On Optimal Movable Antennas in Integrated Sensing and Communications

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***Abstract*—This paper investigates the optimization of movable antenna (MA) arrays for integrated sensing and communications (ICAS) in downlink multiple-input single-output (MISO) systems. We propose an idealized beamsteering model that enables flexible configuration of antenna positions to enhance the signal-to-noise ratio (SNR) for both communication and radar sensing tasks. The study begins with analytical solutions for optimal antenna spacing in simplified scenarios involving one or two users and a radar target, demonstrating the advantages of controlled antenna aliasing and separate beamforming. For more complex scenarios with multiple users and targets, we propose a suboptimal yet efficient two-stage heuristic combining antenna distance optimization with weighted MMSE beamforming and power allocation. The results show that movable antennas can significantly enlarge the achievable SNR region and improve capacity, especially in cases with conflicting beamforming objectives. Numerical simulations validate the proposed methods and highlight the potential of MA technology for future 6G integrated sensing and communication systems.**

***Index Terms*—Joint communications and sensing, movable antenna array, optimization, beamforming**

I. Introduction

Movable antennas (MAs) for wireless communications is an active and promising research area, particularly recognized as a potential cornerstone technology for future networks like 6G [1]. A comprehensive tutorial in [2] on movable antennas for wireless networks highlights their potential to enhance communication and sensing performance by enabling antenna movement, offering a significant shift from conventional fixed antennas. It covers the historical development, promising application scenarios (such as mobile networks, IoT, satellite communication, maritime communication, sensing, and ISAC), new field-response channel models tailored for MAs, and reviews various hardware architectures and practical constraints. It formulates a general optimization framework to exploit the spatial degrees of freedom from antenna movement and delves into major design issues such as antenna movement optimization and channel acquisition. The tutorial also presents existing prototypes and experimental results demonstrating the performance gains of MAs and discusses extensions to other wireless systems and synergy with emerging technologies.

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In multicast communications, [3] investigates the integration of MAs to enhance achievable rates. It proposes a novel MA-assisted multicast transmission architecture based on discretizing the MA motion. The paper develops algorithms like an alternating optimization algorithm based on successive convex approximation to jointly optimize transmit beamforming and antenna positions. For simpler cases, such as the two-user scenario, it derives a closed-form expression for the optimal beamformer and proposes low-complexity methods for MA placement optimization.

For multiuser uplink communications, [4] explores using multiple MAs at the base station (BS) to improve network performance. It models the uplink multiple access channel to capture the effects of MA movement at the BS. The primary objective is to maximize the minimum achievable rate among users by jointly optimizing the MA positions, the receive combining at the BS, and the users' transmit power. To solve this complex non-convex problem, the paper proposes a two-loop iterative algorithm utilizing particle swarm optimization (PSO) and an alternating optimization (AO)-based algorithm to reduce computational complexity.

The application of MAs in physical layer security (PLS) is investigated in [5]. While research in MA-aided PLS is noted as being in its early stages, [5] focuses on a more general MA-aided secure MIMO communication system, building upon prior work that explored MISO scenarios by optimizing MA positions and beamforming to minimize transmit power or maximize secrecy rate

MAs are also being explored for ISAC systems. [6] discusses how MAs demonstrate significant potential in both communication and sensing individually by allowing flexible configuration of wireless channels. However, optimizing antenna positions to effectively enhance the performance of both tasks simultaneously is challenging due to the complex relationship between channel expressions and antenna positions. This paper is highlighted as being among the first works to explore MA-based ISAC, specifically focusing on leveraging an MA array at the BS for flexible beamforming to enhance both communication and sensing performance concurrently.

