Effects of modified schema-based instruction to teach students with autism to solve additive compare problems

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Abstract

This study assessed the effectiveness of a modified schema-based instruction (MSBI) in teaching students with autism spectrum disorder (ASD) how to solve additive compare word problems. The research was conducted in Spain, involving three students with ASD, aged 6, 7, and 9. The research design used was a single-case, multiple-baseline across-students design. The results showed a functional relation between a MSBI and the improvement in the performance of the three participants when solving additive compare problems of type difference amount unknown and compare amount unknown. The improvement was maintained for 4 to 6 weeks following the completion of the intervention. Moreover, two students were able to apply their skills to reference-amount-unknown compare problems, demonstrating successful generalization. However, their generalization to problems involving two operations was somewhat limited by difficulties with one of the operations. Social validity data collected from the students' families and teachers indicated that both groups had positive views on the use of the MSBI for mathematical word problem-solving skills.

Keywords: Modified schema-based instruction, ASD, additive compare problem solving.

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There is a growing interest in investigating the mathematical learning of students with autism spectrum disorder (ASD; e.g., Rockwell et al., 2011; Root, 2019; Cox & Root, 2019). Research on mathematical problem solving with students with ASD has proven to be particularly relevant (Root et al., 2021). Mastering the ability to solve word problems is a crucial part of the curriculum standards in many countries (e.g., National Council of Teachers of Mathematics (NCTM), 2000; Organic Law on Education (LOMLOE), 2020), as it helps students establish connections between mathematics and real-life situations (Bouck et al., 2014). However, since problem-solving entails the utilization of skills beyond basic arithmetic calculations, students with ASD may encounter challenges in this domain owing to specific characteristics associated with the disorder (Bae et al., 2015; Polo-Blanco, Suárez-Pinilla et al., 2024). Specifically, difficulties with communication and language may hinder comprehension of problem statements (Bae et al., 2015), and deficits in executive functioning (such as planning, organization or working memory) may interfere with the problem-solving process (Oswald et al., 2016).

Research conducted on additive word problems with typically developing students has demonstrated that change and combine problems tend to be less challenging than compare problems (Riley et al., 1983). The present study focuses on additive compare word problems, which involve comparing two quantities to establish whether one “has more” or “has less”. These compare problems are categorized into three types, distinguished by the location of the unknown (see Table 1): (1) difference amount unknown (DAU), (2) compare amount unknown (CAU), and (3) reference amount unknown (RAU). In compare problems, the location of the unknown is a factor in the problem's difficulty for students of typical development, with DAU being the least challenging, followed by CAU and then RAU (Riley et al., 1983).

Schema-based Instruction

Considering the relative scarcity of empirical research supporting effective teaching models specifically tailored to students with ASD, as compared to the more extensive body of research for broader groups, such as students with learning disabilities, Spooner et al. (2019) proposed that adapting methodologies proven effective for high-incidence disabilities could provide a promising basis for the establishment of evidence-based practices. The schema-based instruction (SBI) is an instructional method developed to improve problem-solving skills, and it is considered an evidence-based practice for students with learning disabilities, having been demonstrated successful application in this context (e.g., Cook et al., 2020). Specifically, SBI promotes mathematical reasoning by providing visual representations to help students identify the problem's structure and the relation between quantities, as well as heuristics or “attack strategies” (lists of steps the student learns to follow to facilitate problem-solving; Powell & Fuchs, 2018), and explicit instruction on the schema-based problem-solving method (Clausen et al., 2021).

Some researchers applied SBI with students with ASD (e.g., focusing on additive problems, Kasap & Ergenekon, 2017; Rockwell et al., 2011), considering that such a cognitive and behavioral approach could help compensate for some of the deficits in executive functions and language impairments typical of the disorder. In 2011, Rockwell and colleagues employed a SBI approach with a 10-year-old female student diagnosed with ASD to enhance her ability to solve three types of additive problems. The study utilized a single-case multiple probe across behaviors design, with each problem type (change, group and compare) as a distinct behavior. The results showed that the participant’s ability to solve all types of addition and subtraction word problems with unknown in the final position improved following instruction, and that she was able to generalize the acquired abilities to problems with unknowns in the initial and middle position, and maintained them over time. Kasap and Ergenekon (2017) successfully utilized a SBI approach to enhance the problem-solving skills of three students with ASD, aged 9, 11, and 14, in solving compare CAU problems. Two of the students were able to generalize their acquired skills to solve DAU problems as well. The study utilized a single-case, multiple-probe across students as the research method. Root et al. (2021) established SBI as an evidence-based practice for students with ASD, although the authors warned that they considered interventions with multi-component treatment packages in which it is not possible to discriminate the specific contribution of each component.

