8	1.8	3.0	4.3	5.7	7.3	9.3	12.2	17.2	38.2
Jamaica	0.6	1.7	2.5	3.4	4.3	5.6	7.2	9.7	13.6	51.4
Republic of Korea	1.8	4.2	5.9	7.3	8.6	9.7	10.9	12.6	15.0	23.9
Latvia	3.3	5.0	6.3	7.4	8.5	9.5	10.7	12.2	14.7	22.4
Lesotho	0.2	0.8	1.4	2.2	3.3	4.7	6.6	10.2	17.4	53.3
Luxembourg	4.0	5.0	6.0	7.0	8.0	9.0	10.0	12.0	15.0	23.0
Macedonia, FYR	1.7	3.2	4.6	6.5	8.5	10.0	11.9	13.9	15.8	24.0
Malaysia	1.6	2.6	3.5	4.5	5.6	6.9	8.7	11.4	16.2	39.1
Mauritania	2.5	3.7	4.8	5.9	7.1	8.4	9.9	12.0	15.6	30.0
Morocco	2.2	3.8	4.8	5.9	7.0	8.3	9.9	12.1	16.0	30.1
Netherlands	3.6	5.8	6.8	7.7	8.7	9.7	10.9	12.3	14.3	20.3
Niger	0.8	1.7	2.9	4.4	6.0	7.9	10.2	13.2	17.8	35.3
Norway	3.9	5.9	6.8	7.6	8.4	9.3	10.4	11.9	14.1	21.6
Panama	0.6	1.8	2.8	3.9	5.2	6.6	8.6	11.6	16.9	41.9
Paraguay	0.8	1.8	3.0	3.4	5.1	6.0	8.2	11.2	15.7	44.8
Poland	3.0	4.8	5.9	6.9	7.9	9.0	10.3	11.9	14.6	25.8
Portugal	2.0	4.0	5.0	6.0	8.0	9.0	10.0	12.0	16.0	28.0
Romania	3.1	4.8	6.0	7.1	8.1	9.2	10.5	12.1	14.5	24.7
Russian Federation	1.1	3.1	4.5	5.7	6.9	8.3	10.0	12.3	16.2	31.9
Slovenia	4.3	6.0	7.1	7.8	8.7	9.7	10.8	12.0	13.8	19.9
Spain	2.0	5.0	6.0	7.0	8.0	9.0	11.0	13.0	16.0	25.0
Sweden	3.6	5.7	6.8	7.7	8.7	9.7	11.0	12.4	14.4	20.1
Taiwan	3.9	5.2	6.1	7.0	8.0	9.0	10.3	12.0	14.7	23.7
Ukraine	2.2	3.6	4.8	5.7	6.8	8.0	9.5	11.8	15.2	32.3
United Kingdom	3.1	4.8	5.7	6.6	7.7	8.9	10.3	12.1	14.9	26.0
Uruguay	1.8	3.2	4.4	5.5	6.8	8.2	9.9	12.4	16.5	31.3
Venezuela	1.5	2.9	3.9	5.0	6.2	7.6	9.4	11.9	16.3	35.4

Source: UNU-WIDER World Income Inequality Database version 2.0c

Table A2.3. Observed income shares in 2000

	d1	d2	d3	d4	d5	d6	d7	d8	d9	d10
Argentina	1.2	2.3	3.3	4.4	5.6	7.2	9.1	12.2	17.4	37.4
Austria	3.3	5.2	6.2	7.2	8.2	9.3	10.6	12.3	14.8	23.0
Belarus	4.4	6.0	7.0	7.8	8.6	9.5	10.5	11.7	13.8	20.9
Belgium	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0	14.0	25.0
Bolivia	1.1	2.2	3.2	4.2	5.4	6.7	8.6	11.4	16.7	40.6
Bulgaria	2.9	4.9	6.1	7.2	8.2	9.2	10.5	12.1	14.7	24.2
Canada	2.7	4.6	5.8	6.9	8.0	9.1	10.6	12.4	15.1	24.8
Chile	1.3	2.4	3.1	3.9	4.9	6.0	7.6	10.1	15.2	45.3
Colombia	1.2	2.4	3.2	4.0	4.9	6.2	7.7	10.2	14.9	45.5
Costa Rica	1.4	2.9	4.0	5.1	6.2	7.7	9.6	12.1	16.5	34.4
Czech Republic	4.6	5.9	6.8	7.6	8.6	9.4	10.8	12.1	14.2	20.1
Denmark	1.9	4.3	5.4	6.4	7.6	9.0	11.2	13.6	16.1	24.6
Dominican Republic	1.2	2.4	3.3	4.3	5.3	6.8	8.6	11.2	16.0	40.8
Ecuador	1.0	2.0	2.9	3.9	4.9	6.3	8.0	10.6	15.5	44.9
El Salvador	0.7	2.1	3.3	4.5	5.7	7.1	9.0	11.8	17.0	38.8
Estonia	2.4	4.3	5.5	6.5	7.4	8.5	9.9	12.0	15.2	28.5
Finland	4.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0	14.0	20.0
France	4.0	5.0	6.0	7.0	8.0	9.0	11.0	12.0	15.0	22.0
Germany	3.3	5.2	6.2	7.2	8.2	9.3	10.6	12.3	14.9	22.9
Greece	3.0	4.0	6.0	7.0	8.0	9.0	11.0	13.0	16.0	24.0
Guatemala	1.1	2.3	3.2	4.1	5.1	6.4	7.9	10.4	15.2	44.2
Hungary	4.0	5.8	6.9	7.8	8.7	9.5	10.5	11.8	13.8	21.3
Ireland	3.0	5.0	6.0	7.0	8.0	10.0	11.0	12.0	15.0	23.0
Kyrgyz Republic	2.7	3.9	4.9	5.9	7.1	8.5	10.0	12.4	16.4	28.3
Latvia	1.7	4.7	6.1	7.1	7.9	8.7	9.7	11.4	14.5	28.2
Lithuania	2.0	4.4	5.7	6.8	7.9	9.1	10.4	12.3	15.2	26.3
Luxembourg	4.0	6.0	7.0	7.0	8.0	9.0	11.0	12.0	14.0	22.0
Macedonia, FYR	1.7	2.9	4.7	6.5	8.5	10.8	12.5	14.1	15.4	23.0
Mexico	1.0	2.3	3.3	4.2	5.3	6.7	8.5	10.8	15.9	42.0
Netherlands	4.0	6.0	7.0	8.0	9.0	9.0	11.0	12.0	14.0	21.0
Norway	3.9	5.7	6.6	7.4	8.2	9.1	10.3	11.7	13.9	23.4
Panama	0.7	1.6	2.5	3.5	4.7	6.1	8.1	11.2	16.9	44.6
Peru	1.2	2.4	3.5	4.6	5.8	7.4	9.3	12.0	16.6	37.2
Poland	2.1	4.5	5.7	6.8	7.9	9.1	10.5	12.2	15.1	26.1
Portugal	3.0	4.0	5.0	7.0	8.0	9.0	10.0	12.0	15.0	27.0
Romania	3.1	5.0	6.1	7.1	8.1	9.2	10.5	12.2	14.8	23.9
Russian Federation	1.4	3.4	4.6	5.8	6.9	8.3	9.7	11.8	15.4	32.6
Serbia and Montenegro	3.5	4.9	5.8	6.8	7.7	8.7	9.9	11.6	14.9	26.3
Slovak Republic	4.5	5.9	6.8	7.8	8.5	9.3	10.4	11.9	14.2	20.8
Slovenia	4.1	5.8	6.9	7.6	8.6	9.6	10.6	11.9	14.2	20.7
Spain	3.3	4.8	5.8	6.7	7.7	8.7	10.0	11.8	14.8	26.3
Sweden	3.7	5.5	6.5	7.4	8.3	9.3	10.5	12.2	14.4	22.2
Taiwan	3.5	4.9	5.9	6.8	7.8	8.9	10.2	12.1	15.0	24.9
Uganda	2.1	3.2	4.1	4.9	5.9	7.0	8.5	10.8	15.3	38.3
United Kingdom	2.8	4.6	5.6	6.5	7.6	8.7	10.0	11.8	14.5	27.9
United States	1.8	3.5	4.8	6.0	7.3	8.7	10.3	12.5	16.1	29.0
Uruguay	1.7	3.0	4.1	5.2	6.5	7.9	9.7	12.2	16.7	33.0
Venezuela	1.6	3.1	4.2	5.3	6.5	8.0	9.8	12.4	16.5	32.7

Source: UNU-WIDER World Income Inequality Database version 2.0c

Estimated income shares and chi-square statistics for selected countries included in Chapter 2

Table A2.4. Estimated income shares in 1990 (Lamé I distribution)

	d1	d2	d3	d4	d5	d6	d7	d8	d9	d10	X^2 stat	*p*-value
Argentina	1.5	3.0	4.2	5.4	6.6	8.1	9.9	12.3	16.4	32.8	0.9389	0.9958
Brazil	0.6	1.5	2.4	3.5	4.7	6.2	8.3	11.3	16.8	44.7	0.6982	0.9983
Burundi	2.6	4.4	5.6	6.7	7.8	9.0	10.5	12.4	15.3	25.7	0.3535	0.9998
Central African Republic	0.5	1.4	2.3	3.3	4.5	6.1	8.1	11.2	16.7	45.9	0.3816	0.9998
Chile	0.9	2.1	3.2	4.3	5.6	7.1	9.1	11.9	16.8	39.1	2.0244	0.9585
Costa Rica	1.6	3.1	4.3	5.5	6.8	8.2	9.9	12.3	16.3	32.0	0.0426	1.0000
Czech Republic	3.2	5.0	6.2	7.2	8.2	9.3	10.6	12.3	14.8	23.2	3.8459	0.7974
El Salvador	1.2	2.5	3.7	4.9	6.1	7.6	9.5	12.2	16.6	35.7	0.4497	0.9996
Finland	2.2	4.0	5.2	6.3	7.5	8.8	10.4	12.4	15.7	27.5	10.2159	0.1767
Gambia	0.8	1.9	3.0	4.1	5.4	6.9	8.9	11.8	16.8	40.4	14.6181	0.0412
Germany	1.4	2.9	4.1	5.2	6.5	8.0	9.8	12.3	16.4	33.4	9.8408	0.1978
Guinea	1.1	2.4	3.5	4.7	6.0	7.5	9.4	12.1	16.7	36.6	0.1589	1.0000
Honduras	0.8	2.0	3.1	4.2	5.5	7.0	9.0	11.8	16.8	39.9	0.8847	0.9965
Indonesia	2.1	3.8	5.1	6.2	7.4	8.7	10.3	12.4	15.8	28.2	1.2064	0.9908
Jamaica	1.1	2.4	3.5	4.7	5.9	7.5	9.4	12.1	16.7	36.9	0.1371	1.0000
Netherlands	1.8	3.5	4.7	5.9	7.1	8.5	10.1	12.4	16.1	30.1	8.1919	0.3160
Niger	1.9	3.5	4.7	5.9	7.1	8.5	10.1	12.4	16.0	29.9	1.3513	0.9870
Norway	1.7	3.3	4.5	5.7	6.9	8.3	10.1	12.4	16.2	30.9	10.3669	0.1687
Pakistan	1.7	3.2	4.5	5.6	6.9	8.3	10.0	12.4	16.2	31.2	3.6477	0.8193
Paraguay	2.2	3.9	5.2	6.3	7.5	8.8	10.3	12.4	15.7	27.7	0.3935	0.9998
Poland	2.6	4.3	5.6	6.7	7.8	9.0	10.5	12.4	15.4	25.8	1.2982	0.9885
Portugal	1.1	2.5	3.6	4.8	6.1	7.6	9.5	12.1	16.6	35.9	13.6026	0.0587
Romania	2.7	4.5	5.7	6.8	7.9	9.1	10.5	12.4	15.2	25.1	3.6488	0.8192
Senegal	0.6	1.6	2.6	3.6	4.9	6.4	8.5	11.4	16.8	43.6	4.3234	0.7419
Spain	2.2	3.9	5.1	6.3	7.5	8.8	10.3	12.4	15.7	27.9	1.5701	0.9798
Tunisia	2.2	3.9	5.1	6.3	7.4	8.8	10.3	12.4	15.7	27.9	0.4606	0.9996
United Kingdom	1.3	2.7	3.9	5.0	6.3	7.8	9.6	12.2	16.5	34.6	7.4642	0.3822
Venezuela	1.7	3.3	4.5	5.7	6.9	8.3	10.0	12.4	16.2	31.0	0.2279	1.0000
Republic of Yemen	2.0	3.7	4.9	6.1	7.3	8.6	10.2	12.4	15.9	29.0	0.3171	0.9999

Table A2.5. Estimated income shares in 1995 (Lamé I distribution)

	d1	d2	d3	d4	d5	d6	d7	d8	d9	d10	X^2 stat	*p*-value
Algeria	2.4	4.1	5.4	6.5	7.6	8.9	10.4	12.4	15.5	26.7	0.1185	1.0000
Argentina	1.4	2.8	4.0	5.2	6.4	7.9	9.7	12.3	16.5	33.8	0.7145	0.9982
Belarus	3.5	5.3	6.5	7.5	8.4	9.5	10.7	12.2	14.5	22.0	0.3452	0.9998
Belgium	1.6	3.1	4.3	5.5	6.8	8.2	9.9	12.3	16.3	32.0	6.8018	0.4498
Brazil	0.6	1.6	2.6	3.6	4.9	6.4	8.4	11.4	16.8	43.7	0.7989	0.9975
Bulgaria	2.2	3.9	5.1	6.3	7.5	8.8	10.3	12.4	15.7	27.8	3.0514	0.8802
China	1.9	3.5	4.8	5.9	7.1	8.5	10.2	12.4	16.0	29.7	2.4981	0.9272
Colombia	0.8	2.0	3.1	4.2	5.5	7.0	9.0	11.8	16.8	39.8	2.4650	0.9297
Costa Rica	1.5	3.1	4.3	5.5	6.7	8.2	9.9	12.3	16.3	32.2	0.7144	0.9982
Cote d`Ivoire	2.1	3.8	5.0	6.2	7.4	8.7	10.3	12.4	15.8	28.3	0.5157	0.9994
Czech Republic	2.5	4.3	5.5	6.6	7.8	9.0	10.5	12.4	15.4	26.1	6.4225	0.4914
Dominican Republic	1.2	2.5	3.7	4.9	6.1	7.6	9.5	12.2	16.6	35.7	1.3015	0.9884
Ecuador	1.5	3.0	4.2	5.4	6.6	8.1	9.9	12.3	16.4	32.8	2.7549	0.9067
El Salvador	1.1	2.5	3.6	4.8	6.1	7.6	9.5	12.1	16.7	36.2	0.2723	0.9999
Estonia	1.6	3.2	4.4	5.6	6.8	8.2	10.0	12.4	16.2	31.6	2.1007	0.9541
Ethiopia	1.7	3.3	4.6	5.7	7.0	8.4	10.1	12.4	16.1	30.7	2.1532	0.9509
Finland	1.9	3.6	4.8	5.9	7.2	8.5	10.2	12.4	16.0	29.6	13.0396	0.0712
France	1.2	2.5	3.7	4.8	6.1	7.6	9.5	12.1	16.6	35.9	13.7951	0.0549
Germany	1.3	2.7	3.8	5.0	6.3	7.8	9.6	12.2	16.6	34.8	15.3797	0.0314
Greece	1.7	3.3	4.6	5.7	7.0	8.4	10.1	12.4	16.1	30.7	1.8786	0.9663
Honduras	0.9	2.1	3.1	4.3	5.5	7.1	9.1	11.9	16.8	39.3	1.1180	0.9927
Hungary	1.5	3.0	4.2	5.4	6.7	8.1	9.9	12.3	16.3	32.4	15.4670	0.0305
Ireland	1.6	3.2	4.4	5.6	6.8	8.2	10.0	12.3	16.3	31.7	3.5990	0.8246
Israel	1.6	3.1	4.3	5.5	6.8	8.2	10.0	12.3	16.3	31.9	3.2614	0.8598
Jamaica	0.6	1.5	2.4	3.5	4.7	6.3	8.3	11.3	16.8	44.6	2.1385	0.9518
Republic of Korea	2.6	4.3	5.5	6.7	7.8	9.0	10.5	12.4	15.4	25.9	0.6275	0.9988
Latvia	1.8	3.4	4.7	5.8	7.1	8.4	10.1	12.4	16.1	30.2	5.5677	0.5910
Lesotho	0.3	1.0	1.8	2.8	3.9	5.4	7.4	10.6	16.5	50.3	0.8439	0.9970
Luxembourg	2.4	4.1	5.3	6.5	7.6	8.9	10.4	12.4	15.6	26.9	2.1247	0.9526
Macedonia, FYR	3.0	4.8	6.0	7.1	8.1	9.3	10.6	12.3	15.0	23.8	1.9504	0.9625
Malaysia	1.3	2.7	3.9	5.1	6.4	7.8	9.7	12.2	16.5	34.3	1.2044	0.9908
Mauritania	1.8	3.4	4.6	5.8	7.0	8.4	10.1	12.4	16.1	30.5	0.3911	0.9998
Morocco	1.9	3.5	4.7	5.9	7.1	8.5	10.2	12.4	16.0	29.8	0.1220	1.0000
Netherlands	1.7	3.3	4.5	5.7	6.9	8.3	10.1	12.4	16.2	30.9	10.4342	0.1653
Niger	1.0	2.3	3.4	4.6	5.9	7.4	9.3	12.0	16.7	37.2	0.7093	0.9983
Norway	1.3	2.7	3.9	5.1	6.4	7.9	9.7	12.2	16.5	34.3	18.1204	0.0114
Panama	0.8	2.0	3.0	4.2	5.4	7.0	9.0	11.8	16.8	40.1	0.2059	1.0000
Paraguay	0.7	1.8	2.8	3.9	5.2	6.8	8.8	11.7	16.8	41.4	0.5841	0.9991
Poland	1.3	2.7	3.9	5.0	6.3	7.8	9.7	12.2	16.5	34.6	8.7892	0.2681
Portugal	0.6	1.6	2.5	3.6	4.8	6.4	8.4	11.4	16.8	44.0	20.5778	0.0044
Romania	1.7	3.2	4.5	5.6	6.9	8.3	10.0	12.4	16.2	31.2	4.7914	0.6854
Russian Federation	0.9	2.2	3.3	4.4	5.7	7.2	9.2	12.0	16.8	38.4	2.9224	0.8921
Slovenia	1.8	3.4	4.6	5.8	7.0	8.4	10.1	12.4	16.1	30.3	11.9535	0.1021
Spain	1.3	2.7	3.9	5.0	6.3	7.8	9.6	12.2	16.5	34.6	7.9394	0.3380
Sweden	1.1	2.5	3.6	4.8	6.1	7.6	9.5	12.1	16.7	36.2	23.7308	0.0013
Taiwan	2.6	4.4	5.6	6.7	7.8	9.0	10.5	12.4	15.4	25.7	1.0980	0.9931
Ukraine	1.6	3.1	4.3	5.5	6.7	8.2	9.9	12.3	16.3	32.1	0.5082	0.9994
United Kingdom	1.2	2.6	3.7	4.9	6.2	7.7	9.6	12.2	16.6	35.4	9.8192	0.1991
Uruguay	1.8	3.4	4.7	5.8	7.1	8.4	10.1	12.4	16.1	30.2	0.1185	1.0000
Venezuela	1.5	2.9	3.9	5.0	6.2	7.6	9.4	11.9	16.3	35.4	0.2464	0.9999