ISAC has attracted interest for improving the efficiency and scalability of wireless networks [7]. A systematic classification method for both traditional radio sensing and ISAC is pro-

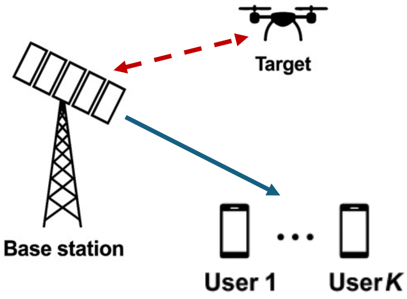

Fig. 1. Joint communications and sensing in the downlink with a movable multiple antenna base station.

vided. Major performance metrics and bounds used in sensing, communications and ISAC, are summarized. Finally, recent advances on ISAC fundamental limits are summarized.

In order to better understand the potential of MA in ISAC, this paper develops a idealized beamsteering model for joint downlink transmission and sensing and compares different beamforming approaches, starting with single-stream beamforming towards multi-stream beamforming with separate beams for communications and sensing. Advantages and limitations are analyzed using closed-form results.

The paper has the following contributions:

- System model and problem formulation for general K-user downlink beamforming and monostatic sensing.
- Characterization of optimal antenna distances for special cases including two-user two-antenna communication downlink (capacity achieving), the one user and one target scenario with single stream (best possible), and the achievable SNR region for two-stream scenario clearly outperforming the λ-half antenna spacing.
- Numerical assessment of general K-user single target scenario with suboptimal but efficient power allocation and antenna distance optimization.

II. System Model and Preliminaries

We consider a downlink ICAS system, where a multi-antenna base station serves K single-antenna users and simultaneously performs monostatic radar sensing of a single target, e.g., a drone. The scenario is illustrated in Fig. 1. The BS is equipped with N_t movable antennas, allowing reconfigurable spatial geometry. The scenario is called multiple-input single-output (MISO) broadcast channel (BC). There are different options to design the transmit signals. Either there are separate signals for sensing and communications and they are multiplexed spatially. Or there is one signal for both that contains the data for the communication users.

A. Communication and Sensing Geometry

Let θ_k denote the azimuth angle from the BS to communication user $k \in \{1, \ldots, K\}$, and let θ_0 denote the angle to the radar target. The BS antenna array geometry is a linear array, which is defined by the position vector $\mathbf{p} = [p_1, p_2, \ldots, p_{N_t}]^T \in \mathbb{R}^{N_t}$, where $p_1 = 0$ and $p_{n+1} - p_n = d_n$ is the distance between adjacent antennas. Please note that the number of free parameters for the optimization corresponds to the positions, or to the inter-antenna distances. In the case with two antennas, there is one free parameter, while with four antennas, there are three free parameters for the optimization. The maximum aperture and minimum spacing are constrained mechanically.

The array steering vector at angle θ is given by

$$\mathbf{a}(\theta) = \left[1, e^{j\frac{2\pi}{\lambda}p_2 \sin(\theta)}, \ldots, e^{j\frac{2\pi}{\lambda}p_{N_t} \sin(\theta)}\right]^T, \quad (1)$$

where λ is the carrier wavelength.

B. Transmit Signal Model

We assume that the BS knows the angles of communication and sensing targets perfectly. The extension to imperfect knowledge can be included by some angular spread corresponding to the uncertainty level. The BS transmits a superposition of user-specific signals and the radar sensing signal via linear beamforming

$$\mathbf{x}(t) = \sum_{k=1}^{K} \mathbf{w}_k s_k(t) + \mathbf{w}_0 s_0(t), \quad (2)$$

where $\mathbf{w}_k \in \mathbb{C}^{N_t}$ is the beamforming vector for user k, and $s_k(t)$ is the corresponding data symbol, with $\mathbb{E}[|s_k(t)|^2] = 1$. The signals from the different users and the sensing signal are assumed to be independent of each other. To maximize communication rate, the data is encoded using point-to-point Gaussian codebooks, while the sensing signal is usually discrete with constant modulus [8]. The sensing beamforming vector is denoted by $\mathbf{w}_0$ with sensing signal $s_0(t)$ with $\mathbb{E}[|s_0(t)|^2] = 1$. The total transmit power is constrained as

