

A methodology for the low-cost optimisation of small wind turbine performance

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Abstract

The increasing use of small wind energy has made it necessary to develop new methods to improve the efficiency of this technology. This improvement is best achieved considering the interaction between the various components, such as the wind rotors, the electrical generators, the rectifiers and the inverters, as opposed to studying the individual components in isolation. This paper describes a methodology to increase the efficiency of Small Wind Turbines (SWTs) equipped with a Permanent Magnet Synchronous Machine (PMSM). To achieve this objective, capacitor banks will be connected between the PMSM and the rectifier. This methodology is motivated by two clear aims. The first one is to operate the SWT with its maximum power coefficient C_p . The second one is to select the most suitable capacitor bank for each wind speed to optimise the energy supplied to the grid. The methodology will be tested on a commercial 3.5 kW SWT, and the results will be studied to determine its feasibility.

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GLOSSARY

Figure glossary

1. INTRODUCTION

2 The small wind industry is experiencing significant growth and is becoming increasingly competitive. Currently, robust equipment that maximises 3 energy production and minimises manufacturing costs is being sought.

5 Some studies to achieve these objectives have focused on:

- 6 • Improving the energy storage system [1],
- 7 • Designing new Permanent Magnet Synchronous Machines (PMSMs) [2],
- 8 • Improving the aerodynamic design of Small Wind Turbines (SWTs) [3,
- 9 4],
- 10 • Controlling the pitch angle [5],
- 11 • Enhancing the inverter operation [6].

12 Previous works have used capacitors behind the rectifier to create a Power
13 Factor Control (PFC) and to improve the SWT efficiency [7, 8, 9].

14 This paper presents a methodology to optimise the efficiency of SWTs
15 based on a PMSM. To test this methodology, it is necessary to create a
16 model of the whole system. The Simulink software package was used for this
17 purpose [10]. To optimise the system, two aspects are considered:

18 1. The aerodynamic factor: the Maximum Power Point Tracker (MPPT) [11]
19 is calculated to extract the maximum energy from the wind [12]. Us-
20 ing the MPPT and the SWT operating surfaces (obtained by means of
21 the Simulink model), the optimal relationship between the continuous
22 voltage of the rectifier v_{dc} and the grid power P_{grid} can be determined.
23 This relationship must be set in the inverter, and it is known as the
24 Maximum Power Characteristic Curve (MPCC).

25 2. The capacitor bank effect: when a capacitor bank is connected, the
26 armature reaction produces a magnetising effect in the PMSM that is
27 added to the magnetic field generated by the magnets. As a result, a
28 higher voltage at the machine terminals is generated [13, 14].

29 This methodology will be tested in a commercial 3.5 kW SWT with a
30 PMSM, and the system improvements will be verified. Finally, an economic
31 study will be conducted, in which the well-known Weibull Distribution (WD)
32 will be employed [15].

33 **2. SYSTEM MODEL**

34 A system model is necessary to use the methodology presented here, and
35 Simulink was used for this purpose [10]. This model makes it possible to
36 analyse the SWT in detail and to evaluate its performance under various
37 operating conditions.

38 First, a PMSM model must be created with the intention of including it in
39 the system model. This model was constructed using the electrical equations

40 of synchronous machines. Eq. (1) defines the flux at the stator of the PMSM:

$$\begin{aligned} \psi_{ea}(t) = & (L_{\sigma_e} + M_{aa})i_{ea}(t) + M_{ea}i_{eb}(t) + M_{ea}i_{ec}(t) + \\ & + \psi_f \cos(\theta_e(t)) \end{aligned} \quad (1)$$

41 where

42

$\psi_{ea}(t)$	Total instantaneous flux in the stator phase “a”,
L_{σ_e}	Leakage inductance of a stator phase,
M_{aa}	Inductance due to the magnetisation flux generated by the phase “a” on itself,
M_{ea}	Inductance due to the magnetisation flux generated by the other phases on the phase “a”,
ψ_f	Flux generated by the rotor,
$\theta_e(t)$	Electrical angle between the rotor and the stator,
$i_{ea}(t), i_{eb}(t), i_{ec}(t)$	Instantaneous current flowing through the stator phases.

43 if $i_{ea}(t) = -(i_{eb}(t) + i_{ec}(t))$, then:

$$\psi_{ea}(t) = (L_{\sigma_e} + M_{aa} - M_{ea})i_{ea}(t) + \psi_f \cos(\theta_e(t)) \quad (2)$$

45 Given the geometry of the stator phases,

$$M_{ea} = M_{aa} \cos\left(\frac{2\pi}{3}\right) = -\frac{M_{aa}}{2} \quad (3)$$

46 and incorporating it into Eq. (2),

$$\psi_{ea}(t) = (L_{\sigma_e} + \frac{3M_{aa}}{2})i_{ea}(t) + \psi_f \cos(\theta_e(t)) \quad (4)$$

47 the total flux at phase “a” can be expressed by the equation

$$\psi_{ea}(t) = L_e i_{ea}(t) + \psi_f \cos(\theta_e(t)) \quad (5)$$

48 where L_e is the stator phase inductance: $L_e = L_{\sigma_e} + \frac{3M_{aa}}{2}$.

49

50 Extending Eq. (5) to the remaining stator phases gives

$$\psi_{eb}(t) = L_e i_{eb}(t) + \psi_f \cos(\theta_e(t) - \frac{2\pi}{3}) \quad (6)$$

51

$$\psi_{ec}(t) = L_e i_{ec}(t) + \psi_f \cos(\theta_e(t) + \frac{2\pi}{3}) \quad (7)$$

52 where

53 $\psi_{eb}(t)$ Total instantaneous flux in the stator phase “b”,
 $\psi_{ec}(t)$ Total instantaneous flux in the stator phase “c”.

