

# Enhanced Predictive Diagnostics for Naval Equipment: Integrating MYT Decomposition for Advanced Process Monitoring

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**ABSTRACT:** The competitiveness in maritime operations demands maintenance strategies that ensure high reliability and availability at minimal cost. While predictive diagnostics have shown promise in detecting deviations from optimal operating conditions, current methodologies often fail to effectively isolate and identify the contributing process variables. This study introduces an enhanced predictive diagnostic approach that integrates MYT (Mason, Young, Tracy) decomposition with traditional statistical monitoring techniques, such as Hotelling's  $T^2$  control charts. By applying this methodology to the auxiliary systems of a 284-meter LNG tanker, we identified that the key variables driving process anomalies were Superheated Steam in Boiler 1 (Tn/h) and Superheated Steam in Boiler 2 (Tn/h). These findings underscore the ability of the proposed method to detect deviations before critical failures occur, providing ship operators with actionable insights to enable precise maintenance scheduling, reduce operational costs, and prevent unscheduled downtime. The demonstrated integration of MYT decomposition into predictive maintenance protocols highlights its potential to optimize monitoring accuracy and decision-making in complex naval systems.

## 1 INTRODUCTION

The maritime industry stands at the forefront of global transportation, with its operational efficiency and reliability directly influencing international trade and economic stability [1]. However, as shipping operations grow increasingly complex, so do the challenges associated with maintaining the performance of critical systems under dynamic and often harsh operating conditions. These conditions include fluctuating weather patterns, such as high winds and rough seas, extreme temperatures that can affect machinery performance, and variability in cargo loads and fuel quality. Such factors not only introduce significant stress on shipboard systems but also demand robust solutions to ensure reliability and safety. This complexity is further amplified by the growing demand for sustainable practices, cost

reduction, and compliance with stringent environmental regulations, compelling operators to adopt innovative maintenance strategies [2].

In recent years, advancements in data acquisition, real-time monitoring, and predictive analytics have laid the groundwork for a paradigm shift in maritime maintenance. Industry 4.0 principles [3] characterized by enhanced connectivity, data-driven decision-making, and automation, have transformed the way systems are monitored and managed. Modern integrated automation systems (IAS) enable the recording of multivariable datasets with high-frequency precision, offering unparalleled insights into the operational state of vessels [4]. Despite these technological strides, traditional monitoring techniques often fall short in addressing the challenges posed by highly correlated datasets, where the sheer

volume of information can obscure critical anomalies and delay corrective actions [5].

To bridge this gap, advanced statistical tools and multivariable analysis methods are becoming indispensable in predictive maintenance frameworks [6-8]. Some techniques such as time series are typically used. Concerning time series, it was applied on a research vessel at the Norwegian University of Science and Technology (NTNU). Relevant variables were selected, unnecessary information or noise was removed, and essential characteristics of the problem were extracted in order to reliably identify the vessel's behaviour [9]. In relation to partial least squares a statistical framework is developed to process the vast amounts of navigation data acquired by the on-board multi-sensor systems and an automatic reporting system is created to monitor fuel consumption [10].

Among these, Hotelling's  $T^2$  control charts have emerged as a robust technique for detecting deviations in multivariate processes [11]. In [12] control of the condition of the oil in the gears of the vessels was analysed by means of Hotelling's  $T^2$  statistic or in [13] where the control of the machining process for industrial components manufactured on conventional lathe machines is monitored.

However, their utility is often limited by their inability to pinpoint the specific variables responsible for detected anomalies [14]. This limitation is particularly critical in maritime systems, where understanding the root cause of deviations is essential for targeted maintenance and operational optimization.

Unlike previous studies that have applied Hotelling's  $T^2$  control charts or multivariate analysis separately in maritime contexts [12], [14], this work introduces a novel integration of Hotelling's  $T^2$  charts with Mason-Young-Tracy (MYT) decomposition [15], to enhance interpretability and diagnostic precision in predictive maintenance systems. While Hotelling's  $T^2$  charts are effective for identifying deviations in multivariate data, they often fall short in isolating the variables responsible for such deviations [14], a gap directly addressed by the MYT methodology. To the best of our knowledge, this is the first time that such an integrated approach has been implemented and validated using real high-frequency operational data from an LNG tanker's auxiliary boiler-turbine system. The proposed framework not only detects early-stage anomalies but also identifies their root causes with clarity, offering a scalable and interpretable solution that aligns with the growing need for data-driven, Industry 4.0-aligned maintenance strategies in the maritime industry [3], [10].