$$\sum_{k=0}^{K} \|\mathbf{w}_k\|^2 \leq P. \quad (3)$$

C. Communication Channel Model

Each user experiences a line-of-sight (LoS) MISO downlink channel modeled as

$$\mathbf{h}_k = \beta_k \mathbf{a}^H(\theta_k), \quad (4)$$

where β_k is the complex path gain. The received signal at user k is

$$y_k(t) = \mathbf{h}_k \mathbf{x}(t) + n_k(t) = \sum_{j=1}^{K} \beta_k \mathbf{a}^H(\theta_k)\mathbf{w}_j s_j(t) + n_k(t), \quad (5)$$

with additive white Gaussian noise $n_k(t) \sim \mathcal{CN}(0, \sigma_k^2)$. The corresponding signal-to-interference-plus-noise ratio (SINR) is

$$\text{SINR}_k = \frac{|\beta_k \mathbf{a}^H(\theta_k)\mathbf{w}_k|^2}{\sum_{j\neq k} |\beta_k \mathbf{a}^H(\theta_k)\mathbf{w}_j|^2 + \sigma_k^2}. \quad (6)$$

D. Sensing Channel Model

The BS operates as a monostatic radar to detect and track the target located at angle θ_0. The radar echo is modeled as

$$\mathbf{y}_{\text{rx}}(t) = \mathbf{H}_{\text{tgt}}(t)\mathbf{x}(t-\tau) + \mathbf{n}_{\text{rx}}(t), \tag{7}$$

where τ is the round-trip delay, and $\mathbf{n}_{\text{rx}}(t) \sim \mathcal{CN}(\mathbf{0}, \sigma_{\text{rad}}^2\mathbf{I})$ is radar receiver noise. The target response is modeled as

$$\mathbf{H}_{\text{tgt}}(t) = \alpha e^{j2\pi f_D t}\mathbf{a}(\theta_0)\mathbf{a}^H(\theta_0), \tag{8}$$

where α is the complex round-trip gain, and f_D is the Doppler shift. We assume that the receiver applies an ideal matched filter to the known signal $\mathbf{x}(t)$. Thereby, the round-trip delay τ is compensated. Furthermore, the radar receiver applies a beamforming vector $\mathbf{v}$, with $\|\mathbf{v}\| = 1$, to the received signal $\mathbf{y}_{\text{rx}}(t)$. The term $\mathbf{a}(\theta_0)\mathbf{a}^H(\theta_0)$ indicates that both transmission and reflection follow the same spatial direction. We further assume that the Doppler shift f_D can be compensated via proper filtering, too.

Under these assumptions, the radar signal-to-clutter-plus-noise ratio (SCNR) is given by

$$\text{SCNR} = \frac{\sum_{k=0}^{K} \left|\mathbf{v}^H\mathbf{H}_{\text{tgt}}\mathbf{w}_k\right|^2}{\sigma_{\text{rad}}^2}. \tag{9}$$

E. Programming Problem

The overall goal is to characterize or optimize the achievable performance region defined by the communication SINRs and the radar SCNR. This leads to the joint optimization problem:

$$\max_{\{\mathbf{w}_k\},\mathbf{p}} \quad \mathcal{F}(\text{SINR}_1, \ldots, \text{SINR}_K, \text{SCNR}) \tag{10}$$

$$\text{s.t.} \quad \sum_{k=0}^{K} \|\mathbf{w}_k\|^2 \leq P, \tag{11}$$

$$\mathbf{p} \in \mathcal{P}, \tag{12}$$

where $\mathcal{F}(\cdot)$ is a performance function (e.g., weighted sum or Pareto boundary, assumed to be monotonically increasing and concave), and $\mathcal{P}$ is the feasible set of antenna positions. Antenna positions are typically constrained by a minimum distance $\delta > 0$ due to mechanical and electromagnetic considerations. The receive beamformer $\mathbf{v}$ for the sensing part is maximum ratio combining $\mathbf{a}(\theta_s)$.