Table A2.6. Estimated income shares in 2000 (Lamé I distribution)

	d1	d2	d3	d4	d5	d6	d7	d8	d9	d10	$\mathcal{X}^2$ stat	*p*-value
Argentina	1.1	2.5	3.6	4.8	6.1	7.6	9.5	12.1	16.6	36.1	0.2167	1.0000
Austria	1.5	3.0	4.2	5.4	6.7	8.1	9.9	12.3	16.3	32.4	8.5843	0.2839
Belarus	3.5	5.4	6.5	7.5	8.5	9.5	10.7	12.2	14.5	21.8	0.4204	0.9997
Belgium	1.6	3.1	4.3	5.5	6.8	8.2	9.9	12.3	16.3	32.0	8.2897	0.3077
Bolivia	0.6	1.6	2.5	3.6	4.8	6.4	8.4	11.4	16.8	44.0	1.2696	0.9892
Bulgaria	3.2	5.1	6.2	7.3	8.3	9.4	10.6	12.3	14.7	22.9	0.1305	1.0000
Canada	1.6	3.1	4.3	5.5	6.8	8.2	9.9	12.3	16.3	31.9	4.4072	0.7319
Chile	0.9	2.1	3.2	4.3	5.6	7.1	9.1	11.9	16.8	39.2	2.1324	0.9522
Colombia	0.8	2.0	3.0	4.2	5.4	7.0	9.0	11.8	16.8	40.1	1.7388	0.9729
Costa Rica	1.4	2.8	4.0	5.2	6.5	7.9	9.8	12.3	16.5	33.6	0.0435	1.0000
Czech Republic	1.8	3.4	4.6	5.8	7.0	8.4	10.1	12.4	16.1	30.5	12.3967	0.0882
Denmark	1.3	2.7	3.9	5.1	6.4	7.8	9.7	12.2	16.5	34.3	5.6354	0.5829
Dominican Republic	1.1	2.5	3.6	4.8	6.1	7.6	9.5	12.1	16.7	36.2	1.0116	0.9946
Ecuador	0.8	1.9	2.9	4.0	5.3	6.8	8.8	11.7	16.8	40.9	0.8249	0.9972
El Salvador	1.0	2.3	3.4	4.5	5.8	7.3	9.3	12.0	16.7	37.6	0.1562	1.0000
Estonia	1.8	3.4	4.6	5.8	7.0	8.4	10.1	12.4	16.1	30.5	0.9494	0.9956
Finland	1.3	2.8	3.9	5.1	6.4	7.9	9.7	12.2	16.5	34.1	21.1411	0.0036
France	1.3	2.7	3.8	5.0	6.3	7.8	9.6	12.2	16.6	34.9	15.9224	0.0258
Germany	1.0	2.2	3.3	4.4	5.7	7.3	9.2	12.0	16.8	38.2	22.2345	0.0023
Greece	1.0	2.3	3.4	4.6	5.8	7.4	9.3	12.0	16.7	37.5	14.9593	0.0365
Guatemala	0.7	1.7	2.8	3.8	5.1	6.7	8.7	11.6	16.8	42.1	1.0207	0.9945
Hungary	1.3	2.8	3.9	5.1	6.4	7.9	9.7	12.2	16.5	34.1	18.8709	0.0086
Ireland	1.7	3.2	4.4	5.6	6.9	8.3	10.0	12.4	16.2	31.3	5.9431	0.5464
Kyrgyz Republic	1.9	3.6	4.8	6.0	7.2	8.5	10.2	12.4	16.0	29.5	0.4480	0.9996
Latvia	1.3	2.7	3.8	5.0	6.3	7.8	9.6	12.2	16.6	34.8	6.1150	0.5264
Lithuania	1.3	2.7	3.9	5.1	6.4	7.8	9.7	12.2	16.5	34.3	5.2576	0.6286
Luxembourg	1.7	3.3	4.5	5.7	6.9	8.3	10.0	12.4	16.2	31.0	10.3195	0.1712
Macedonia, FYR	2.7	4.5	5.7	6.8	7.9	9.1	10.5	12.4	15.3	25.3	2.2452	0.9450
Mexico	1.1	2.4	3.5	4.7	5.9	7.5	9.4	12.1	16.7	36.9	1.1486	0.9921
Netherlands	1.8	3.4	4.6	5.8	7.0	8.4	10.1	12.4	16.1	30.4	10.6862	0.1529
Norway	1.3	2.7	3.9	5.1	6.4	7.9	9.7	12.2	16.5	34.2	15.7213	0.0278
Panama	0.8	2.0	3.1	4.2	5.5	7.0	9.0	11.8	16.8	39.8	1.2506	0.9897
Peru	1.0	2.3	3.4	4.5	5.8	7.3	9.3	12.0	16.7	37.7	0.0407	1.0000
Poland	2.1	3.8	5.0	6.2	7.4	8.7	10.3	12.4	15.8	28.3	0.5491	0.9992
Portugal	0.7	1.8	2.8	3.9	5.2	6.7	8.7	11.7	16.8	41.6	21.7188	0.0028
Romania	1.8	3.4	4.7	5.8	7.1	8.4	10.1	12.4	16.1	30.1	3.9888	0.7811
Russian Federation	1.2	2.5	3.7	4.9	6.2	7.7	9.5	12.2	16.6	35.6	1.2580	0.9895
Serbia and Montenegro	2.7	4.5	5.7	6.8	7.9	9.1	10.5	12.4	15.2	25.1	0.4471	0.9996
Slovak Republic	2.5	4.2	5.4	6.6	7.7	9.0	10.4	12.4	15.5	26.4	4.2981	0.7449
Slovenia	1.9	3.5	4.7	5.9	7.1	8.5	10.2	12.4	16.0	29.8	9.1269	0.2437
Spain	2.0	3.6	4.9	6.0	7.2	8.6	10.2	12.4	15.9	29.1	1.8770	0.9663
Sweden	1.2	2.6	3.7	4.9	6.2	7.7	9.5	12.2	16.6	35.5	18.7027	0.0092
Taiwan	2.3	4.0	5.3	6.4	7.6	8.9	10.4	12.4	15.6	27.1	1.1273	0.9925
Uganda	1.4	2.9	4.1	5.2	6.5	8.0	9.8	12.3	16.4	33.4	1.7333	0.9731
United Kingdom	1.2	2.6	3.7	4.9	6.2	7.7	9.6	12.2	16.6	35.4	7.6310	0.3663
United States	1.2	2.6	3.8	5.0	6.2	7.7	9.6	12.2	16.6	35.1	2.4953	0.9274
Uruguay	1.7	3.3	4.5	5.7	6.9	8.3	10.1	12.4	16.2	30.8	0.3496	0.9998
Venezuela	1.4	2.9	4.1	5.3	6.5	8.0	9.8	12.3	16.4	33.3	0.0453	1.0000

Table A2.7. Estimated income shares in 1990 (Lamé II distribution)

	d1	d2	d3	d4	d5	d6	d7	d8	d9	d10	X^2 stat	*p*-value
Argentina	1.4	2.9	4.2	5.5	6.7	8.2	9.9	12.2	16.1	32.9	1.1216	0.9926
Brazil	0.4	1.4	2.4	3.6	4.9	6.4	8.4	11.3	16.4	44.7	1.0315	0.9943
Burundi	2.5	4.4	5.6	6.8	7.9	9.1	10.5	12.3	15.2	25.8	0.4192	0.9997
Central African Republic	0.4	1.3	2.3	3.4	4.7	6.3	8.3	11.2	16.3	45.9	0.6854	0.9984
Chile	0.8	2.0	3.2	4.4	5.7	7.3	9.2	11.8	16.4	39.2	2.2458	0.9450
Costa Rica	1.5	3.1	4.4	5.6	6.9	8.3	10.0	12.3	16.0	32.1	0.0526	1.0000
Czech Republic	3.1	5.0	6.2	7.3	8.3	9.4	10.6	12.2	14.7	23.3	3.9755	0.7826
El Salvador	1.0	2.5	3.7	5.0	6.3	7.7	9.6	12.1	16.3	35.8	0.5259	0.9993
Finland	2.1	4.0	5.3	6.4	7.6	8.9	10.4	12.3	15.5	27.6	10.470	0.1635
Gambia	0.7	1.9	3.0	4.2	5.5	7.1	9.0	11.7	16.4	40.5	14.690	0.0402
Germany	1.3	2.8	4.1	5.3	6.6	8.1	9.8	12.2	16.2	33.5	10.292	0.1726
Guinea	1.0	2.4	3.6	4.8	6.1	7.6	9.5	12.0	16.3	36.8	0.1532	1.0000
Honduras	0.7	1.9	3.1	4.3	5.6	7.1	9.1	11.8	16.4	40.0	1.0122	0.9946
Indonesia	2.0	3.8	5.1	6.3	7.5	8.8	10.3	12.3	15.6	28.3	1.2726	0.9892
Jamaica	0.9	2.3	3.5	4.8	6.1	7.6	9.4	12.0	16.4	37.0	0.2012	1.0000
Netherlands	1.7	3.4	4.7	5.9	7.2	8.5	10.1	12.3	15.8	30.2	8.3416	0.3034
Niger	1.7	3.5	4.8	6.0	7.2	8.5	10.2	12.3	15.8	30.0	1.4738	0.9832
Norway	1.6	3.3	4.6	5.8	7.0	8.4	10.1	12.3	15.9	31.0	10.566	0.1587
Pakistan	1.6	3.2	4.5	5.7	7.0	8.4	10.0	12.3	16.0	31.3	3.9680	0.7835
Paraguay	2.1	3.9	5.2	6.4	7.6	8.8	10.3	12.3	15.5	27.8	0.4558	0.9996
Poland	2.5	4.3	5.6	6.7	7.9	9.1	10.5	12.3	15.2	25.9	1.3934	0.9858
Portugal	1.0	2.5	3.7	4.9	6.2	7.7	9.5	12.1	16.3	36.1	14.446	0.0438
Romania	2.6	4.5	5.8	6.9	8.0	9.1	10.5	12.3	15.1	25.2	3.7592	0.8071
Senegal	0.5	1.5	2.6	3.7	5.0	6.6	8.6	11.4	16.4	43.6	4.4301	0.7291
Spain	2.1	3.9	5.2	6.3	7.5	8.8	10.3	12.3	15.5	28.0	1.6574	0.9764
Tunisia	2.1	3.9	5.2	6.3	7.5	8.8	10.3	12.3	15.5	28.0	0.4874	0.9995
United Kingdom	1.2	2.6	3.9	5.1	6.4	7.9	9.7	12.1	16.2	34.8	7.8900	0.3424
Venezuela	1.6	3.3	4.6	5.8	7.0	8.4	10.1	12.3	15.9	31.1	0.2564	0.9999
Yemen, Republic of	1.9	3.7	4.9	6.1	7.3	8.7	10.2	12.3	15.7	29.1	0.3554	0.9998

Table A2.8. Estimated income shares in 1995 (Lamé II distribution)

	d1	d2	d3	d4	d5	d6	d7	d8	d9	d10	$\mathcal{X}^2$ stat	*p*-value
Algeria	2.3	4.1	5.4	6.6	7.7	9.0	10.4	12.3	15.4	26.8	0.1846	1.0000
Argentina	1.2	2.8	4.0	5.3	6.6	8.0	9.8	12.2	16.2	34.0	0.7741	0.9977
Belarus	3.4	5.3	6.5	7.5	8.5	9.5	10.7	12.1	14.4	22.0	0.3686	0.9998
Belgium	1.5	3.1	4.4	5.6	6.9	8.3	10.0	12.3	16.0	32.1	6.9976	0.4291
Brazil	0.5	1.5	2.6	3.7	5.0	6.6	8.6	11.4	16.4	43.7	1.0912	0.9932
Bulgaria	2.1	3.9	5.2	6.4	7.5	8.8	10.3	12.3	15.5	27.9	3.1378	0.8720
China	1.8	3.5	4.8	6.0	7.2	8.6	10.2	12.3	15.8	29.8	2.6746	0.9134
Colombia	0.7	1.9	3.1	4.3	5.6	7.2	9.1	11.8	16.4	39.8	2.5435	0.9238
Costa Rica	1.4	3.1	4.3	5.6	6.8	8.2	9.9	12.2	16.0	32.3	0.7226	0.9982
Cote d`Ivoire	2.0	3.8	5.1	6.3	7.5	8.8	10.3	12.3	15.6	28.5	0.6372	0.9988
Czech Republic	2.4	4.3	5.6	6.7	7.8	9.0	10.5	12.3	15.3	26.2	6.6259	0.4688
Dominican Republic	1.1	2.5	3.7	5.0	6.3	7.8	9.6	12.1	16.3	35.8	1.4309	0.9846
Ecuador	1.4	2.9	4.2	5.5	6.7	8.2	9.9	12.2	16.1	32.9	2.7959	0.9032
El Salvador	1.0	2.4	3.6	4.9	6.2	7.7	9.5	12.0	16.3	36.3	0.2814	0.9999
Estonia	1.5	3.2	4.4	5.7	6.9	8.3	10.0	12.3	16.0	31.7	2.1243	0.9526
Ethiopia	1.6	3.3	4.6	5.8	7.1	8.4	10.1	12.3	15.9	30.8	2.2968	0.9416
Finland	1.8	3.5	4.8	6.0	7.3	8.6	10.2	12.3	15.8	29.7	13.4659	0.0615
France	1.0	2.5	3.7	4.9	6.2	7.7	9.5	12.1	16.3	36.0	14.3921	0.0446
Germany	1.1	2.6	3.9	5.1	6.4	7.9	9.7	12.1	16.3	35.0	15.6747	0.0283
Greece	1.6	3.3	4.6	5.8	7.1	8.4	10.1	12.3	15.9	30.8	1.8878	0.9658
Honduras	0.8	2.0	3.2	4.4	5.7	7.2	9.1	11.8	16.4	39.4	1.2429	0.9899
Hungary	1.4	3.0	4.3	5.5	6.8	8.2	9.9	12.2	16.1	32.5	16.0257	0.0249
Ireland	1.5	3.1	4.4	5.6	6.9	8.3	10.0	12.3	16.0	31.8	3.8911	0.7922
Israel	1.5	3.1	4.4	5.6	6.9	8.3	10.0	12.3	16.0	32.0	3.2647	0.8595
Jamaica	0.5	1.4	2.5	3.6	4.9	6.4	8.4	11.3	16.4	44.6	2.1775	0.9494
Korea, Republic of	2.5	4.3	5.6	6.7	7.9	9.1	10.5	12.3	15.2	26.0	0.5575	0.9992
Latvia	1.7	3.4	4.7	5.9	7.2	8.5	10.1	12.3	15.8	30.3	5.7200	0.5728
Lesotho	0.2	0.9	1.8	2.8	4.1	5.6	7.6	10.6	16.1	50.2	1.0091	0.9947
Luxembourg	2.3	4.1	5.4	6.5	7.7	8.9	10.4	12.3	15.4	27.0	2.2985	0.9415
Macedonia, FYR	2.9	4.8	6.1	7.1	8.2	9.3	10.6	12.2	14.8	23.9	1.9617	0.9619
Malaysia	1.2	2.7	4.0	5.2	6.5	7.9	9.7	12.1	16.2	34.5	1.3184	0.9879
Mauritania	1.7	3.4	4.6	5.9	7.1	8.5	10.1	12.3	15.9	30.7	0.4942	0.9995
Morocco	1.8	3.5	4.8	6.0	7.2	8.6	10.2	12.3	15.8	29.9	0.1873	1.0000
Netherlands	1.6	3.3	4.6	5.8	7.0	8.4	10.1	12.3	15.9	31.0	10.6815	0.1531
Niger	0.9	2.3	3.5	4.7	6.0	7.5	9.4	12.0	16.4	37.4	0.7180	0.9982
Norway	1.2	2.7	4.0	5.2	6.5	8.0	9.7	12.2	16.2	34.4	18.9361	0.0084
Panama	0.7	1.9	3.1	4.3	5.6	7.1	9.0	11.7	16.4	40.2	0.2247	1.0000
Paraguay	0.6	1.7	2.9	4.1	5.4	6.9	8.9	11.6	16.4	41.5	0.6832	0.9985
Poland	1.2	2.7	3.9	5.1	6.4	7.9	9.7	12.1	16.2	34.7	9.2266	0.2368
Portugal	0.5	1.5	2.5	3.7	5.0	6.5	8.5	11.4	16.4	44.0	21.8322	0.0027
Romania	1.6	3.2	4.5	5.7	7.0	8.4	10.0	12.3	16.0	31.3	4.9655	0.6642
Russian Federation	0.8	2.1	3.3	4.5	5.8	7.4	9.2	11.9	16.4	38.5	2.8683	0.8969
Slovenia	1.7	3.4	4.7	5.9	7.1	8.5	10.1	12.3	15.9	30.4	12.3434	0.0898
Spain	1.2	2.7	3.9	5.1	6.4	7.9	9.7	12.1	16.2	34.8	8.0120	0.3315
Sweden	1.0	2.4	3.6	4.9	6.2	7.7	9.5	12.0	16.3	36.3	24.6727	0.0009
Taiwan	2.5	4.4	5.6	6.8	7.9	9.1	10.5	12.3	15.2	25.8	1.1999	0.9909
Ukraine	1.4	3.1	4.3	5.6	6.8	8.2	9.9	12.2	16.0	32.3	0.5873	0.9991
United Kingdom	1.1	2.5	3.8	5.0	6.3	7.8	9.6	12.1	16.3	35.5	10.4349	0.1652
Uruguay	1.7	3.4	4.7	5.9	7.1	8.5	10.1	12.3	15.8	30.3	0.1609	1.0000
Venezuela	1.3	2.9	4.2	5.4	6.7	8.1	9.8	12.2	16.1	33.3	0.2976	0.9999