54 The voltage at the stator windings is

$$\begin{aligned} v_{ea}(t) &= R_e i_{ea}(t) + \frac{d\psi_{ea}(t)}{dt} \\ &= R_e i_{ea}(t) + L_e \frac{di_{ea}(t)}{dt} - \psi_f \omega_e(t) \sin(\theta_e(t)) \end{aligned} \quad (8)$$

55 where

56 R_e Equivalent resistance of one stator phase,
 $\omega_e(t)$ Angular speed of the electrical rotor.

57 Eq. (9) establishes the relationship between the angular speeds of the
58 electrical and mechanical rotors:

$$\frac{d\theta_e(t)}{dt} = \omega_e(t) = p\omega_g(t) \quad (9)$$

59 where p is the number of pole pairs and $\omega_g(t)$ is the angular speed of the
60 mechanical rotor.

61 Extending Eqs. (8) and (9) to the remaining phases and using the vectorial
 62 notation gives

$$\begin{bmatrix} v_{ea}(t) \\ v_{eb}(t) \\ v_{ec}(t) \end{bmatrix} = (R_e + L_e \frac{d}{dt}) \begin{bmatrix} i_{ea}(t) \\ i_{eb}(t) \\ i_{ec}(t) \end{bmatrix} - \psi_f \omega_e(t) \begin{bmatrix} \sin(\theta_e(t)) \\ \sin(\theta_e(t) - \frac{2\pi}{3}) \\ \sin(\theta_e(t) + \frac{2\pi}{3}) \end{bmatrix} \quad (10)$$

63 The parameters R_e and L_e are required in Eq. (10). To obtain R_e , a high-
 64 precision multimeter is used. To take into account the relationship between
 65 the resistance and the temperature, Eq. (11) is used [16].

$$R_{T_i} = R_{T_r} \frac{235 + T_i}{235 + T_r} \quad (11)$$

66 Here,

R_{T_i} Resistance at the desired temperature [Ω],
 R_{T_r} Resistance at room temperature [Ω],
 67 T_r Room temperature [$^{\circ}\text{C}$],
 T_i Desired temperature [$^{\circ}\text{C}$].

68 To obtain L_e , the formulation proposed in [17, 18, 19] is used. The PMSM
 69 model is shown in Fig. 1.

70 Figure 1

71 Figure 2

72 After obtaining the PMSM model, the effects of the inverter and the grid
 73 must be incorporated into the system model. For this purpose, a variable
 74 resistor R will be defined (see Fig. 2). This resistor R will control the power
 75 injected into the grid.

76 To consider the efficiency of the inverter, a study of the inverter losses
77 ΔP_{inv} was conducted (see Fig. 3). Thus, the relationship between continu-
78 ous current in the rectifier i_{dc} and the inverter losses ΔP_{inv} was obtained.
79 Therefore, the grid power P_{grid} can be calculated as

$$P_{grid} = P_{dc} - \Delta P_{inv} = v_{dc} i_{dc} - 16.06 e^{0.149 i_{dc}} \quad (12)$$

80 where P_{dc} is the DC power of the rectifier and v_{dc} is the DC voltage of the
81 rectifier.

82 Figure 3

83 The final system model will be used to analyse the SWT performance
84 when certain parameters are modified, such as the rotation speed n , the
85 capacitance value C and the variable resistor R (see Fig. 2). Therefore, the
86 following values can be obtained:

87 • The grid power P_{grid} and the rectifier DC voltage v_{dc} by combining
88 different values of R , C and n .

89 • The MPCC for different operating conditions.

90 **3. MAXIMUM POWER POINT TRACKER (MPPT)**

91 A SWT converts the kinetic energy of wind into electrical energy. A
92 mechanical torque is produced when air passes through the turbine blades.
93 This torque is used by the PMSM to produce three-phase power at a variable
94 frequency. A back-to-back converter then transforms this AC power into AC
95 power with fixed frequency.

96 The equation that relates the wind speed u_h with the mechanical power
 97 at the wind turbine rotor P_w is

$$P_w = \left(\frac{1}{2} \frac{\pi d^2}{4} \rho u_h^3 \right) C_p(\lambda, \beta) \quad (13)$$

98 where

C_p Power coefficient. The maximum value that can be achieved
 is 16/27, which is known as the Betz limit [20],

P_w Mechanical power at the wind turbine rotor [W],

u_h Wind speed [m/s],

99 d Diameter of the swept area [m],

ρ Air density [kg/m^3],

β Pitch angle [$^\circ$],

λ Tip speed ratio.

100 Hence, the mechanical power obtained by the SWT is a function of u_h
 101 and of certain aerodynamic parameters. Eq. (13) depends on the tip speed
 102 ratio λ , and this parameter can be calculated with Eq. (14),

$$\lambda = \frac{\omega \frac{d}{2}}{u_h} = \frac{\frac{2\pi n}{60} \frac{d}{2}}{u_h} \quad (14)$$

103 where

104 ω SWT angular speed [rad/s],

n SWT rotation speed [rpm].

105 Alternatively, Eq. (13) can be expressed in terms of torque by

$$P_w = T_w \omega \quad (15)$$

106 where T_w is the SWT torque [Nm].

107 For each SWT, there is a single value of λ that provides the highest
 108 value of C_p and, according to Eq. (13), the highest mechanical power P_w .
 109 Thus, according to Eq. (14), each u_h has an optimum value of n that keeps
 110 λ constant. Therefore, the example in Fig. 4 shows the mechanical power
 111 P_w with respect to the SWT rotation speed n . The wider line indicates the
 112 maximum mechanical power at various wind speeds and is known as the
 113 Maximum Power Point Tracker (MPPT) [12].

114 With Eq. (15), the associated mechanical torque T_w at various wind
 115 speeds u_h can also be obtained, which is known as the Torque Point Tracker
 116 (TPT).

117 Figure 4

118 **4. METHODOLOGY TO CALCULATE THE OPTIMAL CAPAC-
 119 ITOR BANK AND MPCC**

120 The flowchart to calculate the optimal capacitor bank and its associated
 121 MPCC is shown in Fig. 5. The steps of the methodology are as follows:

122 **Step 1:** Perform several tests on the SWT. At a minimum, a no-load
 123 test and a rated load test should be conducted.