F. Optimal Movable Antenna for Two-User Communications

The idea to optimize the distance between the antennas at a uniform linear array is not novel and has been followed in [9] for the multiple-antenna (multiple-input single-output) interference channel (MISO IFC). For the two-user MISO IFC, the distances between the N_t antennas could be chosen to achieve the capacity region with simple maximum ratio transmission (MRT). The main result is given in terms of the receiver angles θ_1 and θ_2 as follows.

Lemma 1 (Section IV.B in [9])**.** *The optimal antenna distance d for the two-user MISO IFC is given by*

$$d^* = \min_{n\in\mathbb{N}^+} \frac{n\lambda}{N_t|\sin(\theta_1) - \sin(\theta_2)|}, \tag{13}$$

where $n/N_t \notin \mathbb{N}$.

Applying the optimal distance d^* in (13) together with MRT beamforming vectors to our scenario of the MISO BC leads to the following capacity region for communications.

Corollary 1. *The capacity region of the two-user MISO BC with optimal antenna spacing d^* in (13) is given by*

$$\mathcal{C}_{BC} = \bigcup_{(P_1,P_2\geq 0} (R_1, R_2 \mid P_1 + P_2 \leq P), \tag{14}$$

where the rates are computed as

$$R_k = \log_2\left(1 + \frac{\beta_k^2 N_t P_k}{\sigma_k^2}\right). \tag{15}$$

Proof. The proof follows the derivations in [9, Section IV.B], but elaborates on the symmetry between the two interference terms, one from beamformer one created at receiver two and vice versa. Define $\phi(\theta) := \frac{2\pi d}{\lambda}\sin(\theta)$. The requirement to have zero interference at user 2 from user 1 beam is

$$\sum_{\ell=0}^{N_t-1} \exp(j\ell(\phi(\theta_1) - \phi(\theta_2))) = 0. \tag{16}$$

Similarly, the requirement to get zero interference at user 1 from user 2 beam is

$$\sum_{\ell=0}^{N_t-1} \exp(j\ell(\phi(\theta_2) - \phi(\theta_1))) = 0. \tag{17}$$

Both (16) and (17) assume equal amplitude excitation, as typical under MRT with ℓ indexing antenna elements to capture phase progression. Both conditions are the same with flipped sign and they are only satisfied if

$$\phi(\theta_1) - \phi(\theta_2) = \frac{2\pi n}{N_t}, \qquad n \in \mathbb{Z} \setminus \{0\}. \tag{18}$$

From (18), the statement in (13) follows directly. □

The antenna array pattern is shown in Fig. 2. The maximum beam steering gain in the wanted direction with a proper zero at the interference direction can be observed. A generalization of this result beyond the 2-4 user case reported in [9] is reported in [10] where also some optimal closed form solutions for antenna distances and beamforming vectors are derived.

III. Special Cases with One or Two Receivers

A. Single-stream Beamforming

We now consider a simplified instance of the system with $K = 1$ communication user and a single radar target. The BS is equipped with N_t movable antennas. The user is located at azimuth angle θ_c, while the radar target is at angle θ_r. Antennas are linearly placed with spacing $d \geq \delta$.

Let us first focus on the case where only one signal is transmitted, the communication signal with beamforming $\mathbf{w}$. Let the BS antenna positions be $\mathbf{p} = [0, d]^T$, and denote the steering vector at angle θ as:

$$\mathbf{a}(\theta) = \begin{bmatrix} 1 \\ e^{j\frac{2\pi}{\lambda}d\sin(\theta)} \end{bmatrix}. \tag{19}$$