Table A2.9. Estimated income shares in 2000 (Lamé II distribution)

	d1	d2	d3	d4	d5	d6	d7	d8	d9	d10	χ^2 stat	*p*-value
Argentina	1.0	2.4	3.7	4.9	6.2	7.7	9.5	12.0	16.3	36.3	0.3270	0.9999
Austria	1.4	3.0	4.3	5.5	6.8	8.2	9.9	12.2	16.1	32.5	8.9216	0.2583
Belarus	3.5	5.4	6.5	7.5	8.5	9.5	10.7	12.1	14.4	21.9	0.4385	0.9996
Belgium	1.5	3.1	4.4	5.6	6.9	8.3	10.0	12.3	16.0	32.1	8.8140	0.2663
Bolivia	0.5	1.5	2.5	3.7	5.0	6.5	8.5	11.4	16.4	44.1	1.6183	0.9779
Bulgaria	3.2	5.1	6.3	7.3	8.3	9.4	10.6	12.2	14.6	23.0	0.1115	1.0000
Canada	1.5	3.1	4.4	5.6	6.9	8.3	10.0	12.3	16.0	32.0	4.5390	0.7160
Chile	0.8	2.0	3.2	4.4	5.7	7.2	9.2	11.8	16.4	39.3	2.3667	0.9368
Colombia	0.7	1.9	3.1	4.3	5.6	7.1	9.0	11.7	16.4	40.1	1.9157	0.9644
Costa Rica	1.3	2.8	4.1	5.3	6.6	8.0	9.8	12.2	16.2	33.7	0.0920	1.0000
Czech Republic	1.7	3.4	4.7	5.9	7.1	8.5	10.1	12.3	15.9	30.6	12.9461	0.0734
Denmark	1.2	2.7	4.0	5.2	6.5	7.9	9.7	12.2	16.2	34.4	5.7214	0.5726
Dominican Republic	1.0	2.4	3.7	4.9	6.2	7.7	9.5	12.0	16.3	36.3	1.0849	0.9933
Ecuador	0.6	1.8	2.9	4.1	5.4	7.0	8.9	11.7	16.4	41.0	0.9592	0.9955
El Salvador	0.9	2.2	3.4	4.6	6.0	7.5	9.3	11.9	16.4	37.7	0.1486	1.0000
Estonia	1.7	3.4	4.7	5.9	7.1	8.5	10.1	12.3	15.9	30.6	0.9910	0.9950
Finland	1.2	2.7	4.0	5.2	6.5	8.0	9.7	12.2	16.2	34.2	21.8884	0.0027
France	1.1	2.6	3.9	5.1	6.4	7.9	9.7	12.1	16.3	35.0	16.9738	0.0176
Germany	0.8	2.1	3.3	4.6	5.9	7.4	9.3	11.9	16.4	38.3	23.5211	0.0014
Greece	0.9	2.2	3.4	4.7	6.0	7.5	9.4	12.0	16.4	37.6	15.7801	0.0272
Guatemala	0.6	1.7	2.8	4.0	5.3	6.8	8.8	11.6	16.4	42.2	1.2833	0.9889
Hungary	1.2	2.7	4.0	5.2	6.5	8.0	9.7	12.2	16.2	34.3	19.6588	0.0064
Ireland	1.5	3.2	4.5	5.7	7.0	8.4	10.0	12.3	16.0	31.5	6.1086	0.5271
Kyrgyz Republic	1.8	3.6	4.9	6.1	7.3	8.6	10.2	12.3	15.8	29.6	0.6018	0.9990
Latvia	1.1	2.6	3.9	5.1	6.4	7.9	9.7	12.1	16.3	35.0	6.0068	0.5390
Lithuania	1.2	2.7	4.0	5.2	6.5	7.9	9.7	12.2	16.2	34.4	5.2740	0.6266
Luxembourg	1.6	3.3	4.6	5.8	7.0	8.4	10.1	12.3	15.9	31.1	10.7401	0.1504
Macedonia, FYR	2.6	4.5	5.7	6.8	7.9	9.1	10.5	12.3	15.1	25.4	2.2429	0.9452
Mexico	0.9	2.3	3.5	4.8	6.1	7.6	9.4	12.0	16.4	37.0	1.1780	0.9914
Netherlands	1.7	3.4	4.7	5.9	7.1	8.5	10.1	12.3	15.9	30.5	10.9833	0.1393
Norway	1.2	2.7	4.0	5.2	6.5	8.0	9.7	12.2	16.2	34.4	16.5262	0.0207
Panama	0.7	1.9	3.1	4.3	5.6	7.2	9.1	11.8	16.4	39.9	1.3182	0.9879
Peru	0.9	2.2	3.4	4.6	5.9	7.5	9.3	11.9	16.4	37.8	0.1088	1.0000
Poland	2.0	3.8	5.1	6.3	7.5	8.8	10.3	12.3	15.6	28.5	0.5165	0.9994
Portugal	0.6	1.7	2.8	4.0	5.3	6.9	8.8	11.6	16.4	41.7	23.6401	0.0013
Romania	1.7	3.4	4.7	5.9	7.2	8.5	10.1	12.3	15.8	30.3	4.1248	0.7653
Russian Federation	1.1	2.5	3.7	5.0	6.3	7.8	9.6	12.1	16.3	35.7	1.2257	0.9903
Serbia and Montenegro	2.6	4.5	5.8	6.9	8.0	9.1	10.5	12.3	15.1	25.2	0.4921	0.9995
Slovak Republic	2.4	4.2	5.5	6.6	7.8	9.0	10.4	12.3	15.3	26.5	4.4634	0.7251
Slovenia	1.8	3.5	4.8	6.0	7.2	8.6	10.2	12.3	15.8	30.0	9.4270	0.2234
Spain	1.9	3.6	4.9	6.1	7.3	8.7	10.2	12.3	15.7	29.2	2.0171	0.9589
Sweden	1.1	2.5	3.8	5.0	6.3	7.8	9.6	12.1	16.3	35.6	19.6269	0.0064
Taiwan	2.2	4.0	5.3	6.5	7.6	8.9	10.4	12.3	15.4	27.3	1.2375	0.9900
Uganda	1.3	2.8	4.1	5.3	6.6	8.1	9.8	12.2	16.2	33.5	1.8821	0.9661
United Kingdom	1.1	2.5	3.8	5.0	6.3	7.8	9.6	12.1	16.3	35.5	8.0808	0.3255
United States	1.1	2.6	3.8	5.1	6.4	7.8	9.6	12.1	16.3	35.2	2.5716	0.9216
Uruguay	1.6	3.3	4.6	5.8	7.0	8.4	10.1	12.3	15.9	31.0	0.4047	0.9997
Venezuela	1.3	2.9	4.1	5.4	6.6	8.1	9.8	12.2	16.1	33.4	0.1115	1.0000

Table A2.10. Estimated income shares in 1990, 1995 and 2000 (five observations)

		Observed					Estimated (Lamé I)							Estimated (Lamé II)						
country	year	q1	q2	q3	q4	q5	q1	q2	q3	q4	q5	X^2 stat	*p*-value	q1	q2	q3	q4	q5	X^2 stat	*p*-value
Bulgaria	1990	10.5	15.2	17.1	22.6	34.6	9.1	14.2	18.1	22.8	35.8	0.3909	0.8225	9.1	14.3	18.1	22.8	35.7	0.3992	0.8190
China	1990	7.0	11.9	16.1	24.0	41.0	6.7	12.0	16.7	22.8	41.9	0.1132	0.9450	6.6	12.1	16.8	22.7	41.8	0.1411	0.9319
Guinea-Bissau	1990	2.1	6.5	12.0	20.6	58.9	2.8	7.1	12.4	20.7	57.0	0.3140	0.8547	2.6	7.3	12.7	20.8	56.7	0.3106	0.8562
India	1990	9.1	13.1	16.9	21.8	39.1	7.2	12.5	17.0	22.9	40.4	0.6350	0.7280	7.1	12.6	17.1	22.8	40.4	0.6678	0.7161
Lao	1990	9.6	12.9	16.3	21.0	40.2	7.8	13.0	17.4	22.9	38.9	0.6984	0.7053	7.7	13.1	17.4	22.8	38.9	0.7402	0.6907
New Zealand	1990	4.6	10.5	16.3	23.9	44.7	4.9	10.1	15.2	22.4	47.5	0.3856	0.8246	4.8	10.2	15.3	22.3	47.4	0.3319	0.8471
Sweden	1990	7.4	12.7	16.7	25.0	38.2	4.2	9.2	14.4	22.0	50.3	7.5919	0.0225	4.0	9.3	14.6	21.9	50.2	7.6669	**0.0216**
Taiwan	1990	7.5	13.2	17.5	23.2	38.6	7.7	12.9	17.3	22.9	39.2	0.0276	0.9863	7.6	13.1	17.4	22.8	39.1	0.0203	0.9899
Thailand	1990	4.0	8.3	12.5	20.0	55.2	3.2	7.8	13.0	21.2	54.9	0.3481	0.8402	3.0	7.9	13.3	21.2	54.6	0.4800	0.7866
Ukraine	1990	10.4	14.3	17.5	22.8	35.0	9.7	14.6	18.3	22.8	34.6	0.0902	0.9559	9.7	14.7	18.4	22.7	34.5	0.1043	0.9492
United States	1990	3.9	9.6	15.9	24.0	46.6	4.7	9.8	14.9	22.3	48.3	0.3910	0.8224	4.5	9.9	15.1	22.2	48.2	0.3392	0.8440
Australia	1995	3.6	9.3	15.2	24.0	47.9	4.6	9.8	14.9	22.3	48.4	0.4031	0.8175	4.5	9.9	15.1	22.2	48.3	0.3693	0.8314
Egypt	1995	9.8	13.2	16.6	21.4	39.0	6.0	11.3	16.1	22.7	43.9	3.4286	0.1801	5.9	11.4	16.3	22.6	43.8	3.5308	0.1711
Italy	1995	6.8	12.3	17.0	23.2	40.7	3.8	8.7	14.0	21.8	51.7	6.8088	0.0332	3.7	8.9	14.2	21.7	51.5	6.8464	**0.0326**
United States	1995	3.7	9.1	15.2	23.3	48.7	4.0	9.0	14.2	21.9	50.8	0.2712	0.8732	3.9	9.2	14.4	21.9	50.6	0.2156	0.8978
Australia	2000	3.8	9.0	15.0	23.8	48.5	4.5	9.6	14.8	22.2	48.8	0.2817	0.8686	4.4	9.8	15.0	22.1	48.7	0.2685	0.8744
Bangladesh	2000	9.0	12.5	15.9	21.2	41.3	7.2	12.5	17.0	22.9	40.3	0.6601	0.7189	7.1	12.7	17.1	22.8	40.3	0.7119	0.7005
Ethiopia	2000	9.1	13.2	16.8	21.5	39.4	6.7	12.0	16.7	22.8	41.8	1.2049	0.5475	6.6	12.1	16.8	22.8	41.7	1.2444	0.5368
Italy	2000	6.9	12.3	17.2	23.1	40.5	4.3	9.4	14.6	22.1	49.6	4.6146	0.0995	4.2	9.5	14.8	22.0	49.5	4.6198	0.0993
Jamaica	2000	6.7	10.7	15.0	21.7	46.0	4.0	8.9	14.1	21.9	51.0	2.7319	0.2551	3.8	9.1	14.4	21.8	50.9	2.9059	0.2339
Mauritania	2000	6.2	10.6	15.2	22.3	45.7	5.7	11.0	15.9	22.6	44.8	0.1131	0.9450	5.6	11.1	16.0	22.6	44.7	0.1630	0.9217
Philippines	2000	5.4	8.8	13.1	20.5	52.3	4.0	8.9	14.1	21.9	51.2	0.7011	0.7043	3.8	9.0	14.3	21.8	51.0	0.8859	0.6421
Rwanda	2000	5.0	9.0	14.0	20.0	52.0	4.2	9.2	14.4	22.0	50.3	0.4175	0.8116	4.0	9.3	14.6	21.9	50.2	0.5154	0.7728
South Africa	2000	3.5	6.3	10.0	18.0	62.2	0.9	3.5	7.7	16.2	71.7	11.6900	0.0029	0.7	3.5	8.1	16.6	71.2	14.5090	**0.0007**
Thailand	2000	5.5	8.8	13.2	21.5	51.0	4.1	9.1	14.3	22.0	50.5	0.5778	0.7491	4.0	9.2	14.5	21.9	50.4	0.7539	0.6860
Tunisia	2000	6.0	10.3	14.8	21.7	47.3	5.3	10.5	15.5	22.5	46.1	0.1974	0.9060	5.2	10.7	15.7	22.5	46.0	0.2609	0.8777
Turkey	2000	6.1	10.6	14.9	21.8	46.7	5.5	10.8	15.7	22.6	45.5	0.1754	0.9160	5.4	10.9	15.9	22.5	45.4	0.2293	0.8917
Uzbekistan	2000	9.2	14.1	17.9	22.6	36.3	6.1	11.4	16.2	22.7	43.5	3.4708	0.1763	6.0	11.6	16.4	22.7	43.4	3.4878	0.1748

Appendix 3. Parameter estimates for individual countries included in Chapter 2

Table A3.1. Parameter estimates, mean and Gini index in 1990

Country	Gini	a_1	a_2	Mean
Algeria	38.62	0.68	1.53	5021.86
Argentina	44.24	0.65	1.65	6927.9
Armenia	32.57	0.73	1.42	2937.89
Australia	43.76	0.65	1.64	26120.66
Austria	53.1	0.59	1.88	27500.61
Azerbaijan	36.89	0.7	1.5	3101.15
Bahamas	49.52	0.61	1.78	27985.66
Bangladesh	31.32	0.74	1.4	759.2
Barbados	42.91	0.65	1.62	26127.73
Belarus	27.16	0.77	1.33	6434.06
Belgium	32.46	0.73	1.42	26096.48
Bolivia	49.96	0.61	1.79	2713.33
Bosnia and Herzegovina	40.34	0.67	1.57	1144.21
Botswana	56.89	0.56	1.99	6672.89
Brazil	58.3	0.55	2.04	6144.66
Bulgaria	26.51	0.77	1.32	6820.55
Burundi	33.81	0.72	1.44	591.68
Cameroon	47.3	0.63	1.72	1786.24
Canada	39.09	0.68	1.54	27576.96
Cape Verde	43.84	0.65	1.64	1587.4
Central African Republic	59.55	0.55	2.08	680.2
Chile	52.11	0.59	1.85	5519.53
China Version 1	34.99	0.71	1.46	1154.3
Colombia	47.63	0.62	1.73	5410.4
Costa Rica	43.15	0.65	1.63	7330.48
Cote d`Ivoire	40.33	0.67	1.57	1549.43
Croatia	28.76	0.75	1.36	12800.79
Cyprus	36.81	0.7	1.5	13999.56
Czech Republic	29.69	0.75	1.37	15741.7
Denmark	48.66	0.62	1.76	26144.12
Dominican Republic	47.55	0.62	1.73	4709.11
Ecuador	45.84	0.63	1.69	4407.21
Egypt	33.29	0.72	1.43	2538.12
El Salvador	47.99	0.62	1.74	3902.96
Estonia	32.5	0.73	1.42	9300.06
Ethiopia	38.23	0.69	1.53	414.07
Finland	36.65	0.7	1.5	24624.12
France	41.06	0.67	1.58	25813.2
Gambia	53.62	0.58	1.89	1274.09

Table A3.1. Continued

Georgia	33.84	0.72	1.44	6138.17
Germany	45.07	0.64	1.67	26516.53
Ghana	38.16	0.69	1.52	1273.42
Greece	46.3	0.63	1.7	17334.06
Guatemala	57.2	0.56	2	4659.94
Guinea	49.17	0.61	1.77	726.91
Guinea-Bissau	53.37	0.59	1.88	1108.18
Guyana	43.01	0.65	1.62	1994.37
Haiti	55.67	0.57	1.95	1325.6
Honduras	53	0.59	1.87	3076.08
Hong Kong	48.08	0.62	1.74	22241.2
Hungary	40.04	0.67	1.56	12481.44
Iceland	35.58	0.7	1.48	35758.27
India	33.05	0.72	1.43	1430.57
Indonesia	37.73	0.69	1.52	2162.2
Iran	45.51	0.64	1.68	5809.5
Ireland	44.78	0.64	1.66	18682.24
Israel	41.03	0.67	1.58	17747.67
Italy	43.73	0.65	1.64	24792.45
Jamaica	49.48	0.61	1.78	8445.21
Japan	36	0.7	1.48	27717.52
Jordan	43.61	0.65	1.64	3015.1
Kazakhstan	29.36	0.75	1.37	7089.13
Kenya	58.55	0.55	2.05	1181.29
Korea, Republic of	35.38	0.71	1.47	11643.21
Kyrgyzstan	27.77	0.76	1.34	1972.06
Laos	31	0.74	1.4	984.72
Latvia	33.22	0.72	1.43	10108.68
Lesotho	61.07	0.54	2.14	816.17
Lithuania	35.11	0.71	1.47	12499.66
Luxembourg	34.38	0.71	1.45	44678.74
Macedonia	29.36	0.75	1.37	6406.67
Madagascar	45.59	0.64	1.68	886.9
Malawi	66.13	0.5	2.35	512.28
Malaysia	44.27	0.65	1.65	5789.19
Mali	39.53	0.68	1.55	651.81
Mauritania	46.75	0.63	1.71	1594.66
Mauritius	44.52	0.64	1.66	5168.4
Mexico	49.28	0.61	1.77	9392.83
Moldova	30.67	0.74	1.39	2245.83
Morocco	36.41	0.7	1.49	2336.78
Nepal	36.36	0.7	1.49	718.22
Netherlands	40.48	0.67	1.57	27258.31
New Zealand	42.17	0.66	1.61	19782.65

Table A3.1. Continued

Nicaragua	55.65	0.57	1.95	2090.53
Niger	40.23	0.67	1.57	551.58
Nigeria	46.27	0.63	1.7	1167.91
Norway	41.64	0.66	1.59	32591.76
Pakistan	42.05	0.66	1.6	1552.03
Panama	52.91	0.59	1.87	5508.98
Paraguay	37.03	0.69	1.5	3661.36
Peru	44.93	0.64	1.67	3821.67
Philippines	57.83	0.56	2.03	2336.82
Poland	34.02	0.72	1.45	7642.91
Portugal	48.3	0.62	1.75	15200.98
Puerto Rico	54.84	0.58	1.93	17828.28
Romania	32.93	0.72	1.43	6452.79
Russia	31.89	0.73	1.41	12607.88
Rwanda	31.97	0.73	1.41	714.52
Senegal	57.11	0.56	2	1200.02
Serbia	39.86	0.68	1.56	10631.29
Sierra Leone	62.23	0.53	2.19	958.02
Singapore	45.66	0.64	1.68	23624.49
Slovak Republic	26.74	0.77	1.33	11938.08
Slovenia	31.57	0.73	1.41	15493.75
South Africa	65.22	0.51	2.31	5400.58
Spain	37.25	0.69	1.51	20264.09
Sri Lanka	39.12	0.68	1.54	1848.19
Sweden	45.62	0.64	1.68	26300.01
Switzerland	39.45	0.68	1.55	34385.18
Taiwan	31.35	0.74	1.4	13637.65
Tajikistan	33.71	0.72	1.44	2960.79
Tanzania	43.65	0.65	1.64	745.14
Thailand	50.98	0.6	1.82	4403.78
Trinidad &Tobago	39.19	0.68	1.54	10537.52
Tunisia	37.31	0.69	1.51	4073.09
Turkey	44.6	0.64	1.66	6664.95
Turkmenistan	30.67	0.74	1.39	3749.04
Uganda	41.69	0.66	1.6	568.88
Ukraine	24.75	0.78	1.3	8062.6
United Kingdom	46.66	0.63	1.71	23154.23
United States	43.23	0.65	1.63	31388.79
Uruguay	39.91	0.67	1.56	6108.13
Uzbekistan	31.92	0.73	1.41	1940.11
Venezuela	41.79	0.66	1.6	8082.04
Vietnam	35.83	0.7	1.48	900.48
Yemen	38.9	0.68	1.54	1471.36
Zambia	56.04	0.57	1.97	900.08