Modified Schema-based Instruction

In the last decade, researchers have proposed a modified SBI approach (MSBI) to better cater to the specific learning needs of students with moderate or severe disabilities (Clausen et al. 2021; Spooner et al., 2017). MSBI incorporates adaptations to the SBI, particularly for students who exhibit procedural and conceptual learning gaps, and weaknesses in executive functions. According to Spooner et al. (2017), MSBI teaches both procedural and conceptual knowledge, supplementing SBI with established evidence-based practices for teaching problem-solving skills to students with severe disabilities. The aim of MSBI is to ensure that students can: (a) access the problem, (b) comprehend and model the problem conceptually, (c) solve the problem procedurally, and (d) generalize their problem-solving skills. To achieve the first component, authors compiled strategies from previous studies, including structuring the problem in an accessible manner, carefully writing problem statements, and conducting interactive read-alouds of mathematical story problems. To address the conceptual dimension, MSBI considers pre-made schematic diagrams, visual aids, hand motions with a problem type rule to make connections with each schema and enhance metacognition, and task analyses to promote self-instruction. For example, task analyses replace heuristics in SBI. Instead of relying on a mnemonic like 'RUNS' (Read the problem, Use a diagram, Number sentence, and State the answer; Rockwell et al., 2011), a task analysis serves as the guiding heuristic and supports the self-instruction process. This is presented in a student-friendly format known as the 'self-instruction checklist' and includes pictures paired with each step, aiming to assist early readers (Spooner et al., 2017). MSBI also incorporates the use of manipulatives, adapted from SBI research, to visually represent quantities in the problem in the organizer, aiming to deepen students' conceptual comprehension of the problem (Spooner et al., 2017). Adaptations to enhance procedural knowledge include explicit instruction (e.g., model-lead-test) and systematic instruction (e.g., system of least prompts) with error correction. Additionally, generalization of skills can be assessed by solving arithmetic problems of different types than those used during instruction or by applying problem-solving skills to different contexts. Specifically, some of these measures are relevant for students with ASD, as they may encounter challenges comprehending the problem or following the steps necessary to solve it.

In the context of additive problems, Root et al. (2017) examined the effects of MSBI on the word problem-solving skills of three students with ASD and moderate disabilities (aged 7, 9 and 11) with a focus on DAU compare word problems, and comparing the differential effects of concrete versus virtual manipulatives. Results of the multiple probes across students with an embedded alternating treatments design showed a functional relation between the instruction and the students’ ability to successfully solve DAU compare problems. Likewise, Polo-Blanco, González et al. (2024) assessed the effectiveness of a MSBI approach to improve the change problem-solving performance of three students with mild intellectual disabilities (aged 13, 14 and 17), two of them with ASD. Following the intervention, the three students improved their performance when solving change word problems with unknowns in all three locations, and were able to generalize their solving skills to deal with two-step addition and subtraction change word problems.

Objectives of the Study

Previous studies (e.g. Kasap & Ergenekon, 2017 and Root et al., 2017) have focused on addressing a single type of compare problem in instructional settings (CAU and DAU respectively in those works) based on the location of the unknown. The work by Kasap and Engenekon (2017) also evaluated the extent of generalization to another unknown location (DAU). In this paper, we aim to broaden the scope by including two specific types of compare problems in the instruction (namely, DAU and CAU) and by evaluating the generalizations of these skills to RAU problems. The latter is generally considered more complex for typically developing students (Riley et al., 1983). Moreover, with the goal of enhancing our understanding of the application of MSBI for students with high support needs, we also sought to expand the range of participants. Specifically, in this study we included students with ASD facing challenges in mathematical problem-solving who exhibited a higher level of functioning than participants in prior studies (e.g., IQ range 40-60, as considered in Browder et al., 2018, and Root et al., 2017).

The following research questions were considered: (1) Is there a functional relation between the use of an MSBI approach and the acquisition of mathematics skills measured by the accuracy of solving DAU, CAU and mixed (i.e. DAU and CAU) problems for students with ASD? (2) Do students with ASD generalize the acquired skills to solving RAU compare problems and two-step problems (combine and compare)? And (3) Do students with ASD maintain the acquired skills over time?

Method

Participants

Three Spanish elementary students diagnosed with ASD participated in this study. They were participants of a larger project focused on the mathematical learning of children with autism, aimed to both describe the mathematical skills of students with ASD and investigate their connection with cognitive variables (Polo-Blanco, Suárez-Pinilla et al., 2024). Participants of this project were selected from different health, social and educational resources from the autonomous community of Cantabria (Spain).

The present study sought students who met the following criteria: (1) being diagnosed with ASD; (2) experiencing difficulty with solving mathematical word problems, as reported by their tutors and/or families; and (3) having a mathematical age of 5.5 years or higher, as measured by the *Test of Early Mathematics Ability* (TEMA-3; Ginsburg & Baroody, 2007). This criterion was set to ensure that students possessed the necessary foundational knowledge of addition and subtraction, including counting strategies, which are required to effectively engage in problem solving activities. The three students (James, Sam and Alex, pseudonyms) were enrolled in the same inclusive public school and were Spanish-speakers. Sam and Alex attended the same classroom. Based on to their scores on the Childhood Autism Rating Scale (Schopler et al., 1988), administered by the school's educational counselor, they were in the moderate autism range. The diagnoses were confirmed by a psychologist from the research team through a clinical evaluation of the child, a review of their clinical reports, and a detailed medical history with the child's parents to confirm current or past symptoms of ASD. Once the inclusion criteria for the three students were verified, they were evaluated using a pre-test created for this research that included eight one-operation additive problems: three change problems, three compare problems and two combination problems. As the three students struggled the most with solving compare problems, an instructional sequence was developed to specifically target this type of problem structure including variations of the unknown to enhance their problem-solving abilities.