124 **Step 2:** Define a system model. Adjust the model parameters by com-
 125 paring the simulation results with those obtained in Step 1.

126 Figure 5

127 **Step 3:** Calculate the TPT to extract the maximum mechanical power.
 128 Given Eq. (13),

$$P_w = f(d, \rho, n, u_h, \beta) \Rightarrow \{d, \rho \text{ and } \beta \text{ are constants}\} \Rightarrow P_w = f(n, u_h) \quad (16)$$

129 In this way, the maximum mechanical power $P_{w_{max_i}}$ and its associated
 130 rotation speed n_i for each wind speed u_{h_i} can be obtained by deriving P_w
 131 with respect to n and setting the result to zero.

$$\left. \frac{dP_w}{dn} \right|_{u_{h_i}} = 0 \Rightarrow \left[P_{w_{max_i}}, n_i \right]_{u_{h_i}} \quad (17)$$

132 Using Eq. (15), the mechanical torque $T_{w,P_{w_{max_i}}}$ associated with each
 133 $P_{w_{max_i}}$ can be calculated. Thus,

$$T_{w,P_{w_{max_i}}} \Big|_{u_{h_i}}^{n_i} = \frac{\left. P_{w_{max_i}} \right|_{u_{h_i}}^{n_i}}{\omega_i} \quad (18)$$

134 Note that the TPT must remain unchanged for every value of the variable
 135 resistor R . Therefore, the TPT in 3D can be represented as a surface parallel
 136 to axis R .

137 **Step 4:** Calculate a capacitance range $[C_1-C_2]$ that contains the optimal
 138 capacitor C . Using the Simulink model, perform a simulation that varies R
 139 and C for the rated speed of the SWT (see the example in Fig. 6). The initial
 140 ranges of R and C must be sufficiently wide to effectively analyse the SWT
 141 performance.

142 Figure 6

143 Looking at Fig. 6, it can be seen that by increasing the capacitance C (for
 144 a constant value of R) the power injected to the grid P_{grid} increases, up to
 145 reach a maximum capacitance value C_{max} , from which point P_{grid} decreases.

146 However, the value of C_{max} must be controlled because it could also in-
 147 crease the current i_r flowing into the PMSM windings and thereby cause

₁₄₈ damage.

$$i_r = f(R, C, n) \Rightarrow \{n = n_{rated}\} \quad (19)$$

₁₄₉ Therefore, the winding wire diameter should be known to determine the
₁₅₀ maximum current $i_{r,max}$ that can flow through windings. Once the value of
₁₅₁ $i_{r,max}$ is known, the capacitance range $[C_1 - C_2]$ can be obtained by means of
₁₅₂ drawing a perpendicular surface to the axis i_r (see the example in Fig. 7 and
₁₅₃ Fig. 8).

$$i_r \Big|_{n_{rated}} = f(R, C) \Rightarrow [C_1 - C_2] \Big|_{n_{rated}}^{i_r \leq i_{r,max}} \quad (20)$$

₁₅₄ Figure 7

₁₅₅ Figure 8

₁₅₆ **Step 5:** Obtain MPCCs for several values of C_j : $(C_1 \leq C_j \leq C_2)$. The
₁₅₇ greater the number of values of C_j that are tested, the more accurate the
₁₅₈ results will be. For each value of C_j , the following steps must be performed:

₁₅₉ • Step 5.1: Use the system model to determine the operating surface
₁₆₀ $T_w(R, n)$ of the SWT.

$$T_w = f(R, C, n) \Rightarrow \{C = C_j\} \Rightarrow T_w \Big|_{C_j} = f(R, n) \quad (21)$$

₁₆₁ Then, obtain the intersection of that surface with the TPT (see the
₁₆₂ example in Fig. 9). Thus, the relationship $R(n) \Big|_{P_{w_{max}}^{C=C_j}}$ for maximum
₁₆₃ mechanical power extraction from the wind is obtained (see Fig. 10).

₁₆₄ Figure 9

₁₆₅ Figure 10

- 166 • Step 5.2: Project the optimal relationship $R(n)|_{P_{w_{max}}}^{C=C_j}$ perpendicular to
 167 the R - n plane (see the example in Fig. 11).
- 168 • Step 5.3: Use the model to determine the SWT operating surfaces
 169 $P_{grid}(R, n)$ (see the example in Fig. 11) and $v_{dc}(R, n)$ (see the example
 170 in Fig. 12).

$$P_{grid} = f(R, C, n) \Rightarrow \{C = C_j\} \Rightarrow P_{grid} \Big|_{C_j} = f(R, n) \quad (22)$$

$$v_{dc} = f(R, C, n) \Rightarrow \{C = C_j\} \Rightarrow v_{dc} \Big|_{C_j} = f(R, n) \quad (23)$$

172 Then, obtain the intersection of those SWT surfaces with the surface
 173 obtained in Step 5.2. In this way, the optimal relationships $P_{grid}(n)|_{P_{w_{max}}}^{C=C_j}$
 174 and $v_{dc}(n)|_{P_{w_{max}}}^{C=C_j}$ can be determined.

175 • Step 5.4: Finally, the optimal relationship $P_{grid}(v_{dc})|_{P_{wmax}}^{C=C_j}$, also known
 176 as the MPCC, can be determined easily with the values of $P_{grid}(n)|_{P_{wmax}}^{C=C_j}$
 177 and $v_{dc}(n)|_{P_{wmax}}^{C=C_j}$ (see the example in Fig. 13).

Figure 11

Figure 12

Figure 13

181 The additional MPCCs can be obtained by repeating steps 5.1 to 5.4 for
 182 each value of C_j .

183 **Step 6:** Analyse the results and perform an economic study. To perform
184 the analysis, the well-known Weibull Distribution (WD) can be used [12, 15].