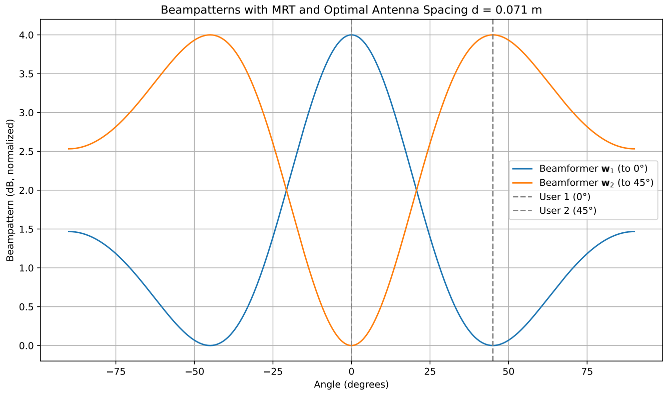

Fig. 2. Antenna array pattern for two users at $0°$ and $45°$, two antennas at the optimal antenna spacing $d/\lambda = 1/\sqrt{2}$. The symmetry of the solution can be clearly observed.

Let the user be located at θ_c and the target at θ_r. The transmit signal is:

$$\mathbf{x}(t) = \mathbf{w}s(t), \quad \text{with } \|\mathbf{w}\|^2 \leq P, \tag{20}$$

where $\mathbf{w} \in \mathbb{C}^2$ is the beamforming vector, and $s(t)$ is the data symbol with unit power. The LoS channel to the user is

$$\mathbf{h} = \beta \mathbf{a}^H(\theta_c), \tag{21}$$

and the received signal is

$$y(t) = \mathbf{h}\mathbf{x}(t) + n(t), \tag{22}$$

with $n(t) \sim \mathcal{CN}(0, \sigma^2)$. The communication SNR is given by

$$\mathrm{SNR}_{\mathrm{com}} = \frac{|\beta \cdot \mathbf{a}^H(\theta_c)\mathbf{w}|^2}{\sigma^2}. \tag{23}$$

The radar echo is modeled as:

$$\mathbf{y}_{\mathrm{rx}}(t) = \alpha \mathbf{a}(\theta_r)\mathbf{a}^H(\theta_r)\mathbf{w}s(t-\tau) + \mathbf{n}_{\mathrm{rx}}(t), \tag{24}$$

with noise $\mathbf{n}_{\mathrm{rx}}(t) \sim \mathcal{CN}(0, \sigma^2_{\mathrm{rad}}\mathbf{I})$. The sensing SNR is:

$$\mathrm{SNR}_{\mathrm{rad}} = \frac{|\alpha|^2 |\mathbf{a}^H(\theta_r)\mathbf{w}|^2}{\sigma^2_{\mathrm{rad}}}. \tag{25}$$

The objective is to jointly maximize both communication and radar SNRs by choosing the antenna spacing d and beamforming vector $\mathbf{w}$:

$$\max_{d,\mathbf{w}} \quad \mathcal{F}(\mathrm{SNR}_{\mathrm{com}}, \mathrm{SNR}_{\mathrm{rad}}) \tag{26}$$

$$\text{s.t.} \quad \|\mathbf{w}\|^2 \leq P, \tag{27}$$

$$d \geq \delta. \tag{28}$$

The result states that both SNRs can be maximized simultaneously with an optimal antenna spacing.

Lemma 2. *The optimal antenna spacing is given by*

$$d^* = \min_{n \in \mathbb{N}+} \left(\frac{n\lambda}{|\sin(\theta_c) - \sin(\theta_r)|} \,\middle|\, d^* \geq \delta \right). \tag{29}$$

The maximum achievable communication SNR is given by

$$\mathrm{SNR}_{\mathrm{com}} = \frac{N_t |\beta|^2 P}{\sigma^2}, \tag{30}$$

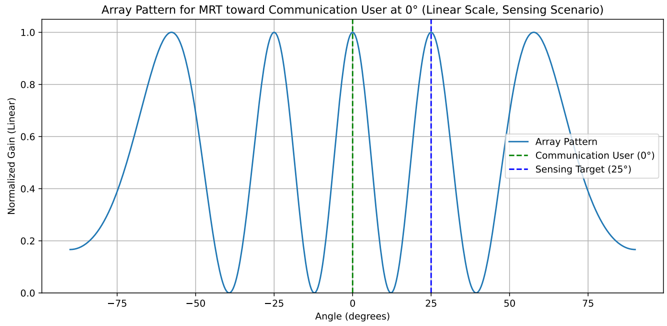

Fig. 3. Antenna array pattern for one user at $0°$ and one target at $25°$, two antennas at the optimal antenna spacing $d/\lambda = 2/\sin(\theta_r)$.