James was a 9-year-old Spanish male enrolled in 4th grade. He was diagnosed with ASD at the age of 6, and with ADHD at the age of 7, both by a child psychiatrist. His IQ was measured at 88 (WISC-V; Wechsler, 2015). James attended a general math class but his curriculum was tailored to 2nd-grade level. His age equivalency in math was 6 years and 10 months according to TEMA-3 (Ginsburg & Baroody, 2007). James received individualized support from specialists for six hours a week at school.

Sam was a 6-year old Spanish male enrolled in 2nd grade. He was diagnosed with ASD at the age of 4 years and 10 months by a child psychiatrist. His IQ was measured to be 87 (WISC-V; Wechsler, 2015), and his math age equivalency was 6 years and 8 months. Sam had some difficulties in oral comprehension in reading. He received individualized support from specialists for two hours a week at school.

Alex was a 7-year old Spanish boy diagnosed with ASD at the age of 6. His IQ was measured to be 96 (WISC-V; Wechsler, 2015), and his math age equivalency was 5 years and 6 months. He did not have issues with oral comprehension but struggled with other aspects of reading, for which the school used picture supports. Alex received four hours of personalized support from specialists per week.

Setting

The study was carried out during school hours at the students' school in an urban area with a medium to high socioeconomic status. The school specializes in providing support for students with ASD and follows an inclusive approach. The study was conducted in a classroom that was specifically adapted to minimize distractions. Each student participated in two to three individualized sessions per week, lasting between 30 and 45 minutes. All sessions were recorded on video for further analysis. The study instructor, who is the first author of the paper, was not a designated teacher at the school but was familiar with the children due to her involvement in the aforementioned larger project. She held a bachelor's degree in primary education teaching and a master's degree in mathematics education. Additionally, she was a second-year Ph.D. student working on her thesis in the field of mathematics learning among students with autism.

Ethical approval: The study received ethical approval by the Clinical Research Ethics Committee of Cantabria, code 2020.252.

Design and Data Collection

A single-case, multiple baseline across-students design (Horner & Baer, 1978) was used to assess the effectiveness of a MSBI instruction (Spooner et al., 2017) on the problem-solving performance of three students with ASD. In order to show a functional relation between the instruction and students' problem-solving performance, they were introduced sequentially into the intervention. The first student began the instruction after reaching a stable baseline (same score in at least three consecutive one-step problem sessions). Subsequently, after scoring 100% in three consecutive instructional sessions, the next student was introduced to the instruction once his baseline was observed to be stable. Finally, when the second student scored 100% in three consecutive sessions, the process was repeated with the third student.

Dependent Variable

The dependent variable was the number of additive compare problems solved correctly by the students independently. Each baseline probe comprised two one-operation (one-step) additive compare problems, one of each type (DAU, CAU). Additionally, a RAU problem was solved during each baseline session as a generalization measure. Examples of the three types of problems (DAU, CAU and RAU) are presented in Table 1. Another generalization baseline probe was administered in a unique session that consisted of four two-step problems using a combination and comparison structure with either DAU or CAU. The intervention was structured in sessions addressing different types of problems, sequenced according to the problem's difficulty, starting with compare story situations, and followed by DAU, CAU and mixed problems (i.e., DAU and CAU). Each probe during the test phase of each instructional session contained four problems of the type addressed in that session. Follow-up sessions required the student to independently solve a probe containing four problems (two DAU and two CAU). Finally, two types of generalization were assessed after the intervention. In the first generalization session, the students took a test that included four RAU problems. In the second generalization session, the students solved a four two-step problems probe, similar to the generalization baseline probe.

Independent Variable

The independent variable was an intervention based on an MSBI approach (Spooner et al., 2017) for teaching problem solving, which is detailed in the procedures section.

Reliability and Fidelity

Inter-rater data was collected randomly across all conditions. An external evaluator, who was blind to the study hypotheses but was familiar with the conditions and the types of problems addressed, re-evaluated 34% of the students' performance (35 out of 104 problems) across all conditions and students by reviewing recorded videos and evaluation probes during independent practice. In particular, 33% of baseline problems for the three students (eight out of 24 problems for James, nine out of 27 for Sam and 10 out of 30 for Alex) were collected. For instruction, 39% (14 out of 36) for James, 30% (12 out of 40) for Sam and 32% (nine out of 28) for Alex were collected. For follow up, 30% (four out of 12) were collected for the three students. For generalization, 50% (four out of eight) for James, 38% (three out of eight) for Sam and Alex were collected. For maintenance, 25% (one out of four) for James and Sam and 50% (two out of four) for Alex were collected. Inter-rater agreement was calculated for each condition by dividing the number of agreements by the total number of data points and multiplying the result by 100. Inter-rater agreement was 100% during baseline, 94% during the intervention, 100% in the follow-up, 100% in the generalization, and 100% during maintenance. The mean interobserver reliability agreement was 100% for baseline, follow up and generalization and 93% for instruction for James; 100% for all conditions for Sam; and 100% for baseline, follow up and generalization and 89% for instruction for Alex.

Procedural fidelity data was collected from 42% (11 out of 26) of the instructional sessions. This data measured the instructor's performance in executing planned behaviors, which included (1) providing the number of problems and agreed-upon materials for each session, (2) offering the stipulated support, (3) allowing the students to independently solve the problems, (4) emphasizing the key aspects for each problem type, and (5) providing verbal praise at the end of each session. Procedural fidelity was calculated for each student by dividing the number of observed teacher behaviors by the number of planned behaviors and multiplying it by 100. The mean procedural fidelity in intervention was 100% for James, 95% for Sam, and 100% for Alex.