Table A3.2. Parameter estimates, mean and Gini index in 1995

Country	Gini	a_1	a_2	Mean
Algeria	35.48	0.71	1.47	4566.97
Argentina	45.62	0.64	1.68	8323.1
Armenia	45.47	0.64	1.68	1915.74
Australia	43.35	0.65	1.63	29476.39
Austria	43.55	0.65	1.64	29359.64
Azerbaijan	46.8	0.63	1.71	1802.73
Bahamas	51.04	0.6	1.82	25667.96
Bangladesh	34.52	0.71	1.46	817.07
Barbados	40.55	0.67	1.57	23630.35
Belarus	27.55	0.76	1.34	4227.79
Belgium	43.16	0.65	1.63	27828.04
Bolivia	54.28	0.58	1.91	2946.63
Bosnia and Herzegovina	36.24	0.7	1.49	1596.41
Botswana	55.75	0.57	1.96	6659.06
Brazil	57.23	0.56	2	6646.4
Bulgaria	37.19	0.69	1.51	6688.9
Burundi	37.1	0.69	1.5	460.87
Cameroon	46.03	0.63	1.69	1445.03
Canada	40.64	0.67	1.57	28486.23
Cape Verde	50.6	0.6	1.81	1809.2
Central African Republic	57.3	0.56	2.01	602.07
Chile	52.28	0.59	1.85	7970.51
China Version 1	39.87	0.68	1.56	1931.26
Colombia	52.87	0.59	1.87	6166.68
Costa Rica	43.39	0.65	1.63	8076.2
Cote d`Ivoire	37.94	0.69	1.52	1377.63
Croatia	35.25	0.71	1.47	9186.35
Cyprus	39.19	0.68	1.54	15605.85
Czech Republic	34.49	0.71	1.46	15079.02
Denmark	47.64	0.62	1.73	28938.57
Dominican Republic	47.96	0.62	1.74	5603.84
Ecuador	44.24	0.65	1.65	4798.52
Egypt	37.65	0.69	1.51	3119.02
El Salvador	48.64	0.62	1.76	4992.75
Estonia	42.6	0.66	1.62	7632.24
Ethiopia	41.34	0.66	1.59	387.78
Finland	39.73	0.68	1.56	22939.54
France	48.22	0.62	1.75	26497.32
Gambia, The	54.7	0.58	1.92	1089.73
Algeria	35.48	0.71	1.47	4566.97

Table A3.2. Continued

Georgia	46.48	0.63	1.7	1928.27
Germany	46.9	0.63	1.71	28486.29
Ghana	36.1	0.7	1.49	1283.39
Greece	41.39	0.66	1.59	17877.86
Guatemala	56.1	0.57	1.97	4970.21
Guinea	43.33	0.65	1.63	718.48
Guinea-Bissau	46.02	0.63	1.69	1091.63
Guyana	44.92	0.64	1.67	2786.87
Haiti	55.64	0.57	1.95	1298.7
Honduras	52.36	0.59	1.86	2956.29
Hong Kong	51.08	0.6	1.82	26606
Hungary	43.73	0.65	1.64	11367.92
Iceland	35.62	0.7	1.48	35337.54
India	35.22	0.71	1.47	1611.27
Indonesia	38.18	0.69	1.52	2891.47
Iran	45.08	0.64	1.67	6351.69
Ireland	42.73	0.66	1.62	22249.03
Israel	42.99	0.65	1.62	20819.27
Italy	47.23	0.63	1.72	26158.75
Jamaica	58.2	0.55	2.04	9258.83
Japan	38.71	0.68	1.54	28970.08
Jordan	42.16	0.66	1.61	3433.41
Kazakhstan	36.03	0.7	1.48	4571.57
Kenya	50.35	0.6	1.8	1120.15
Korea, Republic of	34.14	0.72	1.45	15889.18
Kyrgyzstan	50.67	0.6	1.81	1324.63
Laos	34.93	0.71	1.46	1168.21
Latvia	40.62	0.67	1.57	5995.18
Lesotho	63.93	0.52	2.26	894.93
Lithuania	49.28	0.61	1.77	7314.53
Luxembourg	35.77	0.7	1.48	51287.71
Macedonia	30.78	0.74	1.39	5476.95
Madagascar	43.05	0.65	1.62	776.7
Malawi	56.42	0.57	1.98	497.54
Malaysia	46.28	0.63	1.7	8486.74
Mali	50.27	0.61	1.8	705.68
Mauritania	41.16	0.67	1.58	1469.12
Mauritius	46.84	0.63	1.71	6076.27
Mexico	50.94	0.6	1.82	9122.99
Moldova	42.04	0.66	1.6	1701.29
Morocco	40.11	0.67	1.56	2157.22
Nepal	42.1	0.66	1.6	797.88
Netherlands	41.66	0.66	1.6	29483.84

Table A3.2. Continued

Nicaragua	55.22	0.57	1.94	1823.15
Niger	49.9	0.61	1.79	503.01
Nigeria	50.54	0.6	1.81	1047.76
Norway	46.16	0.63	1.7	38399.04
Pakistan	35.22	0.71	1.47	1705.53
Panama	53.24	0.59	1.88	6330.19
Paraguay	54.78	0.58	1.93	3491.06
Peru	48.39	0.62	1.75	4553.34
Philippines	58.19	0.55	2.04	2362.01
Poland	46.6	0.63	1.71	8772.29
Portugal	57.52	0.56	2.01	16318.6
Puerto Rico	57.26	0.56	2.01	19370.45
Romania	42.08	0.66	1.6	5623.87
Russia	51.34	0.6	1.83	8084.51
Rwanda	38.8	0.68	1.54	615.67
Senegal	42.4	0.66	1.61	1139.83
Serbia	36	0.7	1.48	5152.83
Sierra Leone	55.21	0.57	1.94	800.02
Singapore	45.58	0.64	1.68	31249.67
Slovak Republic	30.81	0.74	1.39	10217.67
Slovenia	40.78	0.67	1.58	15481.71
South Africa	63.59	0.52	2.24	5388.69
Spain	46.64	0.63	1.71	21487.35
Sri Lanka	44.04	0.65	1.65	2288.25
Sweden	48.63	0.62	1.76	25665.2
Switzerland	40.44	0.67	1.57	33006.92
Taiwan	33.94	0.72	1.45	18542.18
Tajikistan	33.79	0.72	1.44	1070.48
Tanzania	39.1	0.68	1.54	671.03
Thailand	53.05	0.59	1.88	6104.65
Trinidad &Tobago	38.59	0.68	1.53	12081.17
Tunisia	41.07	0.67	1.58	4463.13
Turkey	43.33	0.65	1.63	7100.2
Turkmenistan	30.57	0.74	1.39	7240.78
Uganda	36.71	0.7	1.5	707.09
Ukraine	43.34	0.65	1.63	3781.31
United Kingdom	47.63	0.62	1.73	24686.31
United States	46.19	0.63	1.7	33560.13
Uruguay	40.67	0.67	1.57	7975.77
Uzbekistan	37.62	0.69	1.51	1399.86
Venezuela	44.78	0.64	1.66	8873.39
Vietnam	35.85	0.7	1.48	1188.24
Yemen	35.63	0.7	1.48	1636.68
Zambia	52.46	0.59	1.86	962.18

Table A3.3. Parameter estimates, mean and Gini index in 2000

Country	Gini	a_1	a_2	Mean
Algeria	36.16	0.7	1.49	4987.65
Argentina	48.54	0.62	1.75	8908.61
Armenia	47.09	0.63	1.72	2517.15
Australia	43.83	0.65	1.64	34071.96
Austria	43.71	0.65	1.64	33625.05
Azerbaijan	36.54	0.7	1.49	2521.38
Bahamas	55.51	0.57	1.95	31362.62
Bangladesh	32.91	0.72	1.43	907.67
Barbados	5.06	0.95	1.05	28235.56
Belarus	27.34	0.76	1.34	6154.75
Belgium	43.16	0.65	1.63	31700.88
Bolivia	57.58	0.56	2.02	3118.39
Bosnia and Herzegovina	31.13	0.74	1.4	4403.68
Botswana	55.35	0.57	1.94	8701.78
Brazil	57.25	0.56	2.01	6839
Bulgaria	29.25	0.75	1.37	6589.07
Burundi	38.69	0.68	1.53	389.5
Cameroon	44.37	0.64	1.65	1636.18
Canada	43.04	0.65	1.62	33574.98
Cape Verde	57.36	0.56	2.01	2352.28
Central African Republic	48.68	0.62	1.76	605.94
Chile	52.2	0.59	1.85	9339.26
China Version 1	41.25	0.67	1.59	2822.38
Colombia	53.21	0.59	1.88	5793.84
Costa Rica	45.33	0.64	1.68	8863.89
Cote d`Ivoire	45.82	0.63	1.69	1475.91
Croatia	32.56	0.73	1.42	11307.67
Cyprus	44.27	0.65	1.65	17089.32
Czech Republic	41.05	0.67	1.58	16594.92
Denmark	46.24	0.63	1.7	33124.24
Dominican Republic	48.59	0.62	1.76	7287.06
Ecuador	54.19	0.58	1.91	4626.64
Egypt	37.16	0.69	1.51	3657.68
El Salvador	50.32	0.61	1.8	5324.55
Estonia	41.09	0.67	1.58	10983.75
Ethiopia	34.87	0.71	1.46	414.99
Finland	45.96	0.63	1.69	28939.91
France	46.92	0.63	1.71	29667.54
Gambia, The	49.06	0.61	1.77	1224.92
Algeria	36.16	0.7	1.49	4987.65

Table A3.3. Continued

Georgia	47.49	0.62	1.73	2904
Germany	51.02	0.6	1.82	30997.95
Ghana	39.78	0.68	1.56	1477.92
Greece	50.22	0.61	1.8	21041.73
Guatemala	55.51	0.57	1.95	5375.13
Guinea	42.93	0.65	1.62	799.6
Guinea-Bissau	38.56	0.68	1.53	824.35
Guyana	42.36	0.66	1.61	3161.59
Haiti	55.62	0.57	1.95	1387.83
Honduras	51.8	0.6	1.84	2996.55
Hong Kong	56.86	0.56	1.99	28011.21
Hungary	46	0.63	1.69	13261.15
Iceland	40.94	0.67	1.58	37759.56
India	34.35	0.71	1.45	1921.9
Indonesia	33.68	0.72	1.44	2749.85
Iran	44.58	0.64	1.66	7334.96
Ireland	42.27	0.66	1.61	34198.59
Israel	44.63	0.64	1.66	23249.05
Italy	44.77	0.64	1.66	28677.14
Jamaica	46.47	0.63	1.7	8743.89
Japan	40.28	0.67	1.57	29789.76
Jordan	39.71	0.68	1.55	3380.75
Kazakhstan	34.26	0.71	1.45	4910.57
Kenya	49.79	0.61	1.79	1112.5
Korea, Republic of	33.91	0.72	1.45	18728.63
Kyrgyzstan	39.6	0.68	1.55	1635.52
Laos	36.18	0.7	1.49	1368.81
Latvia	46.9	0.63	1.71	7990.29
Lesotho	58.39	0.55	2.04	1079.91
Lithuania	46.17	0.63	1.7	8927.62
Luxembourg	41.81	0.66	1.6	65125.53
Macedonia	33.2	0.72	1.43	6107.92
Madagascar	43.43	0.65	1.63	787.26
Malawi	45.63	0.64	1.68	465.33
Malaysia	47.32	0.62	1.72	9481.72
Mali	41.43	0.66	1.59	728.45
Mauritania	38.81	0.68	1.54	1593.02
Mauritius	47.98	0.62	1.74	7590.09
Mexico	49.48	0.61	1.78	11382.83
Moldova	42.26	0.66	1.61	1558.65
Morocco	40.42	0.67	1.57	2446.35
Nepal	46.43	0.63	1.7	833
Netherlands	40.93	0.67	1.58	35192.22

Table A3.3. Continued

Nicaragua	53.22	0.59	1.88	2067.68
Niger	45.78	0.64	1.69	491.5
Nigeria	47.8	0.62	1.74	1107.7
Norway	46.12	0.63	1.69	45346.25
Pakistan	31.38	0.74	1.4	1779.83
Panama	52.89	0.59	1.87	6959.55
Paraguay	53.42	0.58	1.89	3286.12
Peru	50.44	0.6	1.8	4780.82
Philippines	46.66	0.63	1.71	2461.68
Poland	37.95	0.69	1.52	11283.72
Portugal	54.94	0.58	1.93	19831.95
Puerto Rico	58.79	0.55	2.06	25285.74
Romania	40.57	0.67	1.57	5828.16
Russia	47.86	0.62	1.74	8521.85
Rwanda	45.63	0.64	1.68	632.74
Senegal	41.8	0.66	1.6	1282.19
Serbia	32.97	0.72	1.43	5528.9
Sierra Leone	48.19	0.62	1.75	476.65
Singapore	47.95	0.62	1.74	37210.59
Slovak Republic	34.92	0.71	1.46	12287.96
Slovenia	40.14	0.67	1.56	19189.33
South Africa	69.01	0.48	2.5	5874.92
Spain	39.1	0.68	1.54	26043.01
Sri Lanka	52.77	0.59	1.87	2735.26
Sweden	47.77	0.62	1.73	29992.96
Switzerland	42.34	0.66	1.61	35797.31
Taiwan	36.14	0.7	1.49	23064.55
Tajikistan	34.25	0.71	1.45	1027.31
Tanzania	35.15	0.71	1.47	724.75
Thailand	45.89	0.63	1.69	5651.09
Trinidad &Tobago	38.07	0.69	1.52	15763
Tunisia	40.43	0.67	1.57	5381.11
Turkey	39.64	0.68	1.55	8168
Turkmenistan	33.12	0.72	1.43	8752.23
Uganda	45.04	0.64	1.67	843.5
Ukraine	35.94	0.7	1.48	3730.92
United Kingdom	47.65	0.62	1.73	30155.49
United States	47.19	0.63	1.72	39668.69
Uruguay	41.57	0.66	1.59	8622.27
Uzbekistan	37.08	0.69	1.5	1561.09
Venezuela	44.9	0.64	1.67	8658.2
Vietnam	37.43	0.69	1.51	1538.14
Yemen	34.32	0.71	1.45	2046.72
Zambia	49.9	0.61	1.79	973.49

Table A3.4. Inequality measures for each country in 1990 (Lamé I).

Country	Theil	GE(1.5)	GE(0.5)	GE(0.75)
Algeria	0.2675	0.3081	0.256	0.2588
Argentina	0.362	0.4393	0.3414	0.3464
Armenia	0.1851	0.2039	0.1795	0.1809
Australia	0.3531	0.4264	0.3335	0.3382
Austria	0.5547	0.7521	0.5079	0.5189
Azerbaijan	0.2421	0.2751	0.2327	0.235
Bahamas	0.4696	0.6058	0.4356	0.4437
Bangladesh	0.1704	0.1863	0.1657	0.1668
Barbados	0.338	0.4047	0.32	0.3243
Belarus	0.1263	0.1349	0.1236	0.1243
Belgium	0.1838	0.2023	0.1782	0.1796
Bolivia	0.4795	0.622	0.444	0.4525
Bosnia and Herzegovina	0.2945	0.3442	0.2806	0.284
Botswana	0.6572	0.9483	0.5927	0.6077
Brazil	0.6992	1.0356	0.6267	0.6436
Bulgaria	0.1201	0.1278	0.1177	0.1183
Burundi	0.2006	0.2228	0.194	0.1956
Cameroon	0.4219	0.5295	0.3942	0.4008
Canada	0.2748	0.3177	0.2627	0.2656
Cape Verde	0.3546	0.4284	0.3347	0.3395
Central African Republic	0.7379	1.1198	0.6576	0.6762
Chile	0.5301	0.7084	0.4872	0.4973
China Version 1	0.2159	0.2418	0.2083	0.2101
Colombia	0.4288	0.5403	0.4002	0.407
Costa Rica	0.3421	0.4105	0.3236	0.328
Cote d`Ivoire	0.2944	0.344	0.2805	0.2839
Croatia	0.1423	0.1533	0.139	0.1398
Cyprus	0.2409	0.2735	0.2315	0.2338
Czech Republic	0.1523	0.1648	0.1484	0.1494
Denmark	0.4507	0.575	0.4192	0.4267
Dominican Republic	0.4272	0.5378	0.3988	0.4056
Ecuador	0.3926	0.4847	0.3685	0.3743
Egypt	0.194	0.2147	0.1879	0.1894
El Salvador	0.4365	0.5524	0.4069	0.414
Estonia	0.1843	0.203	0.1787	0.1801
Ethiopia	0.2617	0.3004	0.2507	0.2533
Finland	0.2387	0.2706	0.2295	0.2317
France	0.3063	0.3603	0.2913	0.295
Gambia, The	0.5679	0.7761	0.519	0.5305
Algeria	0.2675	0.3081	0.256	0.2588

Table A3.4. Continued

Georgia	0.201	0.2233	0.1944	0.196
Germany	0.3776	0.4622	0.3552	0.3606
Ghana	0.2607	0.2991	0.2497	0.2524
Greece	0.4016	0.4983	0.3764	0.3824
Guatemala	0.6661	0.9664	0.5999	0.6154
Guinea	0.4617	0.5928	0.4287	0.4366
Guinea-Bissau	0.5614	0.7644	0.5136	0.5249
Guyana	0.3397	0.4071	0.3215	0.3259
Haiti	0.6224	0.8791	0.5642	0.5778
Honduras	0.552	0.7472	0.5056	0.5165
Hong Kong	0.4384	0.5554	0.4086	0.4157
Hungary	0.2897	0.3378	0.2763	0.2796
Iceland	0.2239	0.2518	0.2158	0.2177
India	0.191	0.2111	0.185	0.1865
Indonesia	0.2543	0.2908	0.2439	0.2464
Iran	0.3862	0.475	0.3628	0.3684
Ireland	0.3722	0.4541	0.3504	0.3556
Israel	0.3057	0.3595	0.2908	0.2944
Italy	0.3526	0.4256	0.333	0.3377
Jamaica	0.4687	0.6042	0.4347	0.4428
Japan	0.2296	0.259	0.221	0.2231
Jordan	0.3506	0.4226	0.3311	0.3358
Kazakhstan	0.1487	0.1606	0.145	0.1459
Kenya	0.7069	1.0519	0.6328	0.65
Korea, Republic of	0.2211	0.2483	0.2131	0.2151
Kyrgyzstan	0.1323	0.1418	0.1294	0.1301
Laos	0.1668	0.1819	0.1622	0.1633
Latvia	0.1931	0.2136	0.187	0.1885
Lesotho	0.7881	1.2352	0.6973	0.7182
Lithuania	0.2175	0.2439	0.2099	0.2117
Luxembourg	0.2079	0.2319	0.2009	0.2026
Macedonia	0.1487	0.1607	0.145	0.1459
Madagascar	0.3876	0.4771	0.3641	0.3697
Malawi	0.9776	1.7403	0.8418	0.8726
Malaysia	0.3626	0.4401	0.3419	0.3469
Mali	0.2815	0.3267	0.2688	0.2719
Mauritania	0.4107	0.5123	0.3844	0.3907
Mauritius	0.3674	0.4471	0.3461	0.3512
Mexico	0.4641	0.5968	0.4308	0.4387
Moldova	0.163	0.1775	0.1587	0.1597
Morocco	0.2353	0.2663	0.2263	0.2285
Nepal	0.2345	0.2653	0.2256	0.2278
Netherlands	0.2968	0.3473	0.2827	0.2861

Table A3.4. Continued

Nicaragua	0.6219	0.8781	0.5638	0.5774
Niger	0.2927	0.3418	0.279	0.2823
Nigeria	0.4011	0.4976	0.376	0.382
Norway	0.316	0.3737	0.3001	0.3039
Pakistan	0.3229	0.3834	0.3064	0.3104
Panama	0.5497	0.7432	0.5037	0.5146
Paraguay	0.2441	0.2776	0.2345	0.2368
Peru	0.375	0.4583	0.3529	0.3582
Philippines	0.685	1.0056	0.6152	0.6315
Poland	0.2033	0.2261	0.1966	0.1982
Portugal	0.4428	0.5624	0.4123	0.4196
Puerto Rico	0.5999	0.8359	0.5456	0.5584
Romania	0.1895	0.2093	0.1837	0.1851
Russia	0.177	0.1941	0.1718	0.1731
Rwanda	0.178	0.1954	0.1728	0.1741
Senegal	0.6637	0.9614	0.5979	0.6133
Serbia	0.2868	0.3338	0.2736	0.2768
Sierra Leone	0.8284	1.333	0.7287	0.7516
Singapore	0.3889	0.4791	0.3652	0.3709
Slovak Republic	0.1222	0.1302	0.1197	0.1203
Slovenia	0.1732	0.1897	0.1683	0.1695
South Africa	0.9405	1.6316	0.8141	0.8429
Spain	0.2472	0.2815	0.2373	0.2397
Sri Lanka	0.2752	0.3183	0.2631	0.266
Sweden	0.3882	0.478	0.3646	0.3703
Switzerland	0.2803	0.3251	0.2677	0.2708
Taiwan	0.1707	0.1867	0.166	0.1671
Tajikistan	0.1993	0.2212	0.1928	0.1944
Tanzania	0.3512	0.4236	0.3317	0.3364
Thailand	0.5029	0.6614	0.4641	0.4733
Trinidad &Tobago	0.2763	0.3198	0.2641	0.2671
Tunisia	0.248	0.2826	0.2381	0.2405
Turkey	0.3688	0.4492	0.3474	0.3526
Turkmenistan	0.163	0.1774	0.1586	0.1597
Uganda	0.3169	0.375	0.3009	0.3048
Ukraine	0.1041	0.1099	0.1023	0.1028
United Kingdom	0.4089	0.5095	0.3828	0.3891
United States	0.3436	0.4126	0.3249	0.3294
Uruguay	0.2876	0.3349	0.2744	0.2776
Uzbekistan	0.1773	0.1946	0.1722	0.1734
Venezuela	0.3186	0.3773	0.3025	0.3064
Vietnam	0.2272	0.2561	0.2189	0.2209
Yemen	0.2718	0.3138	0.26	0.2629
Zambia	0.6328	0.8994	0.5727	0.5868

Table A3.5. Inequality measures for each country in 1995 (Lamé I).