185 This distribution is defined as

$$W(u_h) = \frac{K}{u_h} \left(\frac{u_h}{u_m} \right)^K e^{-\left(\frac{u_h}{u_m} \right)^K} \quad (24)$$

186 where K is the shape parameter of the WD, and u_m is the average wind
187 speed.

188 The National Renewable Energy Laboratory (NREL) defines seven differ-
189 ent Wind Classes (WCs). Each WC is a function of the average wind speed
190 u_m and the height at which the SWT is located (10 or 50 m) with respect to
191 the ground [21]. These WCs are shown in Table 1.

192 The Weibull Distributions, Eq. (24), for a value of $K=1.9$ and for the
193 average wind speeds listed in Table 1 (assuming a height of 10 m) are shown
194 in Fig. 14.

195 Table 1

196 Figure 14

197 To compare the results obtained with different values of C_j , four param-
198 eters will be used: $E(u_{h_i})$, E_{tot} , $\Delta\eta\Big|_{C_j}$ and $\Delta\$|_{C_j}$.

199 • $E(u_{h_i})$ is the annual grid energy for each wind speed u_{h_i} and is defined
200 as

$$E(u_{h_i}) = 8760 \left[\frac{\text{hours}}{\text{year}} \right] W(u_{h_i}) P_{grid}(u_{h_i}) \quad (25)$$

201 where $P_{grid}(u_{h_i})$ is the SWT grid power for each wind speed u_{h_i} .

202 • E_{tot} is the annual total energy that can be produced by the SWT. It is
203 obtained from the following equation:

$$E_{tot} = \sum_{i=0}^n E(u_{h_i}) \quad (26)$$

204 • $\Delta\eta|_{C_j}$ represents the annual total energy increment injected into the
 205 grid for different capacitor banks ($C=C_j$) compared to the case without
 206 a capacitor bank ($C=0$).

$$\Delta\eta \Big|_{C_j} = \frac{E_{tot,C=C_j} - E_{tot,C=0}}{E_{tot,C=0}} \quad (27)$$

207 • $\Delta\$|_{C_j}$ represents the annual financial benefit obtained by using differ-
 208 ent capacitor banks ($C=C_j$) compared to the case without a capacitor
 209 bank ($C=0$). In the calculations, an energy cost of 0.2484 US\$/kWh is
 210 assumed (including electricity taxes).

$$\Delta \$ \Big|_{C_i} = (E_{tot,C=C_j} - E_{tot,C=0}) 0.2484 \left[\frac{US\$}{kWh} \right] \quad (28)$$

211 5. SET-UP FACILITY

212 The architecture of the set-up facility is depicted in Fig. 15. The main
213 elements are as follows:

Figure 15

- DC motor to simulate wind effects (5 H.P. and 200 V),
- PMSM of the SWT,
- Rectifier (AC \Rightarrow DC),
- Inverter (DC \Rightarrow AC),
- Six-channel wattmeter,

- Power quality analyser,
- Torque and speed sensors,
- Multimeter to measure the PMSM phase resistance,
- Temperature sensors in windings,
- PC to control the wattmeter, the temperature sensors and the inverter.

225 Fig. 16 shows the connection of the different devices.

Figure 16

227 6. RESULTS

228 This methodology and the set-up facility were used to optimise a com-
229 mercial SWT whose rated parameters are as follows:

- Number of blades: 3,
- Power coefficient: 0.36,
- Diameter: 4.2 m,
- Rated speed of the wind: 11 m/s,
- Rated speed: 250 rpm,
- PMSM rated power: 3500 W,
- Pole pairs: 10.

237 The results obtained were as follows:

238 **Steps 1 and 2:** Various tests were performed, and the results were com-
239 pared with the simulated results. Fig. 17 represents the continuous voltage
240 v_{dc} and the PMSM phase voltage v_r versus the rotation speed n . The simu-
241 lated results show a high degree of agreement with the real values.

242 Figure 17

243 **Step 3:** The TPT was calculated. Table 2 shows the results obtained
244 when the methodology was applied. These results are for the case when no
245 capacitor bank was connected ($C=0$).

246 Table 2

247 **Step 4:** The capacitance range was obtained. If the winding wire diam-
248 eter is 0.8 mm, the maximum allowed current is approximately 11 A. Thus,
249 the capacitance C is calculated to be between $C_1=0$ and $C_2=25 \mu\text{F}$, for every
250 value of R . The capacitances tested were 0, 4, 6, 8, 10, 12, 14, 16, 20, and
251 $24 \mu\text{F}$.

252 **Step 5:** MPCCs for several values of C were calculated. The simulated
253 results without the use of capacitor banks ($C=0$) are shown in Table 2.

254 The remaining MPCCs were determined by repeating this step for the
255 different capacitances. These curves are shown in Fig. 18.

256 Table 3 presents the values of P_{grid} that were obtained for different wind
257 speeds u_{h_i} for each capacitance C_j . This table also shows the capacitance
258 value that produces the highest P_{grid} for each wind speed u_{h_i} (highlighted
259 values).

260

Figure 18

261

Table 3

262 **Step 6:** An analysis of the results and an economic study were performed.

263 Using the WD Eq. (24) and assuming WC=2 (height 10 m) lead to the results
 264 shown in Table 4. This table shows $E(u_{h_i})$, E_{tot} , $\Delta\eta|_{C_j}$ and $\Delta\$|_{C_j}$.

265 As Table 4 shows, the highest benefit (\$41.51 USD) is produced by using a
 266 capacitance of $12 \mu\text{F}$ (4529 kWh). Given that capacitors cost approximately
 267 \$14 USD, the total costs are covered in the first year. Thus, from the second
 268 year on, the capacitor bank would begin to make a profit.

269

Table 4

270 The analysis of the system financial benefits for the remaining WCs
 271 (height=10 m) led to the results shown in Table 5.