Social Validity

Social validity is a critical concept that considers the social importance and appropriateness of intervention effects (Gresham, 1983). In this study, social validity interviews were conducted with the students, their families, and tutors before and after the intervention. In the first interview, conducted before the intervention, students, families and tutors were asked about the students' preferences and dislikes regarding mathematics and which mathematical tasks they enjoyed. Families were also asked about their children's interests, and this information was used to contextualize some of the problems written. In the post-intervention interview, students, families and tutors were asked about the provided intervention, and families or teachers were specifically asked if they considered the intervention to be beneficial to the children.

Procedures

The study was conducted in eight phases (a) baseline, (b) intervention on story situations; (c) intervention instruction on DAU problems, (d) intervention instruction on CAU problems, (e) intervention instruction on mixed (DAU and CAU) problems, (f) follow-up with mixed (DAU and CAU) problems, (g) generalization, and (h) maintenance.

Baseline

During baseline, the performance of each of the three students was evaluated through a series of 6, 7, and 8 initial probes, respectively. During the baseline condition, the students were given only the sheet of paper with the problem statements. They solved the problems individually with only the interviewer present, who provided motivation and reading assistance if necessary. There was no time limit for solving the problems.

Intervention

Each instructional session was approximately 40 minutes long, and followed an MSBI intervention as described by Spooner et al. (2017). The instruction was conducted in Spanish, the native language of both the interventionist and the three participants. To ensure access to the problems, the following measures were implemented: using scenarios based on the students' own interests as themes, presenting the problem in easily understandable language, and reading the problem aloud if necessary. In relation to the latter, students were asked to read the problems independently, and if they encountered difficulties or requested assistance, the instructor read it together with them. Additionally, picture supports were incorporated for the student with reading difficulties. To aid with conceptual comprehension, we used an adapted worksheet, a schematic diagram for representing the problem with manipulatives and a task analysis. To assist with the planning of each session, the instructor had a script outlining the main actions to be taken (such as problem typology, the sequence for presenting them and materials).

The worksheet was divided into four sections that included (see Figure 1, bottom left): the problem statement ("Eva has 7 toy cars. Juan has 2 toy cars. How many toy cars does Juan have more than Eva?") the schematic-diagram, the choice of operation (addition or subtraction) and the solution. There was another schematic diagram in a laminated sheet where students placed manipulatives (blocks) to represent the problem (see Figure 1, top). The task analysis consisted in a visual checklist for problem solving and was accompanied by color-printed picture supports that illustrated the steps to take to solve the problem (see Figure 1, bottom right): (1) Read the problem and underline the data; (2) Circle the largest quantity; (3) Represent the situation; (4) Complete the diagram; (5) Find the operation; and (6) Solve and check.

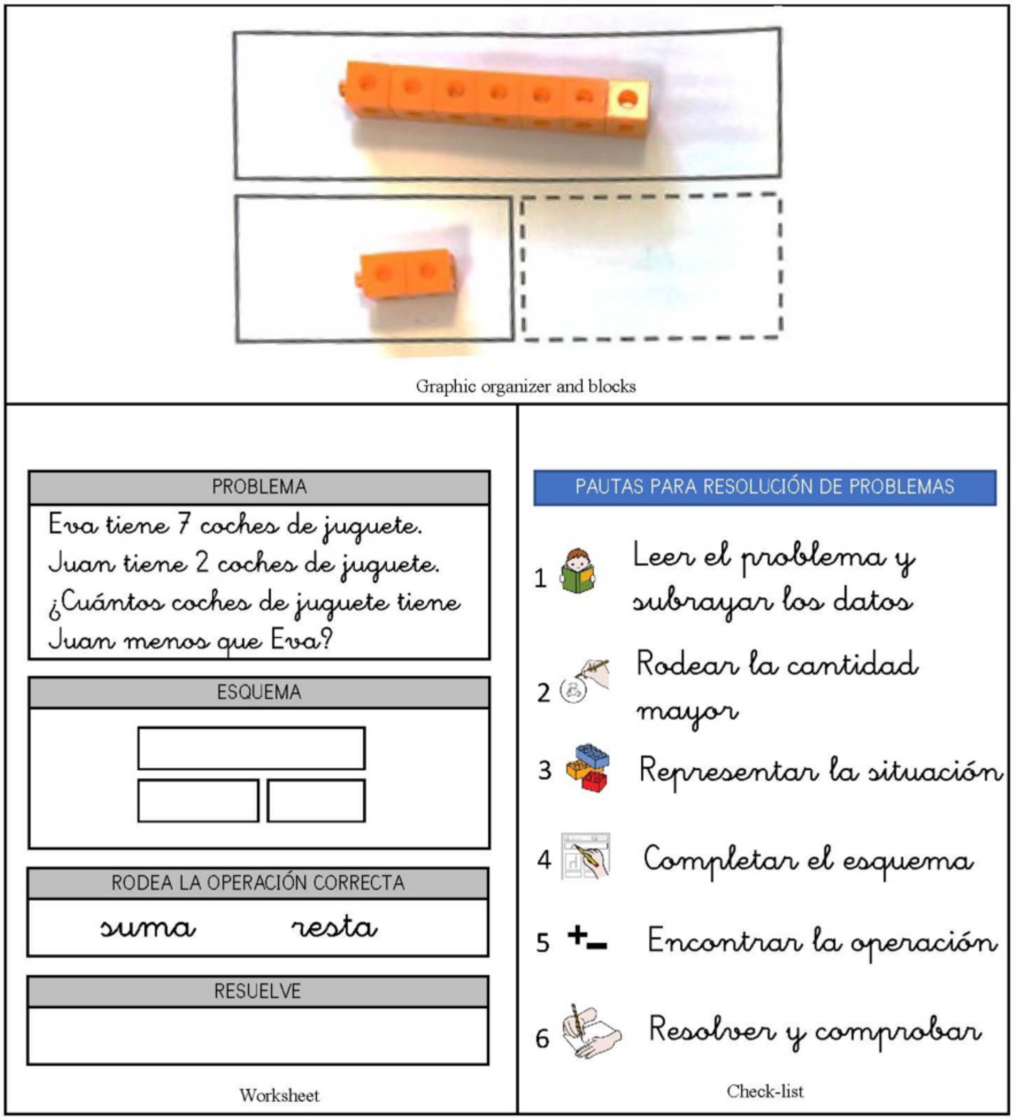
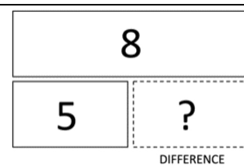
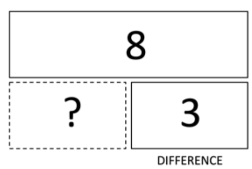
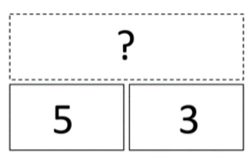