This study introduces an enhanced predictive diagnostic framework that integrates Hotelling's  $T^2$  control charts with Mason-Young-Tracy (MYT) decomposition [15]. The MYT approach dissects multivariable anomalies into their individual components, enabling the precise identification of variables contributing to deviations. By applying this integrated methodology to the auxiliary boiler-turbine system of a 284-meter LNG tanker, we demonstrate its ability to detect early-stage anomalies, isolate their root causes, and provide actionable insights for maintenance planning.

The proposed framework addresses key challenges in modern maritime operations, including the need to manage the complexity of multivariable datasets and the imperative to optimize maintenance interventions. This paper not only validates the efficacy of the methodology through a real-world case study but also highlights its broader implications for advancing predictive maintenance protocols in the maritime sector. In doing so, it underscores the critical role of data-driven diagnostics in enhancing system reliability, reducing operational costs, and supporting the industry's transition toward more sustainable and efficient practices.

By bridging the gap between anomaly detection and root cause analysis, this study represents a significant contribution to the evolving field of predictive maintenance in the maritime industry, offering a blueprint for future research and practical applications in complex naval systems.

The primary objective of this study is to apply the MYT (Mason, Young, Tracy) decomposition technique in conjunction with Hotelling's  $T^2$  control charts to a real-world maritime context, specifically on the auxiliary boiler-turbine system of a 284-meter LNG tanker. This research aims to evaluate the effectiveness of the proposed methodology in detecting operational deviations at an early stage and isolating the specific variables responsible for these anomalies. By doing so, the study seeks to demonstrate the practical applicability of this integrated approach for enhancing predictive maintenance protocols, reducing operational costs, and improving the reliability of complex naval systems.

## 2 MATERIAL AND METHODS

The auxiliary boiler-turbine system of a 284-meter LNG tanker serves as the foundation for this study, designed to explore advanced predictive diagnostic techniques in real-world maritime operations. This system plays a critical role in maintaining the vessel's operational efficiency, ensuring the continuous supply of thermal and mechanical energy necessary for propulsion and auxiliary functions. To achieve this, key performance variables were monitored and analyzed under carefully controlled conditions to establish a robust framework for identifying deviations from normal operations.

This section describes the ship's specifications, the monitored variables, and the methodology employed to create the Historical Database Set (HDS) as a baseline for system behavior. The study's focus extends beyond simple anomaly detection to understanding the underlying causes of deviations using MYT decomposition integrated with Hotelling's  $T^2$  control charts. This combined approach provides a powerful diagnostic tool capable of isolating critical variables responsible for process anomalies, offering actionable insights for predictive maintenance.

### 2.1 System Description

The study was conducted on the auxiliary boiler-turbine system of a 284-meter LNG tanker. The characteristics of ship are listed in Table 1.

Table 1. Ship's specifications.

Type of ship	LNG Tanker
Length overall	284 m
Breadth extreme	42.5 m
Draught	11.4 m
Gross tonnage	98478 Tons
Net tonnage	27143 Tons

Six variables were monitored: shaft power (kW), boiler superheated steam production (Tn/h for two boilers), outlet temperature of superheated steam (in both boilers, °C), and daily fuel consumption (m<sup>3</sup>/day). Data were collected using the ship's integrated automation system over two voyages, each lasting 12 days, under normal operational conditions: vessel speed between 10 - 13.5, average engine room temperature of 27-32°C and average ambient temperature of 16 - 32°C. The voyages covered routes from Malta to Trinidad. The process is show in figure 1.

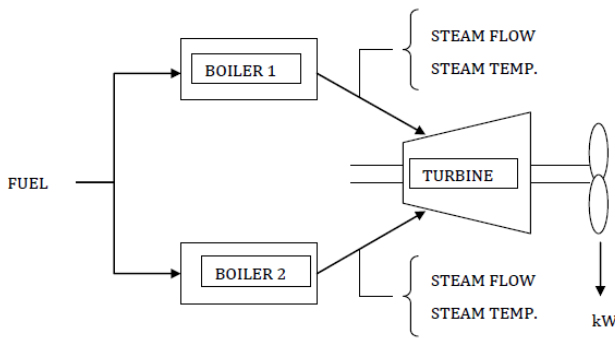


Figure 1. Process system.

## 2.2 Data Purging and Historical Database Creation

To build the historical database (HDS), the n=77 of the preliminary data base, estimated for the multivariate process were monitored using Hotelling's T<sup>2</sup> chart [16] following the expression (1).

$$T^2 = (X_i - \bar{X})^T S^{-1} (X_i - \bar{X}) \quad (1)$$

where:

$X_i = (X_{i1}; X_{i2}; \dots; X_{ip})$  preliminary data,  $\bar{X}$ , is the vector of sample means  $y$   $S^{-1}$ , the inverse of the covariance matrix.