Country	Theil	GE(1.5)	GE(0.5)	GE(0.75)
Algeria	0.2225	0.2501	0.2144	0.2164
Argentina	0.3882	0.478	0.3646	0.3703
Armenia	0.3854	0.4737	0.3621	0.3677
Australia	0.3459	0.4159	0.327	0.3315
Austria	0.3495	0.4211	0.3302	0.3348
Azerbaijan	0.4116	0.5136	0.3852	0.3915
Bahamas	0.5044	0.6639	0.4654	0.4746
Bangladesh	0.2098	0.2341	0.2026	0.2043
Barbados	0.2979	0.3488	0.2837	0.2872
Belarus	0.1301	0.1392	0.1273	0.128
Belgium	0.3423	0.4108	0.3238	0.3283
Bolivia	0.5851	0.8079	0.5333	0.5455
Bosnia and Herzegovina	0.233	0.2634	0.2242	0.2264
Botswana	0.6247	0.8837	0.5661	0.5798
Brazil	0.667	0.9683	0.6007	0.6161
Bulgaria	0.2463	0.2804	0.2365	0.2389
Burundi	0.2451	0.2788	0.2354	0.2378
Cameroon	0.3963	0.4902	0.3717	0.3776
Canada	0.2992	0.3506	0.2849	0.2884
Cape Verde	0.494	0.6463	0.4565	0.4654
Central African Republic	0.6692	0.9727	0.6024	0.618
Chile	0.534	0.7153	0.4905	0.5008
China Version 1	0.287	0.3341	0.2738	0.277
Colombia	0.5488	0.7416	0.503	0.5138
Costa Rica	0.3465	0.4168	0.3275	0.3321
Cote d`Ivoire	0.2574	0.2948	0.2467	0.2493
Croatia	0.2193	0.2461	0.2115	0.2134
Cyprus	0.2763	0.3197	0.2641	0.267
Czech Republic	0.2093	0.2336	0.2022	0.2039
Denmark	0.429	0.5406	0.4004	0.4072
Dominican Republic	0.4357	0.5512	0.4062	0.4132
Ecuador	0.362	0.4393	0.3414	0.3464
Egypt	0.253	0.2891	0.2427	0.2452
El Salvador	0.4502	0.5743	0.4188	0.4263
Estonia	0.3325	0.3968	0.315	0.3192
Ethiopia	0.3109	0.3667	0.2956	0.2993
Finland	0.2847	0.331	0.2718	0.2749
France	0.4411	0.5597	0.4109	0.4181
Gambia, The	0.5962	0.8288	0.5425	0.5551
Algeria	0.2225	0.2501	0.2144	0.2164

Table A3.5. Continued

Georgia	0.4052	0.5038	0.3796	0.3857
Germany	0.4137	0.5169	0.387	0.3934
Ghana	0.2309	0.2607	0.2223	0.2244
Greece	0.3117	0.3678	0.2963	0.3
Guatemala	0.6345	0.9028	0.5741	0.5882
Guinea	0.3454	0.4152	0.3266	0.3311
Guinea-Bissau	0.3961	0.4899	0.3715	0.3774
Guyana	0.3748	0.458	0.3527	0.358
Haiti	0.6217	0.8778	0.5636	0.5773
Honduras	0.5361	0.7189	0.4922	0.5026
Hong Kong	0.5053	0.6655	0.4662	0.4754
Hungary	0.3527	0.4257	0.333	0.3378
Iceland	0.2243	0.2524	0.2162	0.2182
India	0.219	0.2457	0.2112	0.2131
Indonesia	0.2609	0.2994	0.2499	0.2526
Iran	0.3778	0.4625	0.3554	0.3608
Ireland	0.3348	0.4001	0.3171	0.3213
Israel	0.3394	0.4066	0.3211	0.3255
Italy	0.4205	0.5274	0.393	0.3996
Jamaica	0.6962	1.0292	0.6242	0.641
Japan	0.269	0.31	0.2573	0.2602
Jordan	0.3248	0.386	0.308	0.3121
Kazakhstan	0.23	0.2596	0.2214	0.2235
Kenya	0.4883	0.6368	0.4516	0.4603
Korea, Republic of	0.2048	0.228	0.198	0.1996
Kyrgyzstan	0.4956	0.6491	0.4579	0.4669
Laos	0.2152	0.2409	0.2076	0.2095
Latvia	0.2989	0.3502	0.2847	0.2881
Lesotho	0.8905	1.4933	0.7763	0.8024
Lithuania	0.4641	0.5968	0.4308	0.4388
Luxembourg	0.2264	0.255	0.2181	0.2201
Macedonia	0.1643	0.179	0.1598	0.1609
Madagascar	0.3406	0.4083	0.3222	0.3266
Malawi	0.6436	0.921	0.5816	0.5961
Malaysia	0.4012	0.4977	0.376	0.3821
Mali	0.4864	0.6336	0.45	0.4587
Mauritania	0.3078	0.3624	0.2927	0.2964
Mauritius	0.4125	0.5149	0.3859	0.3923
Mexico	0.5021	0.66	0.4634	0.4726
Moldova	0.3228	0.3832	0.3063	0.3103
Morocco	0.2909	0.3393	0.2774	0.2807
Nepal	0.3238	0.3845	0.3071	0.3111
Netherlands	0.3163	0.3742	0.3004	0.3042

Table A3.5. Continued

Nicaragua	0.6103	0.8558	0.5542	0.5674
Niger	0.4781	0.6198	0.4429	0.4513
Nigeria	0.4927	0.6442	0.4554	0.4643
Norway	0.3988	0.494	0.3739	0.3799
Pakistan	0.219	0.2457	0.2112	0.2131
Panama	0.5582	0.7585	0.5108	0.522
Paraguay	0.5984	0.833	0.5444	0.5571
Peru	0.4448	0.5656	0.4141	0.4215
Philippines	0.6958	1.0283	0.6239	0.6407
Poland	0.4076	0.5074	0.3817	0.3879
Portugal	0.6758	0.9864	0.6078	0.6236
Puerto Rico	0.6679	0.97	0.6013	0.6169
Romania	0.3235	0.3842	0.3069	0.3109
Russia	0.5114	0.6759	0.4713	0.4808
Rwanda	0.2703	0.3118	0.2586	0.2614
Senegal	0.3291	0.3921	0.3119	0.3161
Serbia	0.2296	0.2591	0.2211	0.2232
Sierra Leone	0.6102	0.8555	0.5541	0.5673
Singapore	0.3875	0.477	0.364	0.3696
Slovak Republic	0.1646	0.1794	0.1602	0.1613
Slovenia	0.3015	0.3538	0.287	0.2906
South Africa	0.8778	1.4594	0.7666	0.7921
Spain	0.4085	0.5088	0.3825	0.3887
Sri Lanka	0.3585	0.4341	0.3382	0.3431
Sweden	0.45	0.574	0.4186	0.4261
Switzerland	0.2962	0.3465	0.2822	0.2856
Taiwan	0.2022	0.2248	0.1956	0.1972
Tajikistan	0.2003	0.2225	0.1938	0.1954
Tanzania	0.275	0.318	0.2628	0.2658
Thailand	0.5534	0.7498	0.5068	0.5178
Trinidad &Tobago	0.2671	0.3075	0.2556	0.2584
Tunisia	0.3065	0.3606	0.2915	0.2951
Turkey	0.3454	0.4152	0.3265	0.3311
Turkmenistan	0.1619	0.1761	0.1575	0.1586
Uganda	0.2396	0.2718	0.2303	0.2326
Ukraine	0.3457	0.4156	0.3268	0.3313
United Kingdom	0.4289	0.5404	0.4003	0.4071
United States	0.3994	0.4949	0.3744	0.3804
Uruguay	0.2998	0.3514	0.2854	0.2889
Uzbekistan	0.2526	0.2886	0.2423	0.2449
Venezuela	0.3721	0.454	0.3503	0.3555
Vietnam	0.2276	0.2565	0.2192	0.2212
Yemen	0.2245	0.2526	0.2163	0.2183
Zambia	0.5385	0.7232	0.4943	0.5047

Table A3.6. Inequality measures for each country in 2000 (Lamé I).

Country	Theil	GE(1.5)	GE(0.5)	GE(0.75)
Algeria	0.2318	0.2619	0.2231	0.2252
Argentina	0.448	0.5708	0.4169	0.4243
Armenia	0.4177	0.523	0.3905	0.397
Australia	0.3544	0.4282	0.3346	0.3393
Austria	0.3523	0.4252	0.3327	0.3374
Azerbaijan	0.2371	0.2686	0.228	0.2302
Bahamas	0.6181	0.8708	0.5607	0.5742
Bangladesh	0.1893	0.209	0.1834	0.1848
Barbados	0.0042	0.0042	0.0042	0.0042
Belarus	0.128	0.1368	0.1253	0.126
Belgium	0.3424	0.411	0.3239	0.3284
Bolivia	0.6775	0.9899	0.6091	0.6251
Bosnia and Herzegovina	0.1682	0.1836	0.1635	0.1647
Botswana	0.6137	0.8621	0.557	0.5703
Brazil	0.6678	0.9699	0.6013	0.6168
Bulgaria	0.1474	0.1592	0.1439	0.1447
Burundi	0.2686	0.3095	0.257	0.2598
Cameroon	0.3645	0.4429	0.3436	0.3486
Canada	0.3403	0.4079	0.322	0.3264
Cape Verde	0.6709	0.9763	0.6038	0.6195
Central African Republic	0.451	0.5755	0.4195	0.427
Chile	0.5321	0.7119	0.4888	0.4991
China Version 1	0.3094	0.3646	0.2941	0.2978
Colombia	0.5575	0.7572	0.5102	0.5214
Costa Rica	0.3826	0.4696	0.3596	0.3651
Cote d`Ivoire	0.3922	0.4841	0.3681	0.3739
Croatia	0.185	0.2038	0.1794	0.1808
Cyprus	0.3627	0.4402	0.3419	0.3469
Czech Republic	0.306	0.3599	0.2911	0.2947
Denmark	0.4004	0.4964	0.3753	0.3813
Dominican Republic	0.4491	0.5726	0.4179	0.4253
Ecuador	0.5826	0.8034	0.5313	0.5434
Egypt	0.2459	0.2799	0.2361	0.2385
El Salvador	0.4876	0.6356	0.451	0.4597
Estonia	0.3068	0.361	0.2918	0.2954
Ethiopia	0.2144	0.2399	0.2069	0.2087
Finland	0.3949	0.4881	0.3705	0.3763
France	0.4142	0.5176	0.3875	0.3939
Gambia, The	0.4594	0.5891	0.4268	0.4345
Algeria	0.2318	0.2619	0.2231	0.2252

Table A3.6. Continued

Georgia	0.4259	0.5358	0.3977	0.4044
Germany	0.5039	0.663	0.4649	0.4741
Ghana	0.2856	0.3322	0.2725	0.2757
Greece	0.4853	0.6318	0.4491	0.4577
Guatemala	0.618	0.8705	0.5606	0.574
Guinea	0.3382	0.405	0.3201	0.3245
Guinea-Bissau	0.2667	0.307	0.2553	0.258
Guyana	0.3283	0.391	0.3112	0.3154
Haiti	0.6211	0.8765	0.5631	0.5767
Honduras	0.5224	0.6949	0.4806	0.4905
Hong Kong	0.6562	0.9463	0.5919	0.6069
Hungary	0.3958	0.4894	0.3713	0.3771
Iceland	0.3042	0.3574	0.2894	0.293
India	0.2075	0.2314	0.2005	0.2023
Indonesia	0.1989	0.2208	0.1925	0.1941
Iran	0.3684	0.4486	0.347	0.3522
Ireland	0.3268	0.3888	0.3098	0.3139
Israel	0.3693	0.4498	0.3478	0.353
Italy	0.372	0.4538	0.3502	0.3554
Jamaica	0.4051	0.5036	0.3794	0.3856
Japan	0.2936	0.3429	0.2798	0.2831
Jordan	0.2844	0.3305	0.2714	0.2746
Kazakhstan	0.2064	0.2299	0.1994	0.2011
Kenya	0.4756	0.6156	0.4407	0.449
Korea, Republic of	0.2018	0.2243	0.1952	0.1968
Kyrgyzstan	0.2827	0.3282	0.2699	0.273
Laos	0.2321	0.2622	0.2234	0.2255
Latvia	0.4137	0.5169	0.387	0.3934
Lesotho	0.7018	1.0411	0.6288	0.6457
Lithuania	0.3991	0.4944	0.3741	0.3801
Luxembourg	0.3188	0.3777	0.3027	0.3066
Macedonia	0.1928	0.2133	0.1868	0.1882
Madagascar	0.3473	0.4179	0.3282	0.3328
Malawi	0.3884	0.4783	0.3647	0.3704
Malaysia	0.4224	0.5303	0.3946	0.4012
Mali	0.3124	0.3687	0.2969	0.3006
Mauritania	0.2705	0.312	0.2587	0.2616
Mauritius	0.4363	0.5521	0.4067	0.4138
Mexico	0.4685	0.604	0.4346	0.4427
Moldova	0.3266	0.3886	0.3097	0.3138
Morocco	0.2959	0.3461	0.2819	0.2853
Nepal	0.4043	0.5024	0.3788	0.3849
Netherlands	0.3041	0.3573	0.2893	0.2929

Table A3.6. Continued

Nicaragua	0.5577	0.7575	0.5104	0.5216
Niger	0.3914	0.4828	0.3674	0.3732
Nigeria	0.4324	0.546	0.4034	0.4103
Norway	0.3981	0.493	0.3733	0.3793
Pakistan	0.1711	0.1871	0.1663	0.1674
Panama	0.5493	0.7425	0.5034	0.5142
Paraguay	0.5627	0.7667	0.5146	0.526
Peru	0.4906	0.6406	0.4536	0.4623
Philippines	0.4088	0.5092	0.3827	0.3889
Poland	0.2576	0.295	0.2469	0.2495
Portugal	0.6027	0.8411	0.5479	0.5608
Puerto Rico	0.7142	1.0678	0.6387	0.6563
Romania	0.2981	0.3491	0.2839	0.2874
Russia	0.4336	0.5478	0.4044	0.4114
Rwanda	0.3884	0.4784	0.3648	0.3705
Senegal	0.3186	0.3774	0.3025	0.3064
Serbia	0.19	0.2099	0.1841	0.1856
Sierra Leone	0.4405	0.5588	0.4104	0.4176
Singapore	0.4355	0.5508	0.406	0.413
Slovak Republic	0.2149	0.2406	0.2074	0.2093
Slovenia	0.2913	0.3399	0.2777	0.281
South Africa	1.1041	2.1519	0.9341	0.9722
Spain	0.2749	0.3179	0.2628	0.2657
Sri Lanka	0.5464	0.7372	0.5009	0.5117
Sweden	0.4317	0.5449	0.4027	0.4097
Switzerland	0.328	0.3906	0.311	0.3151
Taiwan	0.2315	0.2615	0.2229	0.225
Tajikistan	0.2062	0.2297	0.1992	0.2009
Tanzania	0.2181	0.2445	0.2103	0.2122
Thailand	0.3935	0.486	0.3693	0.3751
Trinidad &Tobago	0.2592	0.2972	0.2484	0.2511
Tunisia	0.2961	0.3463	0.2821	0.2855
Turkey	0.2833	0.3291	0.2705	0.2736
Turkmenistan	0.1918	0.2121	0.1858	0.1873
Uganda	0.3771	0.4615	0.3548	0.3601
Ukraine	0.2288	0.2581	0.2203	0.2224
United Kingdom	0.4293	0.541	0.4006	0.4075
United States	0.4196	0.5259	0.3921	0.3987
Uruguay	0.3147	0.372	0.299	0.3028
Uzbekistan	0.2448	0.2785	0.2351	0.2375
Venezuela	0.3745	0.4576	0.3525	0.3578
Vietnam	0.2497	0.2849	0.2397	0.2421
Yemen	0.2071	0.2309	0.2001	0.2018
Zambia	0.478	0.6196	0.4428	0.4511

Table A3.7. Inequality measures for each country in 1990 (Lamé II)

Country	Theil	GE(1.5)	GE(0.5)	GE(0.75)
Algeria	0.2733	0.3217	0.2601	0.2633
Argentina	0.372	0.4686	0.3476	0.3534
Armenia	0.1883	0.2099	0.182	0.1835
Australia	0.3627	0.4539	0.3395	0.3451
Austria	0.5763	0.8519	0.519	0.5323
Azerbaijan	0.247	0.2858	0.2362	0.2389
Bahamas	0.4855	0.6661	0.4445	0.4541
Bangladesh	0.1732	0.1912	0.1678	0.1691
Barbados	0.3468	0.4292	0.3256	0.3307
Belarus	0.1279	0.1375	0.125	0.1258
Belgium	0.1869	0.2081	0.1807	0.1822
Bolivia	0.496	0.6861	0.4532	0.4632
Bosnia and Herzegovina	0.3013	0.3614	0.2853	0.2891
Botswana	0.6872	1.1225	0.6065	0.625
Brazil	0.7329	1.252	0.6416	0.6624
Bulgaria	0.1216	0.1302	0.119	0.1196
Burundi	0.2042	0.2298	0.1968	0.1986
Cameroon	0.435	0.5739	0.4019	0.4097
Canada	0.2809	0.3323	0.2669	0.2703
Cape Verde	0.3642	0.4563	0.3409	0.3464
Central African Republic	0.7755	1.3838	0.6737	0.6967
Chile	0.55	0.795	0.4976	0.5098
China Version 1	0.2199	0.25	0.2113	0.2134
Colombia	0.4422	0.5867	0.408	0.4161
Costa Rica	0.3511	0.4358	0.3294	0.3346
Cote d`Ivoire	0.3012	0.3612	0.2852	0.289
Croatia	0.1443	0.1566	0.1406	0.1415
Cyprus	0.2458	0.2841	0.2351	0.2377
Czech Republic	0.1545	0.1687	0.1503	0.1513
Denmark	0.4655	0.6286	0.4277	0.4366
Dominican Republic	0.4406	0.5837	0.4066	0.4146
Ecuador	0.4041	0.5211	0.3755	0.3823
Egypt	0.1974	0.2213	0.1905	0.1922
El Salvador	0.4503	0.6011	0.4149	0.4232
Estonia	0.1874	0.2088	0.1812	0.1827
Ethiopia	0.2673	0.3133	0.2546	0.2577
Finland	0.2434	0.281	0.2329	0.2355
France	0.3137	0.3794	0.2963	0.3005
Gambia, The	0.5905	0.8836	0.5304	0.5443
Algeria	0.2733	0.3217	0.2601	0.2633