272

Table 5

273 Table 5 shows that the investment can be recovered during the first year
 274 of use for WC values equal to or greater than 3. The value of the optimal
 275 capacitance varies for each WC. Hence, it is necessary to know the location
 276 of the SWT to choose the capacitor bank that yields the maximum efficiency.

277 Table 6 shows the efficiency increase produced when using different ca-
 278 pacitor banks and WCs. For WC=2 and height=10 m, the efficiency increase
 279 was 3.83%.

280

Table 6

281 Once the simulated results were obtained, an experiment was conducted
282 to verify them. The test consisted of connecting a capacitor bank of $12 \mu\text{F}$
283 (delta connection) to the SWT. The simulated and experimental results are
284 shown in Fig. 19.

285 Figure 19

286 7. CONCLUSIONS

287 The increasing development of small-wind energy systems has made it
288 necessary to study new methods to improve the efficiency of this technology.
289 This paper presents a methodology to increase the SWT efficiency by con-
290 necting a capacitor bank. This methodology involves the use of a Simulink
291 model to simulate the system.

292 The results obtained with the Simulink model are suitable because they
293 show a high degree of agreement with the performance of the real machine.
294 As an example, the efficiency increases predicted by the simulations were the
295 same as those obtained in the real tests. Given the accuracy of the Simulink
296 model, all modifications can be extrapolated to the SWT.

297 It has been shown that connecting different capacitor banks to the system
298 modifies the SWT performance and the MPCC. The power obtained from
299 the SWT increases as a result of connecting capacitor banks and is only
300 limited by the maximum current that can flow through the PMSM windings.
301 To connect the optimal capacitor bank, the precise location of the SWT and
302 the associated average wind speed must be known. In this study, we found
303 that connecting a capacitor bank of $12 \mu\text{F}$ increases the SWT efficiency by

304 3.83%. Thus, in the PMSM design, an added shunt capacitance could be a
305 new parameter to bear in mind.

306 The methodology described in this paper could be used by electrical ma-
307 chine manufacturers to reduce the PMSM size. If the 3.5 kW PMSM is
308 redesigned using a capacitor bank, this new electrical machine will contain
309 less copper (the capacitor bank decreases the current that flows through the
310 PMSM), a smaller magnet size (due to magnetising effect of the capacitors)
311 and fewer magnetic plates to obtain the same electrical power P_{grid} . If these
312 modifications are included in the SWT design, the manufacturing costs would
313 decrease substantially, especially in the case of higher WCs.

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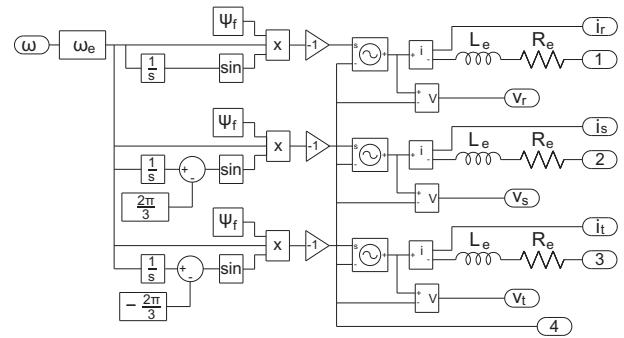


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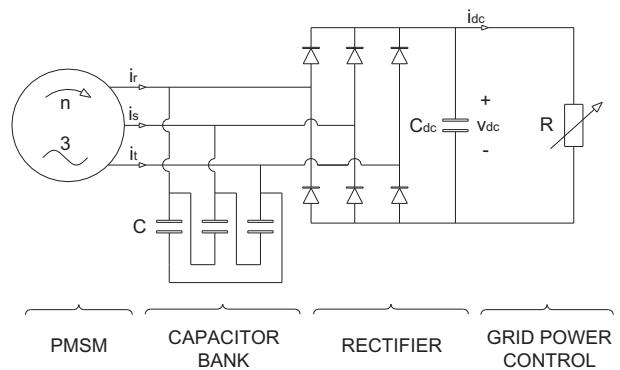


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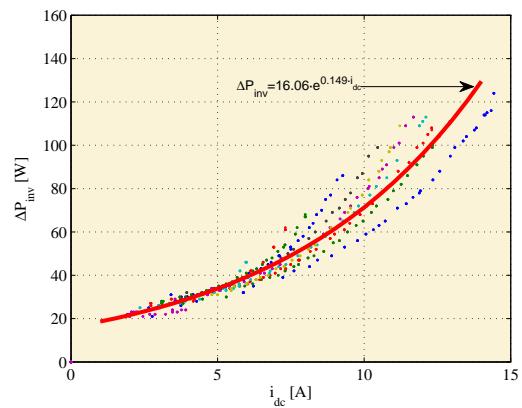


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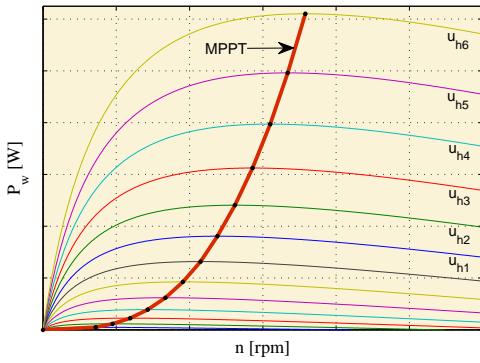


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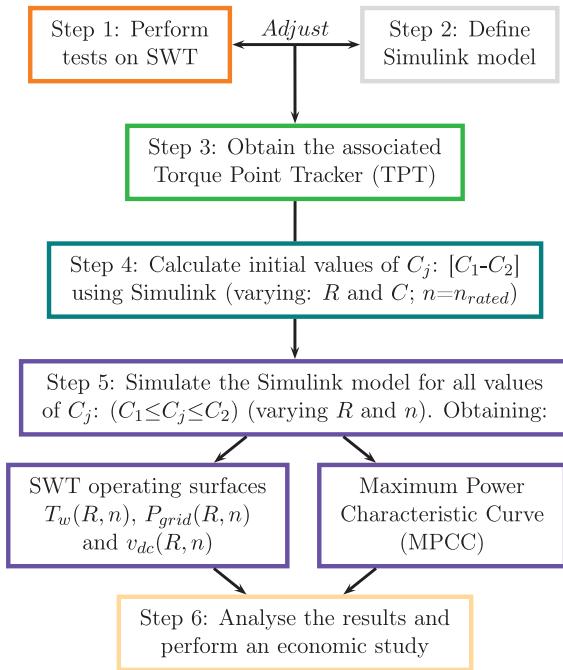