while the maximum achievable radar sensing SNR is

$$\mathrm{SNR}_{\mathrm{rad}} = \frac{N_t |\alpha|^2 P}{\sigma^2_{rad}}. \tag{31}$$

Proof. The proof works similar to the proof of Corollary 1 above. Unlike in (16) and (17), where the phase terms are designed to cancel each other out, we now require all phase terms to be fully aligned. This leads to

$$\sum_{\ell=0}^{N_t-1} \exp(j\ell(\phi(\theta_c) - \phi(\theta_r))) = N_t, \tag{32}$$

which ensures constructive interference for both directions. This is achieved for $\phi(\theta_c) - \phi(\theta_r) = 2\pi m$ for $m \in \mathbb{Z}$. From this, the condition in (29) follows. □

The antenna array pattern is shown in Fig. 3. Note that the distance between antennas could be chosen very large and aliasing in the antenna array pattern occurs. Therefore, the design is sometimes called controlled alias design. This leads to very narrow beams for which the angle estimation should be very accurate. This motivates to look for robust beamforming solutions which are outside the scope of this paper, but might be considered for future works.

One significant disadvantage of the single-stream beamforming approach is that one single waveform does not fit to both objectives as explained before. This cannot be observed looking at the SNR expressions, but considering the mean-square error (MSE) or the Cramer-Rao-bound (CRB). However, separate streams could be beneficial for achievable SINRs, too, as explained in the next section.

B. Two-stream Beamforming

Therefore, we consider the setup, where two separate beams and corresponding signals are formed, one for the communication receiver and one for the radar sensing. The SINR for the communication user is

$$\mathrm{SNR}_{\mathrm{com}} = \frac{|\beta \mathbf{a}^H(\theta_c)\mathbf{w}_c|^2}{|\beta \mathbf{a}^H(\theta_c)\mathbf{w}_r|^2 + \sigma^2}. \tag{33}$$

The SNR at the sensing receiver is given by

$$\mathrm{SNR}_{\mathrm{rad}} = \frac{|\alpha|^2 \left(|\mathbf{a}^H(\theta_r)\mathbf{w}_c|^2 + |\mathbf{a}^H(\theta_r)\mathbf{w}_r|^2 \right)}{\sigma^2_{rad}}, \tag{34}$$

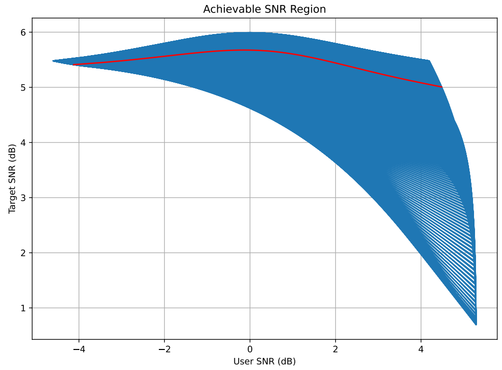

Fig. 4. Achievable SNR region for one user and one target with power allocation and antenna distance allocation. The red line corresponds to the half wavelength spacing.

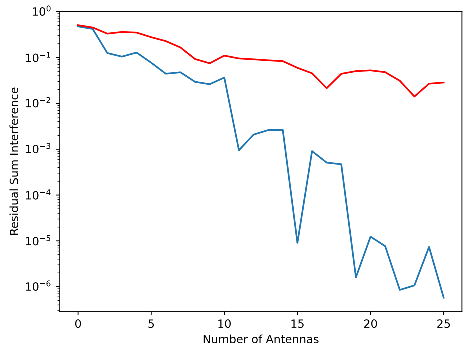

Fig. 5. Residual interference in Gram Matrix with optimized antenna spacings for different number of antennas.

where we used the optimal maximum ratio combining receive beamformer. In this scenario, tuning d alone cannot achieve both interference cancellation and communication power maximization, due to insufficient degrees of freedom.