Table A3.7. Continued

Georgia	0.2046	0.2303	0.1971	0.1989
Germany	0.3883	0.495	0.3619	0.3682
Ghana	0.2662	0.3119	0.2537	0.2567
Greece	0.4136	0.537	0.3836	0.3907
Guatemala	0.6969	1.149	0.614	0.6329
Guinea	0.4771	0.6503	0.4375	0.4468
Guinea-Bissau	0.5836	0.868	0.5249	0.5385
Guyana	0.3485	0.4318	0.3271	0.3323
Haiti	0.6495	1.0242	0.5772	0.5938
Honduras	0.5734	0.8454	0.5166	0.5298
Hong Kong	0.4523	0.6047	0.4166	0.425
Hungary	0.2964	0.3543	0.2809	0.2846
Iceland	0.2282	0.2608	0.2189	0.2212
India	0.1943	0.2174	0.1876	0.1892
Indonesia	0.2596	0.3029	0.2477	0.2506
Iran	0.3973	0.5098	0.3696	0.3762
Ireland	0.3826	0.4856	0.3569	0.363
Israel	0.3131	0.3785	0.2958	0.2999
Italy	0.3621	0.453	0.3391	0.3446
Jamaica	0.4845	0.6642	0.4436	0.4532
Japan	0.234	0.2685	0.2243	0.2267
Jordan	0.3599	0.4496	0.3371	0.3426
Kazakhstan	0.1508	0.1643	0.1468	0.1478
Kenya	0.7414	1.2773	0.6481	0.6692
Korea, Republic of	0.2253	0.257	0.2163	0.2185
Kyrgyzstan	0.1341	0.1446	0.1309	0.1317
Laos	0.1694	0.1866	0.1643	0.1655
Latvia	0.1965	0.2201	0.1896	0.1913
Lesotho	0.8308	1.5737	0.7147	0.7407
Lithuania	0.2216	0.2523	0.2129	0.215
Luxembourg	0.2117	0.2395	0.2037	0.2057
Macedonia	0.1508	0.1643	0.1468	0.1478
Madagascar	0.3988	0.5123	0.3709	0.3776
Malawi	1.0434	2.5973	0.8645	0.9035
Malaysia	0.3725	0.4695	0.3481	0.354
Mali	0.2879	0.3422	0.2732	0.2768
Mauritania	0.4232	0.5535	0.3919	0.3993
Mauritius	0.3775	0.4775	0.3525	0.3585
Mexico	0.4797	0.6551	0.4396	0.449
Moldova	0.1656	0.182	0.1607	0.1619
Morocco	0.24	0.2764	0.2298	0.2323
Nepal	0.2392	0.2753	0.229	0.2315
Netherlands	0.3037	0.3648	0.2874	0.2913

Table A3.7. Continued

Nicaragua	0.6489	1.0229	0.5767	0.5933
Niger	0.2995	0.3587	0.2836	0.2874
Nigeria	0.4131	0.5362	0.3832	0.3903
Norway	0.3238	0.3943	0.3053	0.3097
Pakistan	0.331	0.4051	0.3117	0.3163
Panama	0.5711	0.8402	0.5147	0.5278
Paraguay	0.2491	0.2886	0.2381	0.2408
Peru	0.3855	0.4904	0.3594	0.3656
Philippines	0.7175	1.207	0.6298	0.6498
Poland	0.207	0.2334	0.1993	0.2012
Portugal	0.4571	0.6134	0.4207	0.4292
Puerto Rico	0.625	0.964	0.5579	0.5734
Romania	0.1928	0.2155	0.1862	0.1878
Russia	0.1799	0.1995	0.1742	0.1756
Rwanda	0.181	0.2008	0.1751	0.1766
Senegal	0.6942	1.1417	0.612	0.6307
Serbia	0.2933	0.3499	0.2781	0.2818
Sierra Leone	0.8756	1.746	0.7472	0.7758
Singapore	0.4002	0.5146	0.3721	0.3788
Slovak Republic	0.1238	0.1327	0.1211	0.1217
Slovenia	0.1761	0.1948	0.1706	0.1719
South Africa	1.0012	2.3455	0.8357	0.872
Spain	0.2523	0.2929	0.241	0.2437
Sri Lanka	0.2813	0.3329	0.2673	0.2707
Sweden	0.3995	0.5134	0.3715	0.3781
Switzerland	0.2866	0.3403	0.2721	0.2756
Taiwan	0.1735	0.1916	0.1681	0.1694
Tajikistan	0.2029	0.2282	0.1955	0.1973
Tanzania	0.3606	0.4507	0.3378	0.3432
Thailand	0.521	0.7353	0.4739	0.4849
Trinidad &Tobago	0.2825	0.3345	0.2684	0.2718
Tunisia	0.2532	0.294	0.2418	0.2445
Turkey	0.3791	0.48	0.3538	0.3598
Turkmenistan	0.1655	0.1819	0.1606	0.1618
Uganda	0.3247	0.3957	0.3061	0.3106
Ukraine	0.1053	0.1117	0.1034	0.1039
United Kingdom	0.4213	0.5502	0.3902	0.3976
United States	0.3526	0.4382	0.3308	0.336
Uruguay	0.2942	0.3512	0.2789	0.2826
Uzbekistan	0.1803	0.1999	0.1745	0.1759
Venezuela	0.3265	0.3984	0.3077	0.3122
Vietnam	0.2316	0.2654	0.2221	0.2244
Yemen	0.2778	0.328	0.2642	0.2675
Zambia	0.6607	1.0527	0.5859	0.6031

Table A3.8. Inequality measures for each country in 1995 (Lamé II)

Country	Theil	GE(1.5)	GE(0.5)	GE(0.75)
Algeria	0.2267	0.2589	0.2176	0.2198
Argentina	0.3995	0.5133	0.3715	0.3781
Armenia	0.3965	0.5084	0.3689	0.3754
Australia	0.355	0.4419	0.3328	0.3381
Austria	0.3588	0.4478	0.3361	0.3416
Azerbaijan	0.4241	0.5551	0.3927	0.4001
Bahamas	0.5226	0.7385	0.4752	0.4863
Bangladesh	0.2136	0.2419	0.2055	0.2075
Barbados	0.3049	0.3665	0.2884	0.2924
Belarus	0.1318	0.142	0.1287	0.1295
Belgium	0.3514	0.4362	0.3296	0.3348
Bolivia	0.609	0.9261	0.5452	0.5599
Bosnia and Herzegovina	0.2376	0.2732	0.2275	0.23
Botswana	0.652	1.0306	0.5791	0.5959
Brazil	0.6978	1.1517	0.6148	0.6337
Bulgaria	0.2514	0.2916	0.2402	0.2429
Burundi	0.2501	0.29	0.239	0.2417
Cameroon	0.4079	0.5275	0.3788	0.3857
Canada	0.3064	0.3687	0.2898	0.2938
Cape Verde	0.5115	0.7164	0.4661	0.4767
Central African Republic	0.7002	1.1582	0.6166	0.6357
Chile	0.5542	0.804	0.5011	0.5134
China Version 1	0.2936	0.3502	0.2783	0.282
Colombia	0.57	0.838	0.5139	0.5269
Costa Rica	0.3557	0.4429	0.3334	0.3388
Cote d`Ivoire	0.2628	0.3072	0.2506	0.2535
Croatia	0.2235	0.2546	0.2146	0.2167
Cyprus	0.2825	0.3345	0.2684	0.2718
Czech Republic	0.2132	0.2413	0.2051	0.207
Denmark	0.4424	0.5871	0.4082	0.4163
Dominican Republic	0.4495	0.5996	0.4142	0.4225
Ecuador	0.372	0.4686	0.3476	0.3534
Egypt	0.2583	0.301	0.2465	0.2493
El Salvador	0.465	0.6277	0.4273	0.4362
Estonia	0.341	0.4203	0.3205	0.3254
Ethiopia	0.3186	0.3866	0.3006	0.3049
Finland	0.2912	0.3469	0.2762	0.2798
France	0.4554	0.6103	0.4192	0.4277
Gambia, The	0.621	0.9544	0.5547	0.57
Algeria	0.2267	0.2589	0.2176	0.2198

Table A3.8. Continued

Georgia	0.4174	0.5435	0.3869	0.3941
Germany	0.4264	0.5589	0.3945	0.4021
Ghana	0.2354	0.2704	0.2256	0.228
Greece	0.3194	0.3878	0.3014	0.3057
Guatemala	0.6625	1.0574	0.5874	0.6046
Guinea	0.3545	0.4411	0.3324	0.3377
Guinea-Bissau	0.4078	0.5272	0.3786	0.3855
Guyana	0.3854	0.4901	0.3593	0.3655
Haiti	0.6488	1.0224	0.5766	0.5932
Honduras	0.5564	0.8086	0.5028	0.5153
Hong Kong	0.5236	0.7405	0.476	0.4871
Hungary	0.3622	0.4531	0.3391	0.3446
Iceland	0.2287	0.2614	0.2194	0.2216
India	0.2232	0.2543	0.2143	0.2165
Indonesia	0.2665	0.3122	0.2539	0.2569
Iran	0.3885	0.4953	0.362	0.3683
Ireland	0.3435	0.424	0.3227	0.3276
Israel	0.3481	0.4312	0.3268	0.3319
Italy	0.4335	0.5714	0.4007	0.4084
Jamaica	0.7297	1.2424	0.6392	0.6597
Japan	0.2748	0.3238	0.2615	0.2647
Jordan	0.333	0.4081	0.3134	0.3181
Kazakhstan	0.2345	0.2691	0.2248	0.2271
Kenya	0.5054	0.7044	0.461	0.4714
Korea, Republic of	0.2085	0.2354	0.2008	0.2027
Kyrgyzstan	0.5133	0.7199	0.4675	0.4782
Laos	0.2192	0.2491	0.2107	0.2127
Latvia	0.3061	0.3682	0.2895	0.2935
Lesotho	0.9449	2.0525	0.7966	0.8293
Lithuania	0.4797	0.6551	0.4396	0.449
Luxembourg	0.2308	0.2642	0.2214	0.2236
Macedonia	0.1668	0.1835	0.1619	0.1631
Madagascar	0.3494	0.4331	0.3279	0.333
Malawi	0.6724	1.0833	0.5951	0.6128
Malaysia	0.4131	0.5363	0.3832	0.3903
Mali	0.5034	0.7005	0.4594	0.4697
Mauritania	0.3153	0.3818	0.2978	0.302
Mauritius	0.425	0.5566	0.3934	0.4009
Mexico	0.5201	0.7335	0.4732	0.4841
Moldova	0.3309	0.405	0.3116	0.3162
Morocco	0.2976	0.356	0.282	0.2857
Nepal	0.3319	0.4065	0.3125	0.3171
Netherlands	0.3241	0.3948	0.3056	0.31

Table A3.8. Continued

Nicaragua	0.6362	0.9912	0.5667	0.5827
Niger	0.4946	0.6834	0.452	0.462
Nigeria	0.5102	0.7138	0.465	0.4756
Norway	0.4106	0.532	0.3811	0.3881
Pakistan	0.2232	0.2542	0.2143	0.2165
Panama	0.5801	0.8602	0.522	0.5355
Paraguay	0.6234	0.9601	0.5566	0.572
Peru	0.4593	0.6174	0.4225	0.4312
Philippines	0.7292	1.2411	0.6388	0.6594
Poland	0.4199	0.5478	0.389	0.3963
Portugal	0.7074	1.1784	0.6221	0.6416
Puerto Rico	0.6988	1.1543	0.6155	0.6345
Romania	0.3316	0.4061	0.3123	0.3169
Russia	0.5299	0.7534	0.4812	0.4926
Rwanda	0.2762	0.3258	0.2627	0.266
Senegal	0.3375	0.4149	0.3174	0.3222
Serbia	0.2341	0.2686	0.2244	0.2268
Sierra Leone	0.636	0.9908	0.5666	0.5826
Singapore	0.3987	0.5121	0.3708	0.3775
Slovak Republic	0.1672	0.184	0.1622	0.1634
Slovenia	0.3088	0.3722	0.292	0.296
South Africa	0.9306	1.9849	0.7865	0.8184
Spain	0.4209	0.5494	0.3898	0.3972
Sri Lanka	0.3682	0.4626	0.3444	0.35
Sweden	0.4648	0.6274	0.4271	0.436
Switzerland	0.3031	0.3639	0.2869	0.2908
Taiwan	0.2059	0.232	0.1983	0.2002
Tajikistan	0.2039	0.2295	0.1965	0.1983
Tanzania	0.2811	0.3325	0.2671	0.2705
Thailand	0.575	0.8488	0.5179	0.5311
Trinidad &Tobago	0.2729	0.3211	0.2597	0.2629
Tunisia	0.3139	0.3797	0.2965	0.3007
Turkey	0.3545	0.4411	0.3324	0.3377
Turkmenistan	0.1643	0.1805	0.1595	0.1607
Uganda	0.2444	0.2823	0.2338	0.2364
Ukraine	0.3548	0.4416	0.3327	0.338
United Kingdom	0.4423	0.5869	0.4081	0.4162
United States	0.4112	0.533	0.3816	0.3886
Uruguay	0.307	0.3695	0.2903	0.2943
Uzbekistan	0.2579	0.3005	0.2461	0.2489
Venezuela	0.3825	0.4855	0.3568	0.3629
Vietnam	0.232	0.2658	0.2224	0.2248
Yemen	0.2288	0.2616	0.2195	0.2218
Zambia	0.559	0.8141	0.505	0.5175

Table A3.9. Inequality measures for each country in 2000 (Lamé II)

Country	Theil	GE(1.5)	GE(0.5)	GE(0.75)
Algeria	0.2363	0.2716	0.2264	0.2288
Argentina	0.4627	0.6235	0.4253	0.4341
Armenia	0.4306	0.5662	0.3981	0.4058
Australia	0.364	0.4559	0.3407	0.3463
Austria	0.3618	0.4525	0.3388	0.3443
Azerbaijan	0.2418	0.2788	0.2315	0.234
Bahamas	0.6449	1.0127	0.5736	0.5899
Bangladesh	0.1925	0.2152	0.1859	0.1876
Barbados	0.0042	0.0042	0.0042	0.0042
Belarus	0.1297	0.1395	0.1267	0.1275
Belgium	0.3515	0.4364	0.3297	0.3349
Bolivia	0.7093	1.1835	0.6235	0.6431
Bosnia and Herzegovina	0.1708	0.1884	0.1656	0.1669
Botswana	0.6401	1.0007	0.5698	0.5859
Brazil	0.6987	1.1541	0.6154	0.6344
Bulgaria	0.1496	0.1628	0.1456	0.1466
Burundi	0.2744	0.3233	0.2611	0.2643
Cameroon	0.3746	0.4728	0.3499	0.3558
Canada	0.3491	0.4327	0.3276	0.3328
Cape Verde	0.7021	1.1635	0.618	0.6372
Central African Republic	0.4658	0.6292	0.428	0.4369
Chile	0.5521	0.7995	0.4994	0.5116
China Version 1	0.3169	0.3841	0.2992	0.3034
Colombia	0.5794	0.8585	0.5214	0.5349
Costa Rica	0.3935	0.5036	0.3664	0.3728
Cote d`Ivoire	0.4037	0.5204	0.3751	0.3819
Croatia	0.1882	0.2097	0.1819	0.1834
Cyprus	0.3726	0.4696	0.3482	0.354
Czech Republic	0.3134	0.379	0.2961	0.3002
Denmark	0.4123	0.5348	0.3825	0.3896
Dominican Republic	0.4639	0.6257	0.4263	0.4352
Ecuador	0.6064	0.92	0.5431	0.5577
Egypt	0.2509	0.2911	0.2398	0.2425
El Salvador	0.5046	0.7029	0.4604	0.4707
Estonia	0.3142	0.3801	0.2968	0.301
Ethiopia	0.2183	0.248	0.2099	0.2119
Finland	0.4065	0.525	0.3775	0.3844
France	0.4269	0.5598	0.395	0.4025
Gambia, The	0.4747	0.6458	0.4354	0.4447
Algeria	0.2363	0.2716	0.2264	0.2288

Table A3.9. Continued

Georgia	0.4392	0.5813	0.4055	0.4134
Germany	0.522	0.7373	0.4747	0.4858
Ghana	0.2921	0.3481	0.277	0.2806
Greece	0.5022	0.6982	0.4584	0.4686
Guatemala	0.6447	1.0123	0.5734	0.5898
Guinea	0.347	0.4295	0.3258	0.3309
Guinea-Bissau	0.2725	0.3205	0.2593	0.2625
Guyana	0.3367	0.4137	0.3167	0.3215
Haiti	0.648	1.0206	0.576	0.5926
Honduras	0.5418	0.7778	0.4909	0.5028
Hong Kong	0.6861	1.1196	0.6057	0.6241
Hungary	0.4074	0.5266	0.3783	0.3852
Iceland	0.3115	0.3762	0.2944	0.2985
India	0.2113	0.239	0.2034	0.2053
Indonesia	0.2025	0.2277	0.1952	0.197
Iran	0.3786	0.4793	0.3535	0.3594
Ireland	0.335	0.4112	0.3152	0.32
Israel	0.3795	0.4807	0.3542	0.3603
Italy	0.3824	0.4853	0.3567	0.3628
Jamaica	0.4172	0.5432	0.3867	0.394
Japan	0.3003	0.36	0.2844	0.2882
Jordan	0.2908	0.3463	0.2759	0.2795
Kazakhstan	0.2101	0.2374	0.2023	0.2042
Kenya	0.4919	0.6782	0.4498	0.4596
Korea, Republic of	0.2054	0.2315	0.1979	0.1998
Kyrgyzstan	0.2891	0.3438	0.2743	0.2779
Laos	0.2366	0.2719	0.2267	0.2291
Latvia	0.4263	0.5589	0.3945	0.4021
Lesotho	0.736	1.261	0.6439	0.6648
Lithuania	0.4109	0.5324	0.3813	0.3883
Luxembourg	0.3268	0.3988	0.3079	0.3125
Macedonia	0.1962	0.2198	0.1894	0.191
Madagascar	0.3565	0.4442	0.3341	0.3395
Malawi	0.3996	0.5136	0.3716	0.3783
Malaysia	0.4355	0.5748	0.4023	0.4102
Mali	0.3201	0.3888	0.302	0.3063
Mauritania	0.2764	0.326	0.2629	0.2662
Mauritius	0.4501	0.6007	0.4147	0.423
Mexico	0.4844	0.6639	0.4435	0.4531
Moldova	0.3349	0.411	0.3151	0.3198
Morocco	0.3028	0.3635	0.2866	0.2905
Nepal	0.4164	0.5419	0.3861	0.3933
Netherlands	0.3114	0.376	0.2943	0.2984