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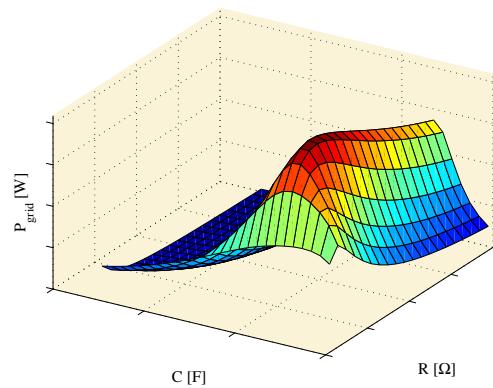


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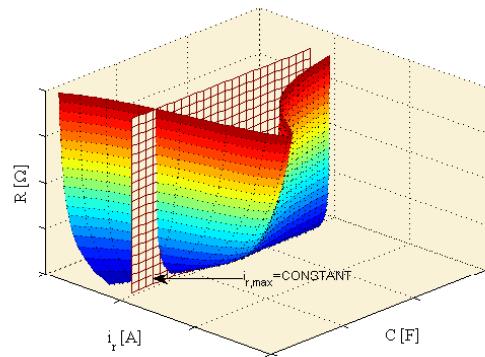


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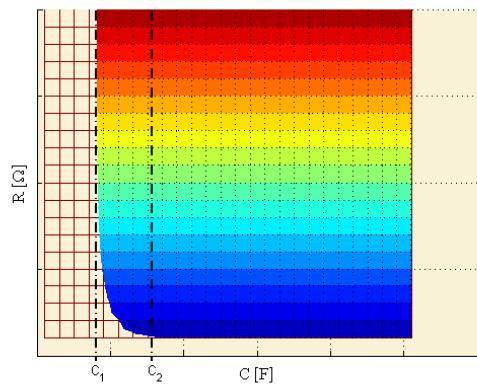


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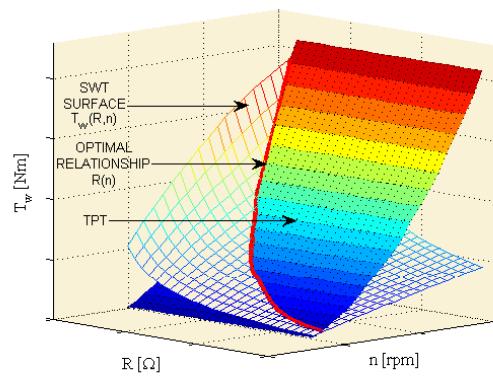


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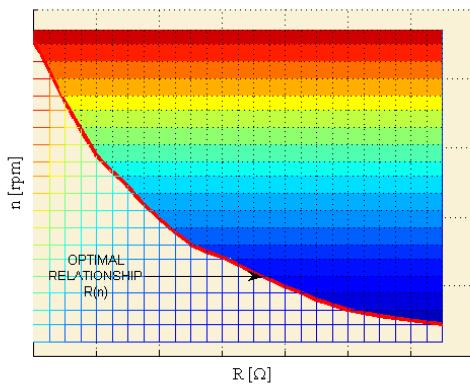


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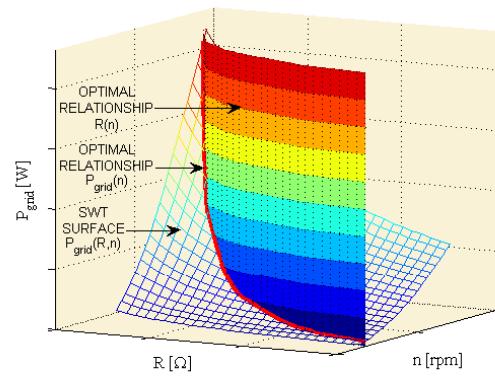


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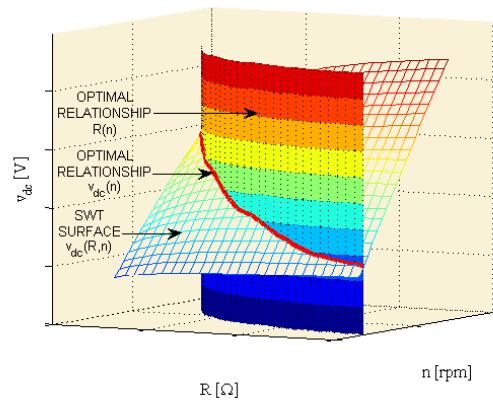


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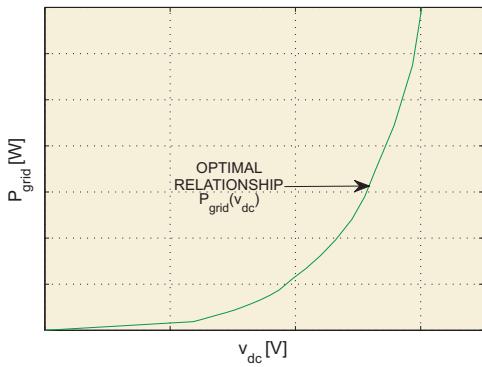