Figure 1. *Materials: Graphic Organizer With Blocks (top), Worksheet (Bottom Left) and Checklist (Bottom Right) used by the students. Pictographic Symbols Adapted From ARASAAC (http://arasaac.org)*

As in similar MSBI studies (e.g., Cox & Root, 2020), each session followed an explicit instruction format (model-lead-test sequence). The intervention was divided into two parts: problem-schema instruction and problem-solving instruction (Jitendra & Hoff, 1996).

Problem-Schema Instruction: Compare Story Situations. Compare story situations are statements of the problems in which there is no unknown (Jitendra & Hoff, 1996). This problem-schema instruction is a critical component of SBI interventions, where students are first taught with all quantities known. During the model phase, the instructor demonstrated how to follow the first four steps of the task analysis shown in Figure 1 (bottom right) to complete the worksheet for the given story. To model the problem-solving process, the instructor read the story and highlighted the three numbers. Next, she asked, “Do we know which of the two has more sweets?” and circled the larger amount. Then, following the task analysis, she filled in the schematic diagram with blocks, placing the larger amount in the top rectangle, the smaller amount in the lower left rectangle, and the difference between the amounts in the box at the lower right. Finally, she represented the numerical data of the problem in the schematic diagram on the worksheet. In the lead phase, the instructor guided the students in representing two additional story-situations. If the student encountered difficulties, the instructor asked guiding questions such as “Do we know who has more objects?” and “How many objects does he/she have?”. The student then used blocks to represent the situation in the schematic diagram and completed the schematic diagram on the worksheet. In the test phase, the student was asked to follow the same steps without any assistance from the instructor. If the student did not achieve 100% accuracy, a problem-schema instructional session was conducted again. Once the student achieved a score of 100%, he proceeded to problem-solving instruction.

Problem-Solving Instruction: Compare Problems. During problem-solving instruction, compare problems with unknown were presented to the students for them to solve. At this point, two new steps were included in the worksheet: (5) find the operation and (6) solve the problem and check (Figure 1). Compare problems were introduced sequentially based on the unknown, as follows: 1) sessions on DAU problems, 2) sessions on CAU problems, and 3) sessions on mixed problems (i.e., DAU and CAU). Word problems with “more than” and “fewer than” were alternated. During modelling, one problem was solved in DAU or CAU sessions, while two problems (one of each) were solved during mixed sessions. In the lead phase, between one and five problems were solved depending on the autonomy shown by the student. In the test phase, the student solved four problems of the type on which the instruction was focused. All problems were presented one by one to the students. The criterion for changing the type of sessions (DAU, CAU, and mixed) was achieving 100% success in test phase during two consecutive sessions.

Problem type	Statement	Schematic Diagram	Op.	Lang.
DAU “more”	Maria has 5 toys, Joan has 8. How many more toys does Joan have than Maria?		S	I
DAU “less”	Maria has 5 toys, Joan has 8. How many fewer toys does Maria have than Joan?			C
CAU “less”	Joan has 8 toys, Maria has 3 fewer toys than Joan. How many toys does Maria have?		S	C
RAU “more”	Joan has 8 toys, Joan has 3 more toys than Maria. How many toys does Maria have?			I
CAU “more”	Maria has 5 toys, Joan has 3 more toys than Maria. How many toys does Joan have?		A	C
RAU “less”	Maria has 5 toys, Maria has 3 fewer toys than Joan. How many toys does Joan have?			I

Notes: DAU= difference amount unknown; CAU=compare amount unknown; RAU= reference amount unknown; Op=operation; S= subtraction; A= addition; Lang=language; I=inconsistent; C= consistent.