Table A3.9. Continued

Nicaragua	0.5796	0.859	0.5216	0.535
Niger	0.4028	0.5189	0.3744	0.3811
Nigeria	0.4461	0.5935	0.4113	0.4195
Norway	0.4099	0.5308	0.3804	0.3874
Pakistan	0.1738	0.192	0.1685	0.1698
Panama	0.5706	0.8393	0.5144	0.5274
Paraguay	0.585	0.8711	0.526	0.5396
Peru	0.5077	0.709	0.4629	0.4734
Philippines	0.4211	0.5499	0.3901	0.3974
Poland	0.263	0.3075	0.2508	0.2537
Portugal	0.628	0.9711	0.5602	0.5758
Puerto Rico	0.7494	1.3016	0.6541	0.6757
Romania	0.3052	0.367	0.2888	0.2927
Russia	0.4473	0.5957	0.4123	0.4206
Rwanda	0.3997	0.5137	0.3717	0.3783
Senegal	0.3266	0.3985	0.3078	0.3123
Serbia	0.1934	0.2162	0.1867	0.1883
Sierra Leone	0.4548	0.6092	0.4187	0.4272
Singapore	0.4492	0.5992	0.414	0.4223
Slovak Republic	0.2189	0.2488	0.2104	0.2125
Slovenia	0.298	0.3566	0.2823	0.2861
South Africa	1.1885	3.7779	0.9602	1.009
Spain	0.281	0.3325	0.267	0.2704
Sri Lanka	0.5674	0.8324	0.5118	0.5247
Sweden	0.4453	0.5921	0.4106	0.4188
Switzerland	0.3364	0.4132	0.3164	0.3212
Taiwan	0.2361	0.2712	0.2262	0.2286
Tajikistan	0.2099	0.2372	0.2021	0.204
Tanzania	0.2222	0.253	0.2134	0.2155
Thailand	0.405	0.5226	0.3763	0.3831
Trinidad &Tobago	0.2648	0.3099	0.2524	0.2554
Tunisia	0.303	0.3637	0.2867	0.2906
Turkey	0.2897	0.3448	0.2749	0.2785
Turkmenistan	0.1952	0.2185	0.1884	0.1901
Uganda	0.3878	0.4941	0.3614	0.3677
Ukraine	0.2333	0.2675	0.2236	0.226
United Kingdom	0.4427	0.5876	0.4085	0.4166
United States	0.4325	0.5696	0.3998	0.4075
Uruguay	0.3225	0.3924	0.3042	0.3086
Uzbekistan	0.2498	0.2895	0.2388	0.2414
Venezuela	0.3851	0.4897	0.359	0.3652
Vietnam	0.255	0.2965	0.2435	0.2462
Yemen	0.2109	0.2384	0.203	0.2049
Zambia	0.4944	0.6831	0.4519	0.4619

Table A3.10. Poverty rates for each country (Lamé I)

Country	$ 1.25 a day			$ 1.45 a day			$ 2 a day		
	1990	1995	2000	1990	1995	2000	1990	1995	2000
Algeria	0.0081	0.0056	0.0052	0.0110	0.0079	0.0073	0.0222	0.0172	0.0156
Argentina	0.0109	0.0097	0.0134	0.0140	0.0124	0.0168	0.0253	0.0219	0.0282
Armenia	0.0097	0.1144	0.0848	0.0140	0.1426	0.1054	0.0331	0.2326	0.1721
Australia	0.0009	0.0006	0.0005	0.0011	0.0008	0.0007	0.0021	0.0015	0.0013
Austria	0.0050	0.0007	0.0005	0.0061	0.0009	0.0007	0.0097	0.0016	0.0013
Azerbaijan	0.0177	0.1382	0.0268	0.0243	0.1700	0.0368	0.0504	0.2683	0.0761
Bahamas	0.0026	0.0039	0.0062	0.0032	0.0048	0.0074	0.0054	0.0079	0.0115
Bangladesh	0.2559	0.2574	0.1898	0.3408	0.3329	0.2553	0.5812	0.5465	0.4612
Belarus	0.0002	0.0010	0.0003	0.0004	0.0016	0.0004	0.0010	0.0045	0.0013
Belgium	0.0000	0.0007	0.0005	0.0000	0.0009	0.0007	0.0001	0.0016	0.0013
Bolivia	0.0961	0.1195	0.1405	0.1172	0.1421	0.1642	0.1835	0.2099	0.2331
Bosnia and Herzegovina	0.1902	0.0719	0.0024	0.2404	0.0975	0.0035	0.3930	0.1918	0.0088
Botswana	0.0540	0.0483	0.0328	0.0641	0.0578	0.0394	0.0954	0.0872	0.0602
Brazil	0.0681	0.0560	0.0543	0.0802	0.0664	0.0644	0.1169	0.0985	0.0954
Bulgaria	0.0001	0.0033	0.0004	0.0002	0.0045	0.0007	0.0007	0.0095	0.0018
Burundi	0.4404	0.6254	0.7235	0.5372	0.7069	0.7899	0.7501	0.8561	0.9016
Cameroon	0.1449	0.1828	0.1359	0.1775	0.2235	0.1697	0.2773	0.3449	0.2761
Canada	0.0002	0.0003	0.0005	0.0003	0.0005	0.0006	0.0006	0.0009	0.0011
Cape Verde	0.1375	0.1749	0.1891	0.1722	0.2094	0.2194	0.2817	0.3105	0.3053
Central African Republic	0.5865	0.6134	0.5496	0.6348	0.6634	0.6130	0.7407	0.7701	0.7499
Chile	0.0428	0.0259	0.0204	0.0521	0.0316	0.0250	0.0821	0.0502	0.0399
China Version 1	0.1303	0.0696	0.0386	0.1754	0.0914	0.0506	0.3292	0.1684	0.0938
Colombia	0.0265	0.0397	0.0448	0.0332	0.0482	0.0543	0.0561	0.0754	0.0844
Costa Rica	0.0082	0.0072	0.0083	0.0107	0.0093	0.0106	0.0197	0.0171	0.0189
Cote d`Ivoire	0.1104	0.1142	0.1750	0.1427	0.1504	0.2147	0.2508	0.2734	0.3338
Croatia	0.0000	0.0010	0.0003	0.0001	0.0014	0.0004	0.0002	0.0031	0.0009
Cyprus	0.0006	0.0008	0.0021	0.0008	0.0011	0.0027	0.0016	0.0022	0.0050
Czech Republic	0.0000	0.0002	0.0011	0.0001	0.0003	0.0015	0.0002	0.0008	0.0029
Denmark	0.0024	0.0017	0.0010	0.0031	0.0021	0.0013	0.0052	0.0037	0.0023
Dominican Republic	0.0328	0.0260	0.0186	0.0411	0.0326	0.0232	0.0692	0.0548	0.0389
Ecuador	0.0299	0.0210	0.0666	0.0379	0.0270	0.0799	0.0656	0.0483	0.1215
Egypt	0.0161	0.0196	0.0127	0.0230	0.0266	0.0174	0.0527	0.0542	0.0362
El Salvador	0.0465	0.0339	0.0371	0.0579	0.0422	0.0457	0.0959	0.0699	0.0739
Estonia	0.0004	0.0070	0.0026	0.0007	0.0091	0.0034	0.0016	0.0169	0.0066
Ethiopia	0.6905	0.7287	0.6831	0.7623	0.7902	0.7629	0.8867	0.8962	0.8959
Finland	0.0001	0.0004	0.0012	0.0002	0.0006	0.0015	0.0004	0.0011	0.0027
France	0.0005	0.0022	0.0014	0.0006	0.0028	0.0018	0.0012	0.0047	0.0031
Gambia, The	0.3053	0.3700	0.2658	0.3523	0.4204	0.3146	0.4759	0.5472	0.4481

Table 3.10. Continued

Georgia	0.0019	0.1221	0.0703	0.0027	0.1509	0.0874	0.0063	0.2418	0.1434
Germany	0.0011	0.0015	0.0029	0.0015	0.0019	0.0036	0.0026	0.0033	0.0059
Ghana	0.1356	0.1136	0.1155	0.1771	0.1521	0.1497	0.3138	0.2855	0.2640
Greece	0.0031	0.0010	0.0046	0.0039	0.0014	0.0057	0.0069	0.0026	0.0094
Guatemala	0.0857	0.0721	0.0620	0.1012	0.0857	0.0740	0.1479	0.1275	0.1112
Guinea	0.4718	0.4211	0.3649	0.5345	0.4928	0.4340	0.6791	0.6627	0.6068
Guinea-Bissau	0.3491	0.2707	0.3012	0.3999	0.3245	0.3732	0.5295	0.4721	0.5655
Guyana	0.0876	0.0588	0.0355	0.1117	0.0745	0.0461	0.1925	0.1278	0.0841
Haiti	0.3165	0.3226	0.3019	0.3619	0.3686	0.3461	0.4803	0.4878	0.4626
Honduras	0.1028	0.1031	0.0969	0.1233	0.1241	0.1171	0.1861	0.1884	0.1795
Hong Kong	0.0029	0.0037	0.0088	0.0036	0.0046	0.0105	0.0061	0.0075	0.0160
Hungary	0.0016	0.0040	0.0046	0.0021	0.0052	0.0059	0.0042	0.0096	0.0104
Iceland	0.0000	0.0000	0.0002	0.0001	0.0001	0.0003	0.0001	0.0001	0.0005
India	0.0661	0.0632	0.0377	0.0929	0.0869	0.0527	0.1978	0.1766	0.1133
Indonesia	0.0439	0.0249	0.0141	0.0591	0.0336	0.0200	0.1167	0.0672	0.0454
Iran	0.0178	0.0144	0.0103	0.0227	0.0184	0.0133	0.0400	0.0326	0.0238
Ireland	0.0020	0.0009	0.0004	0.0026	0.0012	0.0005	0.0047	0.0023	0.0009
Israel	0.0010	0.0011	0.0013	0.0013	0.0015	0.0017	0.0025	0.0027	0.0031
Italy	0.0010	0.0018	0.0009	0.0012	0.0023	0.0012	0.0023	0.0040	0.0022
Jamaica	0.0166	0.0414	0.0102	0.0206	0.0490	0.0129	0.0342	0.0724	0.0225
Japan	0.0001	0.0002	0.0003	0.0001	0.0003	0.0004	0.0003	0.0005	0.0007
Jordan	0.0447	0.0297	0.0222	0.0574	0.0387	0.0295	0.1016	0.0711	0.0573
Kazakhstan	0.0004	0.0063	0.0037	0.0005	0.0087	0.0052	0.0015	0.0187	0.0118
Kenya	0.3867	0.3110	0.3070	0.4333	0.3627	0.3592	0.5494	0.4990	0.4970
Korea, Republic of	0.0006	0.0002	0.0001	0.0008	0.0003	0.0002	0.0018	0.0006	0.0004
Kyrgyzstan	0.0121	0.2592	0.0938	0.0188	0.3051	0.1225	0.0524	0.4309	0.2213
Laos	0.1370	0.1264	0.0997	0.1921	0.1705	0.1340	0.3847	0.3218	0.2554
Latvia	0.0004	0.0079	0.0126	0.0006	0.0105	0.0160	0.0015	0.0204	0.0276
Lesotho	0.5371	0.5345	0.4144	0.5844	0.5788	0.4625	0.6919	0.6801	0.5802
Lithuania	0.0005	0.0202	0.0094	0.0006	0.0251	0.0119	0.0014	0.0417	0.0209
Luxembourg	0.0000	0.0000	0.0001	0.0000	0.0000	0.0001	0.0000	0.0001	0.0002
Macedonia	0.0005	0.0012	0.0016	0.0007	0.0017	0.0024	0.0020	0.0044	0.0056
Madagascar	0.3477	0.3802	0.3778	0.4103	0.4501	0.4467	0.5701	0.6225	0.6174
Malawi	0.7174	0.6765	0.6545	0.7538	0.7243	0.7175	0.8289	0.8212	0.8391
Malaysia	0.0151	0.0104	0.0101	0.0194	0.0132	0.0128	0.0349	0.0231	0.0220
Mali	0.4353	0.4948	0.3948	0.5160	0.5561	0.4690	0.7016	0.6952	0.6488
Mauritania	0.1652	0.1296	0.0921	0.2021	0.1656	0.1212	0.3132	0.2825	0.2223
Mauritius	0.0191	0.0198	0.0160	0.0245	0.0250	0.0201	0.0438	0.0429	0.0341
Mexico	0.0137	0.0180	0.0104	0.0170	0.0221	0.0130	0.0284	0.0359	0.0216
Moldova	0.0142	0.1069	0.1267	0.0211	0.1366	0.1608	0.0525	0.2347	0.2708
Morocco	0.0313	0.0575	0.0464	0.0429	0.0755	0.0611	0.0886	0.1403	0.1140
Nepal	0.3474	0.3569	0.3841	0.4303	0.4270	0.4479	0.6388	0.6036	0.6056
Netherlands	0.0004	0.0004	0.0002	0.0005	0.0005	0.0003	0.0009	0.0010	0.0006
New Zealand	0.0010	0.0013	0.0018	0.0013	0.0017	0.0023	0.0025	0.0031	0.0040

Table 3.10. Continued

Nicaragua	0.1956	0.2227	0.1726	0.2283	0.2592	0.2043	0.3208	0.3606	0.2962
Niger	0.5365	0.6396	0.6294	0.6160	0.6975	0.6941	0.7811	0.8141	0.8218
Nigeria	0.2500	0.3372	0.2859	0.3007	0.3910	0.3387	0.4425	0.5296	0.4813
Norway	0.0003	0.0008	0.0006	0.0004	0.0010	0.0007	0.0009	0.0017	0.0013
Pakistan	0.1256	0.0555	0.0304	0.1597	0.0765	0.0442	0.2700	0.1571	0.1044
Panama	0.0466	0.0398	0.0336	0.0564	0.0483	0.0409	0.0880	0.0752	0.0642
Paraguay	0.0125	0.1004	0.0975	0.0171	0.1195	0.1168	0.0356	0.1774	0.1761
Peru	0.0342	0.0380	0.0441	0.0436	0.0474	0.0542	0.0763	0.0786	0.0872
Philippines	0.1955	0.1971	0.0846	0.2263	0.2278	0.1054	0.3128	0.3139	0.1731
Poland	0.0011	0.0103	0.0012	0.0016	0.0131	0.0017	0.0037	0.0227	0.0034
Portugal	0.0055	0.0191	0.0104	0.0069	0.0227	0.0126	0.0117	0.0342	0.0196
Puerto Rico	0.0119	0.0149	0.0132	0.0143	0.0177	0.0156	0.0223	0.0268	0.0232
Romania	0.0013	0.0115	0.0083	0.0019	0.0150	0.0111	0.0046	0.0281	0.0215
Russia	0.0002	0.0226	0.0131	0.0002	0.0277	0.0164	0.0006	0.0447	0.0279
Rwanda	0.2987	0.4613	0.5052	0.3890	0.5445	0.5743	0.6280	0.7297	0.7270
Senegal	0.3650	0.2145	0.1711	0.4122	0.2658	0.2153	0.5311	0.4161	0.3515
Serbia	0.0021	0.0047	0.0020	0.0028	0.0065	0.0029	0.0056	0.0141	0.0069
Sierra Leone	0.4955	0.4893	0.6546	0.5414	0.5435	0.7139	0.6485	0.6687	0.8304
Singapore	0.0016	0.0010	0.0012	0.0020	0.0012	0.0015	0.0036	0.0022	0.0026
Slovak Republic	0.0000	0.0002	0.0005	0.0000	0.0003	0.0006	0.0001	0.0008	0.0014
Slovenia	0.0001	0.0012	0.0007	0.0001	0.0016	0.0009	0.0003	0.0031	0.0018
South Africa	0.1354	0.1207	0.1631	0.1541	0.1384	0.1830	0.2069	0.1889	0.2373
Spain	0.0003	0.0023	0.0003	0.0004	0.0029	0.0004	0.0008	0.0051	0.0007
Sri Lanka	0.0705	0.0758	0.1178	0.0931	0.0962	0.1411	0.1738	0.1648	0.2116
Sweden	0.0013	0.0025	0.0016	0.0017	0.0031	0.0021	0.0030	0.0053	0.0036
Switzerland	0.0002	0.0002	0.0003	0.0002	0.0003	0.0004	0.0004	0.0006	0.0008
Taiwan	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002	0.0004	0.0004	0.0004
Tajikistan	0.0117	0.1402	0.1586	0.0167	0.1903	0.2129	0.0380	0.3603	0.3912
Tanzania	0.4065	0.4149	0.3285	0.4767	0.4957	0.4127	0.6460	0.6854	0.6290
Thailand	0.0525	0.0410	0.0197	0.0642	0.0497	0.0250	0.1021	0.0777	0.0436
Trinidad &Tobago	0.0019	0.0012	0.0006	0.0025	0.0017	0.0008	0.0051	0.0034	0.0017
Tunisia	0.0103	0.0154	0.0096	0.0140	0.0203	0.0128	0.0291	0.0387	0.0247
Turkey	0.0123	0.0090	0.0035	0.0158	0.0117	0.0047	0.0283	0.0214	0.0094
Turkmenistan	0.0033	0.0005	0.0006	0.0050	0.0007	0.0009	0.0126	0.0019	0.0021
Uganda	0.5297	0.3602	0.3635	0.6062	0.4433	0.4284	0.7679	0.6494	0.5914
Ukraine	0.0000	0.0288	0.0099	0.0001	0.0372	0.0138	0.0002	0.0670	0.0295
United Kingdom	0.0020	0.0022	0.0016	0.0026	0.0028	0.0020	0.0045	0.0048	0.0034
United States	0.0005	0.0010	0.0009	0.0007	0.0012	0.0012	0.0013	0.0022	0.0020
Uruguay	0.0068	0.0045	0.0046	0.0090	0.0060	0.0060	0.0178	0.0116	0.0115
Uzbekistan	0.0261	0.1077	0.0819	0.0378	0.1425	0.1098	0.0885	0.2623	0.2100
Venezuela	0.0054	0.0076	0.0081	0.0071	0.0097	0.0104	0.0135	0.0175	0.0186
Vietnam	0.2282	0.1310	0.0872	0.2950	0.1748	0.1163	0.4912	0.3231	0.2198
Yemen	0.1085	0.0638	0.0322	0.1419	0.0872	0.0452	0.2555	0.1756	0.0980
Zambia	0.4532	0.3903	0.3577	0.5056	0.4450	0.4140	0.6300	0.5802	0.5568

Table A3.11. Poverty rates for each country (Lamé II)