From the perspective of optimizing the distance d between the two antennas, we have the following four goals

$$\begin{aligned} &|\mathbf{a}^H(\theta_c)^H\mathbf{w}_c|^2\uparrow \quad |\mathbf{a}^H(\theta_c)^H\mathbf{w}_r|^2\downarrow \\ &|\mathbf{a}^H(\theta_r)^H\mathbf{w}_c|^2\uparrow \quad |\mathbf{a}^H(\theta_r)^H\mathbf{w}_r|^2\uparrow . \end{aligned} \tag{35}$$

Therefore, we suggest an achievable solution in the following and choose

$$\mathbf{w}_c = \mathbf{a}(\theta_c) \quad \text{and} \quad \mathbf{w}_r = \mathbf{a}(\theta_r). \tag{36}$$

Finally, we optimize over the distance $d \geq \delta$. Depending on d, we get the whole Pareto boundary of the achievable SNR region. This region is illustrated in Fig. 4 with the same angles as in the illustrations before, where the advantage of MA is clearly visible since the achievable region is much larger than the standard $\lambda/2$ antenna spacing. In particular for the user SINR, the choice of antenna distance matters significantly.

IV. Multi-User and Multi-Beam Scenario

In this section, we move forward and propose an approach for joint beamforming, antenna distance, and power optimization in programming problem (12). We consider a specific setup to illustrate the benefits of MA.

The approach is a two-stage approach where we first optimize the antenna distances to create channels with favourable propagation conditions for sensing and communications channels and then optimize the beamforming vectors and power allocation to maximize a certain objective function.

For the first step, the ultimate goal is to create orthogonal channels between all users to achieve the capacity region of the MISO BC. At the same time, the goal is to have all channels of the users parallel to the target channel to maximize the received signal power at the target. These objectives are inherently conflicting, necessitating a compromise solution.

Most important for the communications users is to achieve a certain minimum SINR to support their services. Thus, the antenna spacing is optimized to make the Gram matrix of the K users' beamsteering vectors as diagonal as possible.

The following steps are performed to obtain the optimized antenna distances and beamforming vectors:

1) Optimize antenna distances to obtain *good* interference situation between communications users.
2) Optimize beamforming vectors for communication users without considering the target by applying the WMMSE algorithm.
3) Check if SCNR with communications signals alone is above threshold to perform sensing task and if not, apply additional sensing signal in the null-space of the communication users with minimum required transmit power.
4) (Optional) iterate the last two steps until convergence.

It is not claimed that these steps lead to the optimal antenna distances and beamforming vectors, but it provides a suboptimal heuristic solution which exploits the benefits of MA as well as smart beamforming optimization.

A. Antenna Distance Optimization

In order to obtain a reasonable solution, we will assume that the number of antennas is larger than the number of users K. The Gram matrix is given by

$$\mathbf{G} = \begin{bmatrix} \mathbf{a}^H(\theta_1) \\ \mathbf{a}^H(\theta_2) \\ \cdots \\ \mathbf{a}^H(\theta_K) \end{bmatrix} \cdot [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_K)] . \tag{37}$$

This leads to the following programming problem:

$$\min_{\mathbf{p}} \sum_j \sum_{i \neq j} |\mathbf{G}_{ij}|^2, \tag{38}$$

which we solve by using the Sequential Least Squares Programming (SLSQP) [11]. The solution method does not converge to the global minimum, but provides an efficient suboptimal solution.