Table 1. *Examples of Compare Problem Types With Their Schematic Diagram*

DAU Problems. During the model phase in DAU problems, the instructor read the problem and said: “We know the two quantities (compared and reference quantities), let's underline them on the statement and circle the larger one." She then placed on the top and bottom left of the schematic diagram as many blocks as these quantities indicated, respectively. To determine the difference, she employed a matching strategy, lining up both quantities in two rows of blocks. She told the student that the number of blocks that were not paired off indicated the difference between quantities, and therefore the solution to the problem. Then, she wrote the number of blocks in symbolic representation in the schematic diagram on the worksheet and selected the subtraction operation and solved it.

During the lead phase, the student was guided to identify the three amounts (known and unknown) and to represent them with the blocks in the schematic diagram. After that, the student represented the data in the schematic diagram on the worksheet, and selected the operation that yielded the result (in this case, always subtraction). All the steps in the solution were done with the support of the task analysis.

The DAU "more" problem type received special attention due to the inconsistent language used in these types of problems (Lewis & Mayer, 1987). Specifically, these problems are solved using addition when the statement uses the phrase "fewer than," or using subtraction when it uses "more than" (see Table 1). Note also that Spanish comparison terms such as "más" can be translated to both "more" and "plus," while "menos" can be translated as "fewer" or "minus". This can be particularly confusing for students, as these terms may suggest the operation to be performed. During the test phase, the students solved four DAU problems with no help or feedback from the instructor, following the steps in the task analysis.

CAU Problems. In the CAU problems instruction, the instructor emphasized during the model phase that the difference between the quantities in the problems was known. For instance, in the case of the CAU problem "fewer than" illustrated in Table 1, the instructor would highlight that the difference between the quantities is already known, and might ask: “Who has more, Joan or Maria?” and underline “Joan” in the statement. Next, she would write “8” at the top of the diagram and ask, “How many does Maria have? Since we do not know how many, we write a question mark here (bottom left part of the diagram).” The instructor would then guide the student to identify the operation. During the test phase, the student solved four CAU problems without the instructor’s help.

Mixed (DAU and CAU) Problems. During the model phase, the instructor demonstrated how to solve one DAU problem and one CAU problem, emphasizing how to discriminate between these two types of problems. To achieve this, the instructor would ask questions such as: “Is this one of those problems where we don't know how many more?” (referring to DAU), or “Is this one of those problems where we do know?” (referring to CAU). Then, the corresponding problem type was solved, with the instructor focusing on the key aspects mentioned above.

Follow-Up

Follow-up condition took place immediately after the completion of instruction and included three evaluation sessions. Unlike in the instructional sessions with mixed problems, the student did not have access to the support materials provided during the intervention (such as worksheets, schematic diagrams, and guideline sheets). Instead, the student was given only the problem statement on a sheet of paper, as was the case in the baseline probes.

Generalization

Generalization condition occurred immediately after the follow-up sessions. Two generalization sessions were conducted to evaluate the students' ability to apply the learned skills to different type of problems, namely: (1) RAU problems and (2) two-step problems.

Maintenance

Finally, between 4 and 6 weeks after the end of the intervention (4 for James and Sam, and 6 for Alex), the maintenance of the acquired skills was evaluated. This was done in one session by having each student solve four compare problems (two DAU and two CAU). The problems were presented in a format similar to that of the follow-up condition.

Results

Figure 2 shows the number of additive compare problems that were solved correctly by the three students independently. In total, 21 sessions were held with James and Alex, and 23 sessions with Sam. The results for each student in each condition are described below.