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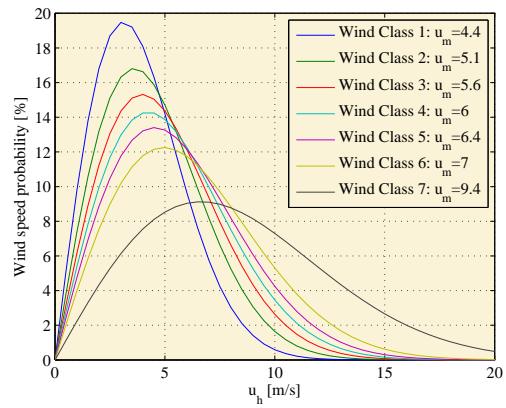


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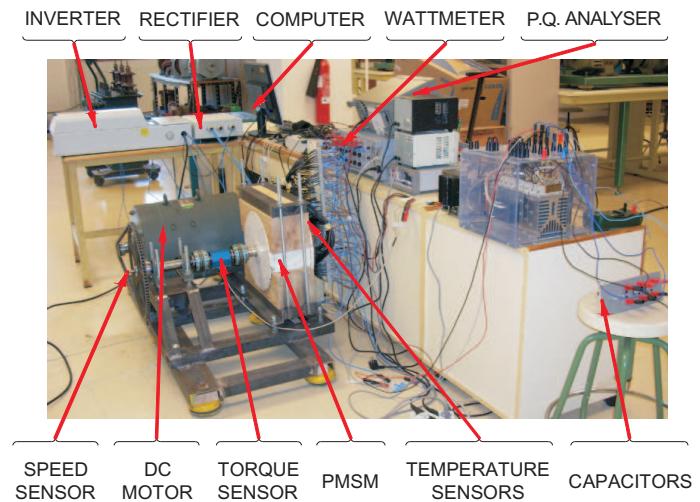


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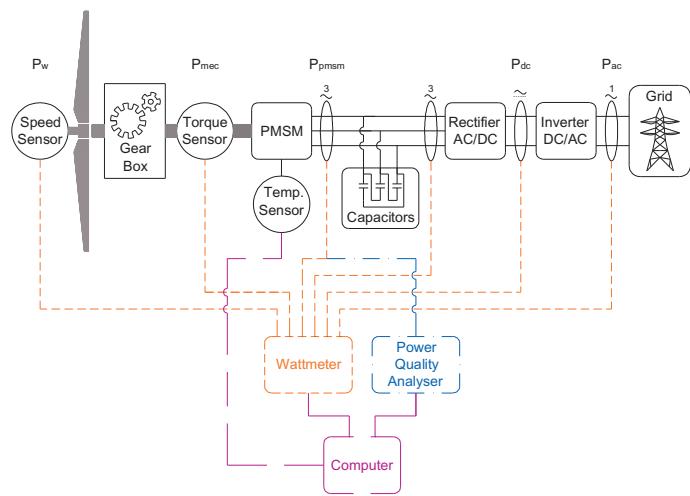


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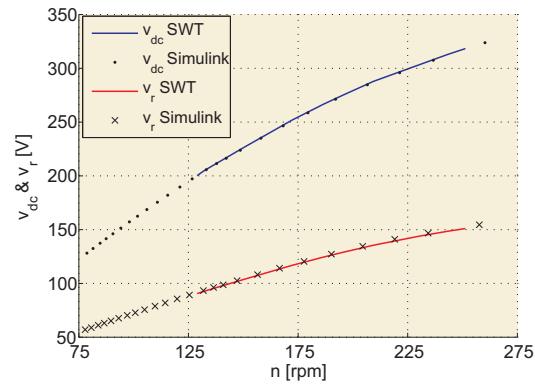


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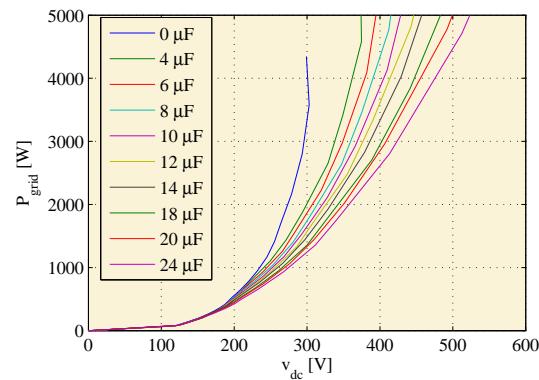


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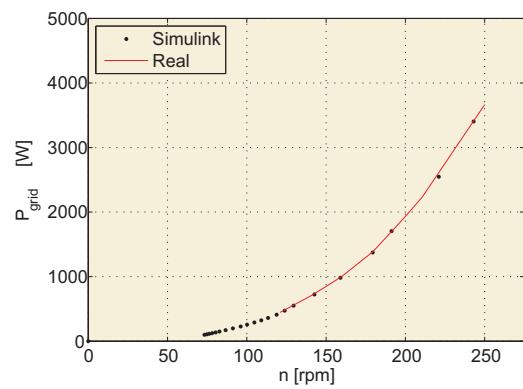


Figure 19:

β	Pitch angle [°].
ΔP_{inv}	Inverter losses [W].
$\Delta\eta$	Increment of E_{tot} using different capacitor banks [%].
$\Delta \$$	Annual benefit [\$].
λ	Tip speed ratio.
C	Capacitance of the capacitor banks [F].
C_{dc}	DC bus capacitor [F].
C_p	Power coefficient.
d	Diameter of turbine blades [m].
E_{tot}	Annual total energy extracted from the SWT [kWh].
$E(u_h)$	Annual grid energy for each wind speed [kWh].
i_{dc}	DC Rectifier current [A].
i_r, i_s, i_t	Phase RMS current of the PMSM [A].
n	SWT rotation speed [rpm].
P_{dc}	Rectifier DC power [W].
P_{grid}	Grid power [W].
P_w	Mechanical power at the wind turbine rotor [W].
R	Variable resistor to simulate inverter and grid effects [Ω].
T_w	Torque in SWT shaft [Nm].
u_h	Wind speed [m/s].
v_{dc}	DC Rectifier voltage [V].
v_r, v_s, v_t	Phase RMS voltage of the PMSM [V].
$W(u_h)$	Weibull distribution.