In Fig. 5, the residual interference after antenna distance optimization is shown in blue for a scenario with five users at angles $(-20°, 0°, 25°, 45°, 75°)$. It can be seen that the residual interference decreases on average with more transmit

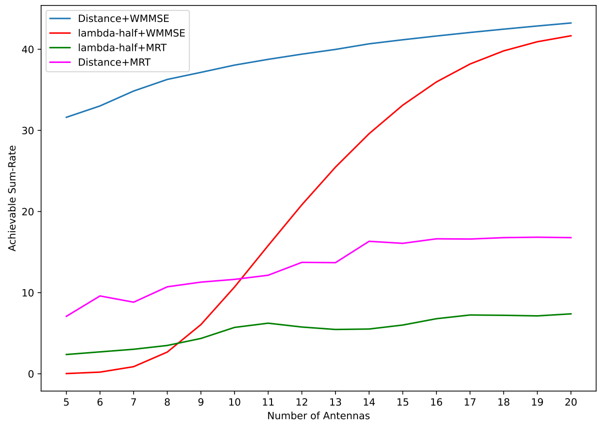

Fig. 6. Achievable Communications Sum-Rate Performance for $K = 5$ users at angles $0°, 5°, 10°, -7°, -10°$ and varying number of transmit antennas.

antennas. However, due to the suboptimal SLSQP approach, the curve is not monotonic. As a baseline for comparison, the residual interference for the $\lambda/2$ antenna spacing is shown in red, too. The gain due to the non-uniform linear array is clearly visible.

B. Beamforming Optimization

In the second step, we optimize the beamforming vectors with the antenna distances fixed. One popular algorithm to maximize the sum rate for the communications users is the weighted MMSE beamforming [12] algorithm. It exploits the representation of the mutual information in terms of a weighted MSE expression.

In Fig. 6, we compare the sum rate performance of two baseline schemes with the performance of the proposed first two steps, namely lambda-half spacing with MRT and lambda-half spacing with WMMSE. The SNR operating point is chosen at a large value of 20 dB to focus on the impact of interference.

C. Target SCNR Optimization

In the final step, we compute the achieved SCNR with the communication signals of the K users alone. This is dominated by the received signal power and the check can be implemented by the threshold SCNR_{th}

$$\text{SCNR}_{th} \cdot \sigma_{rad}^2 \leq \sum_{k=1}^{K} |\beta_k|^2 |\mathbf{a}^H(\theta_k)\mathbf{w}_k|^2. \quad (39)$$

If the condition in (39) is satisfied, the sensing beamforming vector $\mathbf{w}_0$ is set to zero. Otherwise, we choose it with squared norm

$$\Delta = \text{SCNR}_{th} \cdot \sigma_{rad}^2 - \sum_{k=1}^{K} |\beta_k|^2 |\mathbf{a}^H(\theta_k)\mathbf{w}_k|^2$$

as

$$\mathbf{w}_0 = \sqrt{\Delta} \frac{\Pi^{\perp}_{\mathbf{h}_1,\ldots,\mathbf{h}_K} \mathbf{h}_0}{||\Pi^{\perp}_{\mathbf{h}_1,\ldots,\mathbf{h}_K} \mathbf{h}_0||^2}, \quad (40)$$

to make sure that the required SCNR is achieved.

V. Conclusions and Future Works

In this paper, we have considered the optimization of multiple-antenna downlink transmission in ISAC with MA. The beneficial impact of MA on both interference suppression in the interference network as well as signal enhancement for ISAC is analytically demonstrated. For an idealized scenario with two receivers, it is analytically proved that MA can achieve the capacity of the MISO BC for communication and the largest SNR region for ISAC with sensing target.

There are several open questions left for future works. This includes the assumptions made in the system model about the knowledge of the perfect channel state information, the compensation of the round-trip delay and Doppler shift at the radar receiver, and the ideal beamsteering-based channel model. Another important direction is the charaterization of the sensing performance for the multi-user setup. We have studied only SCNR-based measures. A complete numerical investigation of the gains of MA optimization for other sensing performance metrics is left for future works.

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