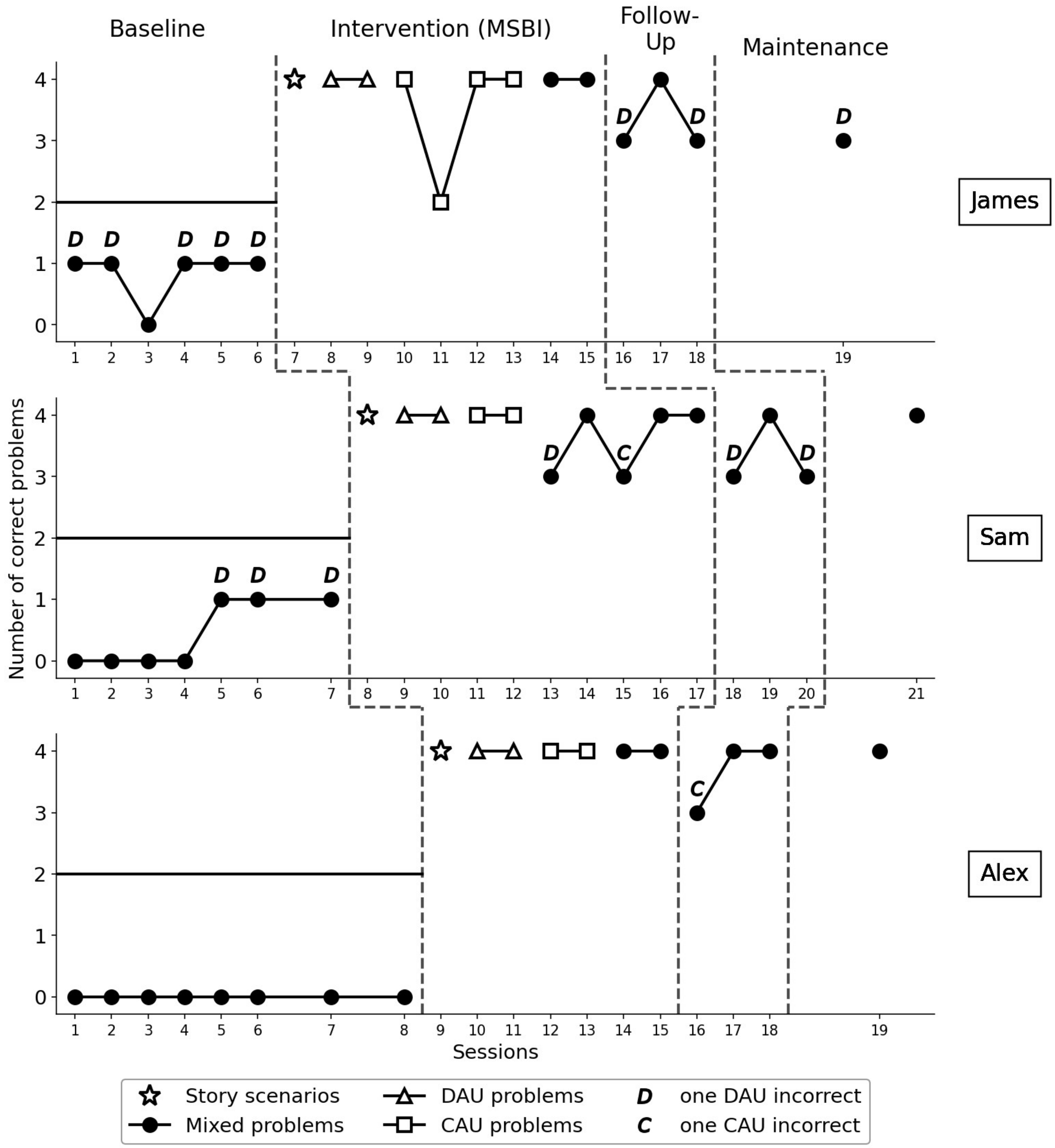


Figure 2. *Number of Correct Problems.*

Using guidelines for visual analysis of SCRD data (Kratochwill et al., 2013), we visually examined data from each study for: (a) level, (b) trend, (c) variability, (d) immediacy of effect, (e) overlap, and (f) consistency of data patterns. During the baseline condition, all three students maintained a stable pattern. Generalization data is presented in Figure 3, where the performance for both generalization measures (RAU and two-step problems, respectively) is shown for the three students.

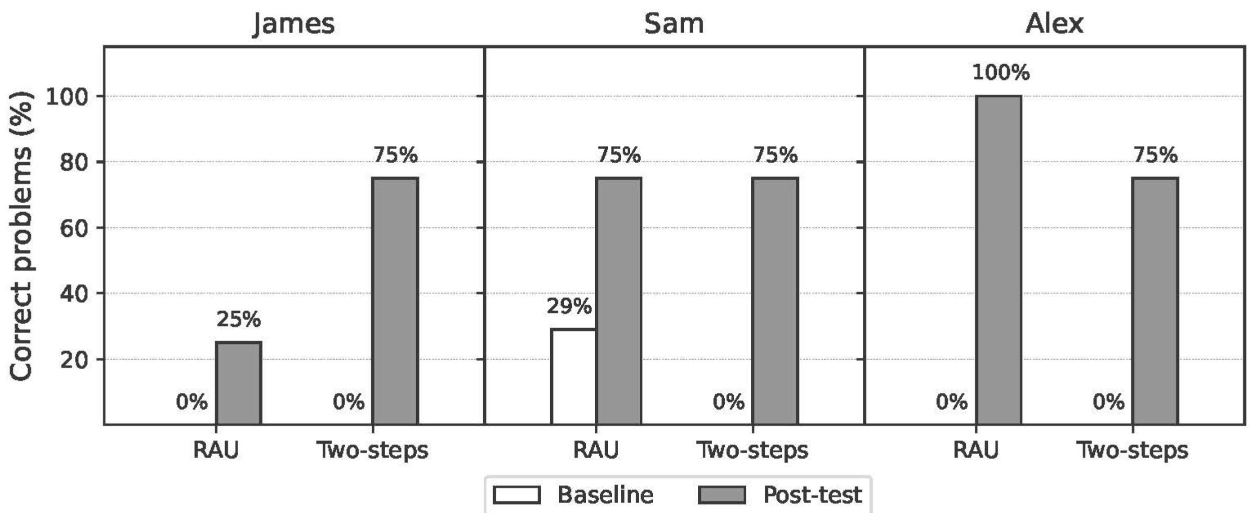


Figure 3. *Performance for both Generalization Measures: RAU and Two-Step Problems*

James

James had a mean baseline performance of 0.42 correct one-step additive compare problems. In particular, he solved correctly five out of six CAU problems and none of the DAU ones. Upon introduction of the intervention, he showed an immediate increase solving all one-step compare DAU problems correctly. The mean intervention performance was 3.8. There was no overlap of data between baseline and intervention for DAU and mixed problems sessions. The follow-up condition had a slight variability with a mean performance of 3.3 correct problems. During follow-up James solved 2 problems incorrectly, both of type DAU and both involving inconsistent language. He solved 3 out of 4 problems of the maintenance probe correctly. Concerning generalization, during baseline James did not solve any of the RAU problems correctly, and none of the two-step generalization probe. After intervention, he solved 25% of the RAU problems correctly, indicating difficulties again with the inconsistent language used in these types of problems, and he solved 75% problems correctly in the two-step problems generalization probe (see Figure 3).