Depending on the circumstances, the T<sup>2</sup> statistic can be described by three different probability functions: the Beta, the F and the chi-square distributions. When  $\mu, \sigma$  are estimated, the Beta distribution is used in the purging process of a Phase I operation, whereas the F distribution is used in the development of the control process in a Phase II operation. When  $\mu, \sigma$  are known, the chi-square has applications in both Phase I and Phase II operations [17].

During the purging process, the atypical observations of the process, obtained in the generation the preliminary database, were detected and eliminated, in order to avoid possible errors in results.

In this case, with a mean and standard deviation  $\mu, \sigma$  estimated, for the calculation of the UCL (Upper Control Limit), the  $\beta$  distribution of  $\alpha=0.05$ , was used in the process of purging outliers from those

observations that were outliers in the process. The level of  $\alpha$  is typical value for this type of process.

The UCL was determined by the following expression:

$$UCL = \left\{ \frac{(n-1)^2}{n} \right\} \beta_{\left\{ \alpha; \frac{p}{2}; (n-p-1)/2 \right\}} \quad (2)$$

where:

n: Is the size of the data set, p: Number of variables,  $\beta_{\left\{ \alpha; p/2; (n-p-1)/2 \right\}}$ , is the  $\alpha$ th, quantile of the beta distribution,  $\beta_{\left\{ p/2; (n-p-1)/2 \right\}}$

If the value of T<sup>2</sup>, which was monitored for an observation, exceeded the UCL, the observation was removed from the preliminary database.

With the remaining observations, we calculated a new vector of means and covariance matrix and again, outliers, produced by errors in the measurements, were detected and eliminated, this process was repeated, until a homogeneous set of observations was obtained. The final data set was the (HDS), from the normal operation mode of the process, consisting of 54 samples.

The preliminary data base, consisting of 77 samples is showed in Table 2. In Table 3, the detected outliers are represented in each step until the HDS was obtained.

Table 2. Part of Preliminary data base.

1-shaft power (kW)					
2- superheated steam in Boiler 1 (Tn/h)					
3- superheated steam in Boiler 2 (Tn/h)					
4- outlet temperature of superheated steam in boiler 1 (°C)					
5- outlet temperature of superheated steam in boiler 2 (°C)					
6- daily fuel consumption (m <sup>3</sup> /day)					
1	2	3	4	5	6
8009	25.9	25.6	514	510	162
8232	26.1	27.2	514	510	162
8085	28.5	28.4	514	510	162
8126	27.7	27.8	515	512	168
7841	27.5	27.5	515	512	168
7685	27.5	27.4	515	512	168
8622	28.8	28.8	515	512	168
8340	27.1	27.4	515	512	168
8469	26.4	26.6	515	512	168
8520	26.5	26.7	514	496	164
8286	27.3	27.6	514	496	164
8380	26.4	26.6	514	496	164
8287	26.4	26.7	514	499	168
8461	26.9	27.2	514	499	168
8358	26.8	27	514	499	168
8490	27	27.2	514	499	168

Table 3. Steps to get the HDS.

No. of observations	UCL	No. outliers detected
77	12.043	6
71	11.995	4
67	11.959	4
63	11.918	2
61	11.895	2
59	11.871	1
58	11.858	1
57	11.845	1
56	11.832	2
54	11.803	0

### 2.3 Process Control and MYT decomposition application

In this step, it was tested to see if a new entry of data generated a signal, with respect to the historical data set (HDS). Considering a continuous steady-state process where the observation vector are independent and the parameters of the underlying normal distribution are unknown and must be estimated. We assume the process is being monitored by observing a single vector of 23 new valid samples acquired after having analyzed them according to the criteria of the normal condition of the operation.