Figure 20: Glossary

Table 1:

Wind	u_m	u_m
Class	(10 m)	(50 m)
1	0 - 4.4	0 - 5.6
2	4.4 - 5.1	5.6 - 6.4
3	5.1 - 5.6	6.4 - 7
4	5.6 - 6	7 - 7.5
5	6 - 6.4	7.5 - 8
6	6.4 - 7	8 - 8.8
7	7 - 9.4	8.8 - 11.9

Table 2:

		Step 3		Step 5	
u_{hi}	n	T_w	R	P_{grid}	v_{dc}
11.9	276.9	194.5	20	4346.1	299.2
10.1	233.2	138.8	30	2804.7	293.5
9.2	213.3	115.4	35	2170.3	278.8
8.4	194.9	97.7	40	1702.6	264.1
7.4	172.2	74.6	50	1171.0	245.1
6.0	139.2	50.2	65	644.0	208.3
5.0	116.0	33.9	85	361.2	180.1
4.1	95.0	23.3	105	197.2	150.9
3.1	72.7	13.0	150	75.1	118.5

Table 3:

$u_{hi} [m/s]$	$P_{grid} [W]$					
	$0\mu F$	$6\mu F$	$8\mu F$	$10\mu F$	$12\mu F$	$14\mu F$
5.0	360	361	360	358	358	359
6.0	643	645	651	648	647	650
7.0	995	1010	1008	1014	1010	1007
8.0	1464	1503	1496	1504	1500	1492
9.0	2054	2122	2141	2120	2143	2135
10.0	2763	2888	2877	2888	2900	2870
11.0	3572	3784	3817	3807	3846	3804
12.0	4401	4834	4822	4895	4855	4922

Table 4:

u_{h_i}	$E(u_{h_i})$ [kWh]					
	0 μF	6 μF	8 μF	10 μF	12 μF	14 μF
5	441.1	442.5	441.0	438.9	438.9	440.2
6	622.5	624.6	629.3	627.6	626.3	629.0
7	696.4	707.0	705.6	709.2	706.9	704.7
8	682.1	699.5	697.0	700.1	698.5	695.1
9	589.7	609.1	614.6	608.6	614.9	612.8
10	454.4	474.8	473.0	474.8	476.7	471.8
11	313.4	332.1	335.0	334.1	337.4	333.8
12	174.2	211.5	211.0	214.1	212.4	215.3
E_{tot}	4362	4517	4523	4526	4529	4522
$\Delta\eta \Rightarrow$		3.56	3.68	3.77	3.83	3.65
$\Delta\$ \Rightarrow$		38.65	39.92	40.82	41.51	39.56

Table 5:

WC	$\Delta\$$ (Height = 10 m)				
	$6\mu F$	$8\mu F$	$10\mu F$	$12\mu F$	$14\mu F$
1	15.4	15.9	16.1	15.8	15.1
2	38.6	39.9	40.9	41.5	39.7
3	66.2	67.1	69.3	70.4	68.3
4	94.0	94.3	98.2	99.3	97.5
5	124.2	125.5	131.3	132.2	131.1
6	175.1	176.4	185.6	185.8	186.3
7	356.8	357.3	380.2	375.4	385.5

Table 6:

WC	$\Delta\eta$ [%] (Height = 10 m)				
	$6\mu F$	$8\mu F$	$10\mu F$	$12\mu F$	$14\mu F$
1	2.21	2.25	2.25	2.26	2.13
2	3.62	3.68	3.77	3.83	3.67
3	4.76	4.83	4.99	5.06	4.92
4	5.67	5.74	5.97	6.03	5.93
5	6.54	6.61	6.91	6.95	6.90
6	7.73	7.79	8.19	8.20	8.22
7	10.96	10.97	11.68	11.53	11.84

Figure Captions

FIGURES

Figure 1: PMSM schematic generated using Matlab.

Figure 2: Wind power generation system.

Figure 3: DC current of the rectifier i_{dc} vs. inverter losses ΔP_{inv} .

Figure 4: Turbine rotation speed n vs. mechanical power P_w for different wind speeds u_h .

Figure 5: Flowchart of the methodology.

Figure 6: Grid power P_{grid} vs. resistance R and capacitance C for the rated speed n_{rated} .

Figure 7: Winding current i_r vs. resistance R and capacitance C .

Figure 8: Relationship $R(C)$ not exceeding the restriction of $i_{r,max}$ for the rated speed of the SWT.

Figure 9: Intersection between the TPT and the operating surface $T_w(R,n)$ of the SWT.

Figure 10: Optimal relationship $R(n)|_{P_{wmax}}$.

Figure 11: Intersection between the optimal relationship $R(n)$ and the operating surface $P_{grid}(R,n)$ of the SWT.

Figure 12: Intersection between the optimal relationship $R(n)$ and the operating surface $v_{dc}(R,n)$ of the SWT.

Figure 13: Maximum Power Characteristic Curve (MPCC): Grid power P_{grid} vs. rectifier DC voltage v_{dc} .

Figure 14: Weibull distribution for different WCs at a height of 10 m.

Figure 15: System: PMSM + Rectifier + Inverter + Capacitor bank.

Figure 16: SWT block diagram.

Figure 17: Real and simulated results.

Figure 18: MPCCs obtained with different capacitor banks.

Figure 19: Simulated and experimental results ($C=12\mu F$).

Figure 20: Glossary.

TABLES

Table 1: Wind classification according to average speed. Adapted from [21].

Table 2: Results of steps 3 and 5 without a capacitor bank.

Table 3: Grid power improvement achieved by connecting capacitor banks (delta connection).

Table 4: Annual benefits produced by placing different capacitors at a location with WC=2, Height = 10 m.

Table 5: Annual financial benefits produced by using different capacitors for all WCs at a height of 10 m.

Table 6: Efficiency increase produced by using different capacitors for all WCs at a height of 10 m.