# The kink effect of the nuclear charge radii in some isotopic chains and the nucleon-nucleon tensor force within nonlinear relativistic models in the Hartree-Fock approximation.

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**Abstract.** Relativistic nonlinear models in the Hartree-Fock approach, with  $\sigma$ ,  $\omega$ ,  $\rho$  and  $\pi$  mesons, are used to explore the influence of the nucleon-nucleon tensor force (*tf*) on the behaviour of the nuclear charge radii of the Pb isotopic chain. It is found that a part of its effect on the charge radii is channelled through its opposite effect to the spin-orbit interaction. The kink effect seems to be produced by the combination of the geometrical features of the 1*i*<sub>11/2</sub> neutron orbital and its binding energy.

## 1 Introduction

The marked change in trend of the evolution of the charge radii of some isotopic families of nuclei versus the mass number A is known as kink effect (KE). This is a consequence of the shell structure of nuclei. The fact that the density-dependent Hartree-Fock model with standard Skyrme functionals [1] or Gogny forces [2] were not able to reproduce this effect for the Pb isotopic chain, for example, whereas the relativistic models in the simple mean-field approach did reasonably well [3, 4], has increased the interest in understanding the mechanism responsible for the KE [5-8]. However, it seems that the full theoretical understanding has not been reached yet.

The aim of this work is to consider a nuclear relativistic Lagrangian and use the Hartree-Fock approximation, without pairing correlations, to explore the influence of the NN tensor force (tf) on the behaviour of the nuclear charge radii of the Pb isotopic chain.

## 2 The model

The effective Lagrangian density *L* includes the nucleons and the  $\sigma$ ,  $\omega$ ,  $\rho$  and  $\pi$  meson fields ( $\varphi$ ,  $\sigma$ ,  $\omega_{\mu}$ ,  $\rho_{\mu}$  and  $\pi$ , respectively), and the electromagnetic field  $A_{\mu}$  [9]:

$$L = L_0 (\varphi, \sigma, \omega_\mu, \rho_\mu, \pi, A_\mu) + L_{\text{int.}} - U_{\text{NL.}}$$
(1)  
 $L_0$  describes the free system and

$$\begin{split} L_{int.} &= -g_{\sigma}\bar{\varphi}\sigma\varphi - g_{\omega}\bar{\varphi}\gamma^{\mu}\omega_{\mu}\varphi - g_{\rho}\bar{\varphi}\gamma^{\mu}\rho_{\mu}\cdot\tau\varphi - \\ & \frac{f_{\pi}}{m_{-}}\bar{\varphi}\gamma_{5}\gamma^{\mu}\partial_{\mu}\pi\cdot\tau\varphi - e\bar{\varphi}\gamma^{\mu}\frac{1+\tau_{3}}{2}A_{\mu}\varphi, \end{split}$$
(2)

represents the interaction of nucleons with the boson fields. The nonlinear potential energy density

$$U_{NL} = \frac{1}{3} \bar{b} M (g_{\sigma} \sigma)^3 + \frac{1}{4} \bar{c} (g_{\sigma} \sigma)^4$$
(3)

takes into account the  $\sigma$ -meson self-interactions.  $g_{\sigma}, g_{\omega}, g_{\rho}$  and  $f_{\pi}$  are the  $\sigma, \omega, \rho$  and  $\pi$  meson-nucleon coupling constants, respectively.  $\tau$  is the usual isovector operator,

 $\overline{b}$  and  $\overline{c}$  are dimensionless parameters.  $\frac{f_{\pi}^2}{2\pi} \cong 0.08$ . The quantities  $m_{\sigma}$ ,  $g_{\sigma}$ ,  $g_{\omega}$ ,  $g_{\rho}$ ,  $\overline{b}$  and  $\overline{c}$  are adjusted to nuclear ground state experimental data [9], see Table 1.

**Table 1.** Columns 3-8 give the fitted parameters of the model without ( $\eta$ =0) and with ( $\eta$ =1) tensor force contribution.  $m_{\sigma}$  is the  $\sigma$  meson mass, whereas  $g_{\sigma}$ ,  $g_{\omega}$ ,  $g_{\rho}$ ,  $\bar{b}$  and  $\bar{c}$  are the dimensionless parameters entering eqs. (2) and (3). The last 4 columns give the nuclear matter saturation values for the nuclear density ( $\rho_0$ ), the binding energy per particle (-E/A), the symmetry energy coefficient ( $a_4$ ) and the compressibility modulus (K).

Model	η	$m_{\sigma}$ ,	$oldsymbol{g}_{\sigma}$	gω	$g_ ho$	$\overline{b}$
		MeV				$\times 10^{3}$
HF0	0	441.7	5.015	9.510	0.67	-5.26
HF1	1	443.3	5.322	10.39	0.72	-4.36

Model	Ē	ρ₀,	E/A,	<i>a</i> 4,	К,
	$\times 10^{3}$	fm <sup>-3</sup>	MeV	MeV	MeV
HF0	-8.95	0.146	-16.3	36.8	294
HF1	-7.26	0.149	-16.3	35.5	285

## 3 Results

### 3.1 Effect of the tensor force

Fig. 1 shows the neutron single-particle energies of <sup>208</sup>Pb. It can be seen that the *tf* reduce the spin-orbit splittings. Fig. 2 shows the charge radius isotope shift  $\Delta \langle r_c^2(Pb) \rangle = \langle r_c^2({}^{A}Pb) \rangle - \langle r_c^2({}^{208}Pb) \rangle$ . The *tf* reduces the difference between the radii of the two spin-orbit partners but it has little influence on  $\Delta \langle r_c^2 \rangle$  both for the i-conf. (...1 $i_{11/2}^{N-126}$ ), where the KE is observable, and for the g-conf. (...2 $g_{9/2}^{N-126}$ ), where it is not.

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**Figure 1**: Neutron sp energies of <sup>208</sup>Pb for models HF0 ( $\eta$ =0) and HF1 ( $\eta$ =1), and Experiment data



**Figure 3**: The sp r.m.s. charge radii of the proton nodeless orbitals of the Pb isotopic chain for the HF0 ( $\eta$ =0) and HF1 ( $\eta$ =1) models and *i*-conf.

Figs. 3 and 4 show the charge radii of the proton nodeless orbitals of the Pb isotopic chain. The effect of the *tf* on  $\langle r_{ci} \rangle$  for the two orbitals of a spin-orbit doublet is opposite to each other and decreases as *l* increases. The effect of the *tf* is observable and it seems (only) qualitatively equivalent to that of reducing the spin-orbit interaction.

# 4 Conclusions

The effect of the tensor force (tf) on the nuclear charge radii of the nodeless proton orbitals of the Pb isotopic chain is opposite to that of the spin-orbit interaction and it decreases as *l* increases. For orbitals with nodes, the effect is more involved and less important. The global effect of the *tf* on the nuclei charge radii is very small.



**Figure 2**: Charge radius isotope shift  $\Delta \langle r_c^2 \rangle$  for the Pb isotopic chain with respect to that of the <sup>208</sup>Pb for models HF0 ( $\eta$ =0) and HF1 ( $\eta$  =1) and for the neutron *i*- and *q*-confs.



Figure 4: The same as Fig. 3, but for the g-conf.

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