Country	$ 1.25 a day			$ 1.45 a day			$ 2 a day		
	1990	1995	2000	1990	1995	2000	1990	1995	2000
Algeria	0.0111	0.0077	0.0073	0.0145	0.0104	0.0098	0.0268	0.0208	0.0192
Argentina	0.0153	0.0142	0.0194	0.019	0.0175	0.0234	0.0315	0.0283	0.0361
Armenia	0.0121	0.1190	0.0918	0.0169	0.1453	0.1112	0.0368	0.2293	0.1732
Australia	0.0018	0.0014	0.0012	0.0022	0.0017	0.0015	0.0038	0.0029	0.0025
Austria	0.0093	0.0014	0.0012	0.0110	0.0018	0.0015	0.016	0.003	0.0025
Azerbaijan	0.0216	0.1417	0.0312	0.0286	0.1712	0.0413	0.0550	0.2632	0.0798
Bahamas	0.0051	0.0073	0.0116	0.0061	0.0087	0.0135	0.0093	0.013	0.0191
Bangladesh	0.2515	0.2527	0.1875	0.3352	0.3267	0.2508	0.5797	0.5432	0.4561
Belarus	0.0004	0.0014	0.0004	0.0006	0.0022	0.0007	0.0015	0.0058	0.0018
Belgium	0.0001	0.0014	0.0012	0.0001	0.0018	0.0014	0.0002	0.0030	0.0025
Bolivia	0.1038	0.128	0.1494	0.1235	0.1487	0.1709	0.1848	0.2107	0.2333
Bosnia and Herzegovina	0.1885	0.0756	0.0034	0.2363	0.1001	0.0048	0.3853	0.1896	0.0109
Botswana	0.0671	0.0606	0.0441	0.0772	0.0702	0.0511	0.1073	0.0987	0.0723
Brazil	0.0820	0.0694	0.0676	0.0938	0.0797	0.0777	0.1283	0.1103	0.1075
Bulgaria	0.0002	0.0049	0.0007	0.0004	0.0065	0.001	0.001	0.0125	0.0025
Burundi	0.4348	0.6244	0.7259	0.5338	0.7088	0.7941	0.7535	0.8605	0.9054
Cameroon	0.1481	0.1828	0.1387	0.1783	0.2209	0.1702	0.2718	0.3368	0.2705
Canada	0.0005	0.0007	0.0010	0.0007	0.0009	0.0013	0.0012	0.0017	0.0022
Cape Verde	0.1400	0.1771	0.1933	0.1724	0.2089	0.2208	0.2759	0.3035	0.2995
Central African Republic	0.5764	0.6055	0.5422	0.6272	0.6584	0.6084	0.7401	0.7720	0.7523
Chile	0.0529	0.0350	0.0288	0.0624	0.0412	0.0340	0.0917	0.0606	0.0500
China Version 1	0.1310	0.0745	0.0445	0.1739	0.0953	0.0564	0.3229	0.1679	0.0981
Colombia	0.0339	0.0501	0.0556	0.0410	0.0589	0.0652	0.0642	0.0857	0.0945
Costa Rica	0.0119	0.0106	0.0124	0.0149	0.0133	0.0153	0.0252	0.0223	0.0249
Cote d`Ivoire	0.1135	0.1164	0.1756	0.1439	0.1506	0.2126	0.2462	0.2680	0.3260
Croatia	0.0001	0.0017	0.0005	0.0001	0.0023	0.0007	0.0003	0.0045	0.0015
Cyprus	0.0010	0.0015	0.0039	0.0014	0.0020	0.0048	0.0027	0.0036	0.0079
Czech Republic	0.0001	0.0005	0.0021	0.0001	0.0006	0.0027	0.0003	0.0013	0.0047
Denmark	0.0048	0.0034	0.0021	0.0057	0.0042	0.0026	0.0088	0.0065	0.0042
Dominican Republic	0.0406	0.0335	0.0255	0.0492	0.0405	0.0307	0.077	0.0632	0.0474
Ecuador	0.0370	0.0269	0.0779	0.0453	0.0334	0.0908	0.0728	0.0552	0.1298
Egypt	0.0193	0.0238	0.0162	0.0266	0.0312	0.0214	0.0563	0.0589	0.0409
El Salvador	0.0548	0.0422	0.0462	0.0662	0.0507	0.0551	0.1027	0.0782	0.0829
Estonia	0.0008	0.0102	0.0043	0.0011	0.0129	0.0054	0.0024	0.0220	0.0096
Ethiopia	0.6918	0.7311	0.6845	0.7659	0.7944	0.7665	0.8908	0.9004	0.8994
Finland	0.0003	0.0009	0.0025	0.0004	0.0011	0.003	0.0008	0.0020	0.0049
France	0.001	0.0043	0.0029	0.0012	0.0052	0.0035	0.0022	0.0081	0.0056
Gambia, The	0.2989	0.3605	0.2611	0.3435	0.4096	0.3074	0.4650	0.5375	0.4384
Georgia	0.0029	0.1265	0.078	0.0040	0.1534	0.0944	0.0083	0.2382	0.1468
Germany	0.0023	0.0030	0.0058	0.0029	0.0037	0.0069	0.0047	0.0059	0.0103
Ghana	0.1367	0.1154	0.1181	0.1759	0.1519	0.1503	0.3073	0.2799	0.2588

Table A3.11. Continued

Greece	0.0055	0.002	0.0082	0.0067	0.0025	0.0098	0.0108	0.0044	0.0148
Guatemala	0.0983	0.0845	0.0742	0.1129	0.0976	0.0859	0.1559	0.1365	0.1211
Guinea	0.4623	0.4128	0.3569	0.5264	0.4854	0.4258	0.6778	0.6620	0.6036
Guinea-Bissau	0.3405	0.2654	0.2950	0.3896	0.3170	0.3659	0.5198	0.4635	0.5618
Guyana	0.0928	0.0658	0.0417	0.1155	0.0810	0.0524	0.1912	0.1313	0.0893
Haiti	0.3097	0.3155	0.296	0.3528	0.3591	0.3377	0.4690	0.4766	0.4512
Honduras	0.1117	0.1116	0.1055	0.1307	0.1311	0.1243	0.1884	0.1902	0.1818
Hong Kong	0.0053	0.0070	0.0158	0.0064	0.0084	0.0182	0.0100	0.0125	0.0254
Hungary	0.0028	0.0066	0.0077	0.0035	0.0082	0.0094	0.0064	0.0137	0.0151
Iceland	0.0001	0.0001	0.0005	0.0001	0.0001	0.0006	0.0003	0.0003	0.0011
India	0.0693	0.067	0.0417	0.0951	0.0898	0.0566	0.1952	0.1751	0.1148
Indonesia	0.0487	0.0295	0.0171	0.0637	0.0385	0.0235	0.1187	0.0717	0.0492
Iran	0.0237	0.0197	0.0148	0.0292	0.0242	0.0183	0.0473	0.0395	0.0301
Ireland	0.0037	0.0019	0.0008	0.0046	0.0023	0.0010	0.0075	0.0040	0.0018
Israel	0.0018	0.0022	0.0026	0.0024	0.0027	0.0032	0.0041	0.0046	0.0053
Italy	0.0019	0.0036	0.0019	0.0024	0.0044	0.0024	0.0040	0.0070	0.0039
Jamaica	0.0235	0.0549	0.0150	0.0281	0.0628	0.0183	0.0429	0.0862	0.0292
Japan	0.0002	0.0004	0.0006	0.0003	0.0006	0.0008	0.0005	0.0010	0.0014
Jordan	0.0514	0.0356	0.0270	0.0639	0.0448	0.0347	0.1063	0.0768	0.0626
Kazakhstan	0.0006	0.0085	0.0052	0.0009	0.0114	0.0071	0.0021	0.0225	0.0147
Kenya	0.3765	0.3040	0.3002	0.4217	0.3537	0.3503	0.5383	0.4895	0.4877
Korea, Republic of	0.0010	0.0004	0.0002	0.0014	0.0005	0.0003	0.0028	0.0011	0.0007
Kyrgyzstan	0.0142	0.2552	0.0975	0.0213	0.2984	0.1247	0.0548	0.4208	0.2180
Laos	0.1368	0.1273	0.1022	0.1896	0.1692	0.1348	0.3791	0.3156	0.2506
Latvia	0.0008	0.0112	0.0181	0.0011	0.0143	0.0220	0.0023	0.0254	0.0348
Lesotho	0.5247	0.5210	0.4033	0.5735	0.5664	0.4505	0.6872	0.6732	0.5703
Lithuania	0.0008	0.0275	0.0139	0.0011	0.0330	0.0170	0.0023	0.0505	0.0273
Luxembourg	0.0000	0.0001	0.0003	0.0001	0.0001	0.0003	0.0001	0.0001	0.0005
Macedonia	0.0008	0.0018	0.0025	0.0011	0.0025	0.0035	0.0028	0.0059	0.0074
Madagascar	0.3396	0.3719	0.3695	0.4014	0.4420	0.4385	0.5645	0.6199	0.6145
Malawi	0.7123	0.6726	0.6528	0.7517	0.7235	0.7189	0.8324	0.8257	0.8442
Malaysia	0.0203	0.0152	0.0151	0.0251	0.0186	0.0183	0.0416	0.0298	0.0289
Mali	0.4281	0.4853	0.3868	0.5104	0.5483	0.4618	0.7031	0.6945	0.6479
Mauritania	0.1668	0.1318	0.0957	0.2010	0.1656	0.1233	0.3061	0.2767	0.2189
Mauritius	0.0249	0.0263	0.0223	0.0308	0.0320	0.0270	0.0508	0.0508	0.0421
Mexico	0.0199	0.0255	0.0160	0.0239	0.0303	0.0191	0.0366	0.0453	0.0292
Moldova	0.0168	0.1107	0.1294	0.0241	0.1386	0.1613	0.0555	0.2310	0.2654
Morocco	0.0357	0.0628	0.052	0.0474	0.0803	0.0664	0.0917	0.1415	0.1169
Nepal	0.3407	0.3491	0.3752	0.4238	0.4189	0.4389	0.6384	0.6005	0.6013
Netherlands	0.0008	0.0009	0.0006	0.0010	0.0011	0.0007	0.0018	0.0020	0.0012
New Zealand	0.0020	0.0026	0.0035	0.0025	0.0032	0.0043	0.0043	0.0053	0.0069
Nicaragua	0.1983	0.2228	0.1761	0.2281	0.2562	0.2051	0.3138	0.3514	0.2903
Niger	0.5313	0.6359	0.6265	0.6139	0.6970	0.6943	0.7852	0.8189	0.8267
Nigeria	0.2458	0.3291	0.2799	0.2941	0.3813	0.3307	0.4335	0.5209	0.4723

Table A3.11. Continued

Norway	0.0008	0.0017	0.0013	0.0010	0.0021	0.0016	0.0017	0.0034	0.0026
Pakistan	0.1283	0.0595	0.0337	0.1603	0.0798	0.0476	0.2647	0.1565	0.1057
Panama	0.0572	0.0505	0.0437	0.0672	0.0591	0.0514	0.0977	0.0857	0.0748
Paraguay	0.0159	0.1105	0.1070	0.0209	0.1283	0.1250	0.0402	0.1813	0.1794
Peru	0.0412	0.0464	0.0535	0.0507	0.0559	0.0637	0.0828	0.0864	0.0956
Philippines	0.1994	0.2011	0.0914	0.2273	0.2288	0.1110	0.3066	0.3077	0.1740
Poland	0.0018	0.0152	0.0021	0.0025	0.0185	0.0027	0.0053	0.0295	0.0052
Portugal	0.0092	0.0294	0.0175	0.0111	0.0338	0.0203	0.0172	0.0468	0.0289
Puerto Rico	0.0193	0.0241	0.0225	0.0225	0.0277	0.0257	0.0320	0.0384	0.0351
Romania	0.0021	0.0156	0.0117	0.0029	0.0197	0.0149	0.0062	0.0339	0.0265
Russia	0.0003	0.0310	0.0188	0.0004	0.0367	0.0227	0.0010	0.0545	0.0355
Rwanda	0.2934	0.4546	0.4975	0.3832	0.5400	0.5689	0.6279	0.7323	0.7288
Senegal	0.3557	0.2118	0.1709	0.4012	0.2606	0.2126	0.5200	0.4080	0.3439
Serbia	0.0035	0.0066	0.0030	0.0045	0.0089	0.0041	0.0082	0.0175	0.0089
Sierra Leone	0.4824	0.4781	0.6523	0.5287	0.5334	0.7146	0.6403	0.6648	0.8355
Singapore	0.0031	0.0020	0.0026	0.0038	0.0025	0.0031	0.0062	0.0041	0.0049
Slovak Republic	0.0000	0.0004	0.0008	0.0001	0.0005	0.0011	0.0002	0.0012	0.0023
Slovenia	0.0002	0.0022	0.0013	0.0002	0.0028	0.0017	0.0005	0.0050	0.0030
South Africa	0.1513	0.1363	0.1806	0.1682	0.1523	0.1981	0.2150	0.1976	0.2455
Spain	0.0006	0.0043	0.0006	0.0007	0.0052	0.0008	0.0014	0.0084	0.0014
Sri Lanka	0.0752	0.0819	0.1255	0.0968	0.1013	0.1470	0.1729	0.1655	0.2117
Sweden	0.0026	0.0048	0.0033	0.0032	0.0058	0.0041	0.0053	0.009	0.0064
Switzerland	0.0004	0.0005	0.0008	0.0005	0.0007	0.0010	0.0009	0.0012	0.0017
Taiwan	0.0002	0.0002	0.0003	0.0003	0.0003	0.0004	0.0007	0.0007	0.0008
Tajikistan	0.0145	0.1402	0.1578	0.0199	0.1880	0.2098	0.0418	0.3541	0.3849
Tanzania	0.3980	0.4076	0.3222	0.4690	0.4897	0.4064	0.6444	0.6864	0.6284
Thailand	0.0623	0.0516	0.0259	0.0738	0.0605	0.0318	0.1099	0.0880	0.0511
Trinidad &Tobago	0.0031	0.0021	0.0011	0.0041	0.0028	0.0015	0.0074	0.0051	0.0028
Tunisia	0.0134	0.0199	0.0131	0.0177	0.0254	0.0168	0.0337	0.0445	0.0299
Turkey	0.0171	0.0129	0.0054	0.0212	0.0161	0.0070	0.0349	0.0271	0.0127
Turkmenistan	0.0045	0.0008	0.0011	0.0065	0.0012	0.0015	0.0151	0.0027	0.0032
Uganda	0.5239	0.3534	0.3552	0.6033	0.4369	0.4196	0.7715	0.6493	0.5870
Ukraine	0.0001	0.0350	0.0128	0.0001	0.0437	0.0171	0.0003	0.0732	0.0338
United Kingdom	0.0039	0.0043	0.0033	0.0047	0.0052	0.0039	0.0076	0.0081	0.0062
United States	0.0012	0.0021	0.002	0.0015	0.0025	0.0025	0.0025	0.0041	0.0039
Uruguay	0.0096	0.0069	0.0070	0.0124	0.0088	0.0089	0.0223	0.0155	0.0155
Uzbekistan	0.0295	0.1102	0.0855	0.0413	0.1431	0.1120	0.0906	0.2572	0.2070
Venezuela	0.0082	0.0114	0.0121	0.0103	0.0141	0.0149	0.0179	0.0231	0.0244
Vietnam	0.2244	0.1319	0.0907	0.2892	0.1735	0.1184	0.4860	0.3168	0.2165
Yemen	0.1113	0.0677	0.0362	0.1428	0.0902	0.0492	0.2507	0.1742	0.1002
Zambia	0.4417	0.3804	0.3489	0.4944	0.4344	0.4041	0.6235	0.5727	0.5492

Appendix 4

Countries included in Chapter 3

Afghanistan	Egypt	Liberia	Rwanda
Albania	El Salvador	Lithuania	Saudi Arabia
Algeria	Estonia	Luxembourg	Senegal
Argentina	Fiji	Malawi	Sierra Leone
Armenia	Finland	Malaysia	Slovakia
Australia	France	Mali	Slovenia
Austria	Gabon	Malta	South Africa
Bahrain	Gambia	Mauritania	Spain
Bangladesh	Germany	Mauritius	Sri Lanka
Belgium	Ghana	Mexico	Sudan*
Belize	Greece	Moldova (Rep. of)	Swaziland
Benin	Guatemala	Mongolia	Sweden
Bolivia	Guyana	Morocco	Switzerland
Botswana	Haiti	Mozambique	Syrian Arab Republic
Brazil	Honduras	Myanmar	Tajikistan
Brunei Darussalam	Hong Kong	Namibia	Tanzania (United Rep.)
Bulgaria	Hungary	Nepal	Thailand
Burundi	Iceland	Netherlands	Togo
Cameroon	India	New Zealand	Tonga
Canada	Indonesia	Nicaragua	Trinidad and Tobago
Central African Republic	Iran (Islamic Rep.)	Niger	Tunisia
Chile	Ireland	Norway	Turkey
China	Israel	Pakistan	Uganda
Colombia	Italy	Panama	Ukraine
Congo	Jamaica	Papua New Guinea	United Arab Emirates
Congo (Democratic Rep.)	Japan	Paraguay	United Kingdom
Costa Rica	Jordan	Peru	United States
Côte d'Ivoire	Kenya	Philippines	Uruguay
Cuba	Rep. of Korea	Poland	Venezuela
Cyprus	Kuwait	Portugal	Viet Nam
Denmark	Lao People's Democratic Rep.	Qatar	Yemen
Dominican Republic	Latvia	Romania	Zambia
Ecuador	Lesotho	Russian Federation	Zimbabwe

Parameter estimates of the bidimensional distributions included in Chapter 3

Table A.4.1. Parameter estimates for the Sarmanov-Lee distribution with classical beta marginals

	Education		Health		Income		*w* parameters		
	a	*b*	*a*	*b*	*a*	*b*	Educ/Inc	Health/Inc	Educ/Health
1980	2.1630	2.7261	5.7030	2.8515	3.2529	2.8149	2.6902	3.3501	3.2944
1985	2.4723	2.6910	5.9646	2.6583	3.5715	3.1289	2.7738	3.6091	3.1524
1990	2.6331	2.5126	5.5423	2.2991	3.4830	2.9740	2.8975	3.8143	3.1350
1995	2.6808	2.1603	4.9960	1.9703	3.3117	2.7090	3.1247	4.0405	3.1268
2000	2.7446	1.8708	4.7117	1.7127	3.2128	2.4244	3.3639	4.0024	3.1542
2005	2.9151	1.6569	4.5615	1.4723	3.3059	2.3022	3.6501	3.9061	3.2032
2010	3.1493	1.5754	4.6958	1.3079	3.4623	2.2527	3.8344	3.8838	3.2258

Appendix 5

Regions and countries included in Chapter 4

Western Europe, North America, and Oceania: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Japan, Korea (republic of), Luxembourg, Malta, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States.

Arab States: Algeria, Bahrain, Egypt, Israel, Jordan, Morocco, Qatar, Saudi Arabia, Sudan, Syrian Arab Republic, Tunisia, Yemen, United Arab Emirates.

East Asia and the Pacific: Brunei Darussalam, China, Fiji, Hong Kong, Indonesia, Kiribati, Lao People's Democratic Republic, Malaysia, Mongolia, Myanmar, Papua New Guinea, Philippines, Thailand, Tonga, Viet Nam.

Europe and Central Asia: Albania, Armenia, Bulgaria, Cyprus, Estonia, Latvia, Lithuania, Moldova (Republic of), Montenegro, Romania, Russian Federation, Slovenia, Slovakia, Tajikistan, Turkey, Ukraine.

Latin America and the Caribbean: Argentina, Belize, Bolivia (Plurinational State of), Brazil, Chile, Colombia, Costa Rica, Cuba, Dominican Republic, Ecuador, El Salvador, Guatemala, Guyana, Haiti, Honduras, Jamaica, Mexico, Nicaragua, Panama, Paraguay, Peru, Trinidad and Tobago, Uruguay, Venezuela (Bolivarian Republic of).

South Asia: Afghanistan, Bangladesh, India, Iran (Islamic Republic of), Nepal, Pakistan, Sri Lanka.

Sub-Saharan Africa: Benin, Botswana, Burundi, Cameroon, Central African Republic, Congo, Democratic Republic of the Congo, Cote d'Ivoire, Gabon, Gambia, Ghana, Kenya, Lesotho, Liberia, Malawi, Mali, Mauritania, Mauritius, Mozambique, Namibia, Niger, Nigeria, Rwanda, Senegal, Sierra Leone, South Africa, Swaziland, Tanzania (United Republic of), Togo, Uganda, Zambia, Zimbabwe.