The  $T^2$  values, for the new data input, were calculated, following the expression (3).

$$T^2 = (X_i - \bar{X})' S^{-1} (X_i - \bar{X}) \quad (3)$$

where  $\bar{X}$  is the vector of sample means and  $S^{-1}$  the inverse of the covariance matrix, obtained from the HDS and  $X_i$  the new data entry.  $X_i = (X_{i1}; X_{i2}; \dots; X_{ip})'$ . Here, the  $T^2$  statistic [18] follows the F distribution. For the calculation of the UCL (Upper Control Limit), the F distribution of  $\alpha=0.05$ , for Type II errors, was used [18]. The level of  $\alpha$  can be variable, making more or less strict the method. The chosen alpha level is normally used in industrial processes. The UCL is computed as:

$$UCL = \left\{ \frac{p(n+1)(n-1)}{n(n-p)} \right\} F_{\{\alpha; p; (n-p)\}} \quad (4)$$

Where  $p$ , is the number of variables,  $n$ , is the size of the HDS and  $F_{\{\alpha; p; (n-p)\}}$ , is the  $\alpha$ th, quantile of  $F_{\{p; (n-p)\}}$ .

The values of  $T^2$  which exceeded the UCL, were declared as signals and this concluded that the observation was out of rank with respect to the mode of normal operation of the process.

Once the  $T^2$  statistical detected samples which were out of rank in the process from normal operating conditions, the MYT decomposition was used [19, 20] to identify the variables with more weight, responsible for state out of rank for each sample.

The general decomposition for "p" variables of the Hotelling's  $T^2$  statistic, follow the equation:

$$T^2 = T_1^2 + T_{2,1}^2 + T_{3,1,2}^2 + T_{4,1,2,3}^2 + \dots + T_{p,1,\dots,p-1}^2 = T_1^2 \sum_{j=1}^{p-1} T_{j+1,1,\dots,j}^2 \quad (5)$$

The final  $T^2$  value,  $T_1^2$ , is Hotelling's statistic for the first variable. It reduces to the square of the univariate t statistic for the initial variable:

$$T_1^2 = \frac{(X_1 - \bar{X}_1)^2}{S_1^2} \quad (6)$$

where,  $\bar{X}_1$  and  $S_1$  is the mean and standard deviation of variable  $X_1$ .

The statistic  $T_{p,1,\dots,p-1}^2$  is the  $p^{th}$  component of the vector  $X_i$  adjusted by the estimates of the mean and standard deviation of the conditional distribution of  $X_p$  given  $X_1, X_2, \dots, X_{p-1}$ . It is given by

$$T_{p,1,\dots,p-1}^2 = \frac{(X_{ip} - \bar{X}_{p,1,\dots,p-1})^2}{S_{p,1,\dots,p-1}} \quad (7)$$

where:

$$\bar{X}_{p,1,\dots,p-1} = \bar{X}_p + b_p' (X_i^{(p-1)} - X^{(p-1)}),$$

$\bar{X}_p$  is the sample mean of  $n$  observations on the  $p^{th}$  variable,

$b_p = S_{XX}^{-1} s_{Xp}$  is a  $(p-1)$  - dimensional vector estimating the regression coefficients of the  $p^{th}$  variable regressed on the first  $p-1$  variables,

$$S_{p,1,\dots,p-1}^2 = S_X^2 - S_{Xp}' S_{XX}^{-1} S_{Xp}$$

$$\text{and } S = \begin{pmatrix} S_{XX} & S_{Xp} \\ S_{Xp}' & S_X^2 \end{pmatrix}$$

### 3 RESULTS

$T^2$  values were calculated according to Eq. (3), for each one of the 23 new observations, and they were monitored in a control chart, according to Fig. 2 with a upper control limit previously calculated, according to the expression Eq. (4), valued in  $UCL = 15.4833$ , with  $\alpha=0.05$ , to detect changes significant in the normal operation condition.

The control chart shows that in the observation 5, there is a value, over the UCL, which indicates, that in that interval of time, the process had a deviation from its normal operation mode.

This situation does not mean that the process was failing, but that at that moment it deviated from its normal operating conditions. But if such a negative trend is repeated over time, it would be an indication of the need to take corrective maintenance action to restore process operability.

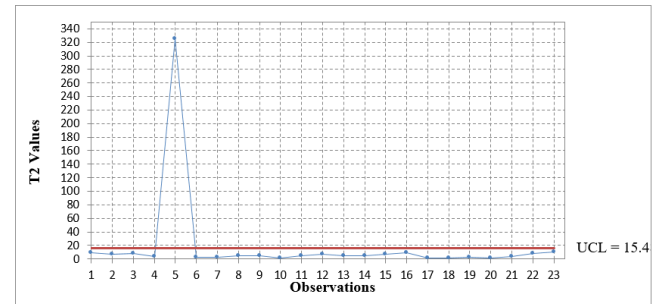


Figure 2. Control Chart.

In the next stage, it was identified which were the variables that had produced the state out of rank of each observation.

Through MYT decomposition technique, each  $T^2$  value was decomposed for each one of the signals to detect, which was the variable which had contributed most strongly to the state out of rank of process, the unconditional terms was calculated following Eq. (6), and the conditional terms were calculated following Eq. (7), the decomposition is listed in Table 4. It shows that the variables (2 and 3) Superheated steam in Boiler 1 (Tn/h) and Superheated steam in Boiler 2 (Tn/h), caused the state out of rank of process.

Table 4. MYT Decomposition.

No. Observation	No. Variable	Variable
5	2	Superheated steam in Boiler 1 (Tn/h)
	3	Superheated steam in Boiler 2 (Tn/h)

The variables related to superheated steam production in both boilers serve as key indicators of the system's thermal and energy performance. These variables are fundamentally connected to the equilibrium between energy demand and the system's capacity to fulfill that demand under normal operational conditions.

Simultaneous deviations in both variables often point to a potential interdependence or imbalance in the coordinated operation of the boilers, which can compromise the system's ability to maintain stable performance. Superheated steam production plays a critical role in energy transfer to the turbine system, and any deviation from established production limits can lead to efficiency losses and destabilization of the overall system.

The proposed method proved effective in detecting observations that deviated from the normal process conditions. For the ship's engineers, identifying that the process was out of rank provided an early warning, enabling them to remain vigilant and prepared to address significant changes in the system.

Additionally, the application of MYT decomposition facilitated the identification of the specific variables causing the deviation—namely, superheated steam production in Boiler 1 (Tn/h) and Boiler 2 (Tn/h). This insight was instrumental for the ship's engineers, as it allowed them to focus on correcting the deviations and restoring the process to its normal operating conditions.

Traditional monitoring methods operate on a fundamentally different principle: they rely on univariate thresholds and detect anomalies only after a specific variable exceeds its acceptable range. In such reactive systems, no early warning is available, and the failure must occur—or be imminent—before any corrective action can be triggered. Conversely, the methodology proposed here identifies multivariate deviations before any individual variable breaches its limits, enabling earlier detection and diagnosis. This predictive capability underscores the utility and effectiveness of the MYT-enhanced Hotelling's  $T^2$  approach, offering significant advantages over conventional techniques in managing complex systems.

#### 4 CONCLUSIONS

This study has demonstrated the effectiveness of integrating MYT decomposition with Hotelling's  $T^2$  control charts for advanced monitoring of naval systems. This methodology not only enabled the early detection of deviations in the operational performance of the boiler-turbine system of a 284-meter LNG tanker but also precisely identified the variables responsible for these anomalies, providing a robust framework for predictive maintenance. The combination of advanced statistical tools and multivariable decomposition

techniques offers significant advantages for managing complex systems, including early anomaly detection, which allows deviations to be identified before they manifest as critical failures, providing sufficient time to implement corrective actions. Moreover, the MYT decomposition highlighted the variables with the greatest impact on the out-of-range state, facilitating a more focused and efficient diagnosis and enabling resource optimization by prioritizing maintenance interventions based on the responsible variables, thereby reducing the need for generalized inspections and minimizing operational costs.

The ability to identify specific deviations in key variables, such as superheated steam production in both boilers, demonstrates the practical value of this approach in the maritime industry, underscoring the importance of integrating advanced monitoring tools into maintenance protocols, especially in systems where operating conditions are dynamic and failures can have significant consequences. Additionally, the proposed approach supports the transition toward more proactive, data-driven maintenance strategies, aligning with Industry 4.0 objectives and promoting greater reliability and operational efficiency in ships.

Advancing the current state of knowledge, this work operationalizes an integrated diagnostic approach that not only detects anomalies but also explains them in a multivariate context. The application of MYT decomposition within the maritime domain—particularly in combination with Hotelling's  $T^2$  control charts and using real onboard datasets—has not been previously demonstrated in such a comprehensive manner. By bridging the interpretability gap in multivariable monitoring, this contribution offers a methodological innovation with immediate applicability for condition-based maintenance strategies, reinforcing the novelty and practical value of the proposed framework in enhancing reliability and operational intelligence in the maritime sector.

This work validates the utility of MYT decomposition as a complementary tool to traditional techniques in multivariable monitoring of maritime systems. The findings highlight its potential to optimize decision-making, reduce operational costs, and enhance the reliability of vessels in a highly competitive environment. In conclusion, the proposed methodology represents a significant advancement in monitoring and predictive maintenance strategies for naval systems. Its ability to manage the complexity of multivariable data and provide accurate diagnostics positions it as a key tool for improving operational efficiency in the modern maritime industry.

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