Sam

Sam had a mean baseline score of 0.21 correct one-step problems. Specifically, he solved all DAU problems incorrectly. When solving the problems, he either provided one of the problem numbers as solution, or performed addition to solve it. This resulted in the correct answer in three of the CAU problems (one in each of the last three baseline sessions). Upon introduction of the intervention on DAU problems, Sam demonstrated an immediate increase, with a mean intervention performance of 3.8 problems and no overlap of data between baseline and intervention for DAU and mixed problems. There was an increased level across ten instructional sessions with slight variability during instruction in mixed-problems, when he exhibited difficulties discriminating between problem types. During the follow-up condition, there was a slight variability and a mean performance of 3.3 correct problems. Sam maintained the acquired skills solving all 4 problems correctly in the maintenance probe. Concerning generalization, during baseline he had 29% correct RAU problems and 0% correct two-steps problems (see Figure 3). After instruction, he scored 75% in both generalization probes (RAU and two-step problems) .

Alex

Alex had a baseline performance of 0 correct responses on all types of problems (DAU, CAU). He answered all problems by providing random numbers, or a given number of the problem data. Upon introduction of the intervention, Alex displayed an immediate increase in compare problems of type DAU, and later of type CAU, with a mean score of 4 correct problems and no overlap of data between baseline and intervention in all types of problems, and a stable trend across all seven instructional sessions. The follow-up condition had slight variability with a mean performance of 3.7 problems across three sessions. He maintained the skills solving correctly all problems in the maintenance probe. Concerning generalization, Alex scored 0% in the RAU baseline probe and 100% in the RAU postest probe. He obtained 0% correct in the baseline two-step generalization probe and 75% in the two-step postest probe (see Figure 3).

Social Validity

In the first interview, the three students stated that problem solving was what they liked least about mathematics, and the families of the three indicated that understanding problem statements was the part that gave them the most trouble. The teachers also noted that the difficulties solving problems originated from not understanding the situation described, which resulted from not identifying the arithmetic operation needed to solve them.

During the post-intervention interview, students expressed their enjoyment of the instructional sessions, using phrases such as “I feel more confident in solving problems now” and “I would love to do math with you again” in reference to the instructor. The families of the three students reported that participating in the study had a positive impact on their children. Furthermore, the teachers at the school noted the improved problem-solving strategies of the students, who demonstrated increased confidence and interest in math when working on problems in their general education classroom. Following the completion of the intervention, the instructor provided the families and the school counselor with a report detailing each student's performance during the sessions, along with a brochure containing all the materials used. The counselor expressed great interest in this documentation and stated that she intended to use it to train the special education teachers at the school in the methodology, which could benefit even more students with special educational needs.

Discussion

The aim of this study was to investigate the impact of a MSBI intervention designed to teach students with ASD how to solve additive compare mathematical problems (e.g. DAU and CAU problems). The results showed a functional relation between the MSBI intervention and the improvement in the performance of the three participating students when solving additive compare problems. This study makes a valuable contribution to the field of teaching problem-solving skills to students with ASD who face difficulties in this area and provides a comprehensive analysis of the MSBI methodology for compare problems, building on previous studies with students with ASD (e.g. Root et al., 2017). Specifically, it extends the work of Root et al. (2017), who focused solely on DAU problems, by evaluating the impact of MSBI intervention on compare problem-solving skills for all types of problems with varying unknowns. Our results are also consistent with those of Kasap and Ergenekon (2017), as some students were able to generalize their skills to other types of compare problems beyond those covered in the intervention (to DAU problems in their research, and to RAU in ours). Furthermore, this study was conducted in Spain and involved participants with a higher level of functioning compared to previous research (e.g., Browder et al., 2018; Root et al., 2017), thereby contributing to a broader geographic representation and an increased overall participant count. This effort aims to expand the scope of research studies and geographic locations for the validation of MSBI as an evidence-based practice (see Clausen et al., 2021).

The results indicate that SBI can be adapted to better meet the needs of students with ASD. Specifically, the utilization of schematic diagrams with manipulatives, a frequently used support in MSBI studies (Clausen et al., 2021), has been shown to be advantageous for students, consistent with previous MSBI research (Root et al., 2017). In this study, the use of manipulatives was relevant in representing and aligning quantities, allowing students to utilize the matching strategy to facilitate comparisons and helping them determining the necessary operation (addition or subtraction) to solve the problem. These results align with those of Bouck et al. (2014), who indicate that the use of manipulatives aids students who struggle with numerical fluency. Furthermore, the incorporation of picture supports for the word problems was shown to be beneficial for the student who experienced difficulty with reading.

The results from the maintenance tests demonstrated that the skills were retained by the students. Moreover, two of the students were able to generalize their newly acquired skills to RAU problems. It is worth mentioning that the impact of instruction was especially noticeable when addressing DAU problem-solving. Prior to the intervention, all three students had solved all these problems incorrectly. However, their performance improved significantly after receiving instruction. Despite this, two of the students still faced some difficulties with this type of problem throughout the study. In particular, all DAU problems that these students solved incorrectly involved inconsistent language, which provides valuable insight into where future instruction should focus, as suggested by Polo-Blanco, González et al. (2024).

Limitations and Future Research

There are some limitations to this study that should be considered when interpreting the results. Firstly, the fact that there were multiple components to the intervention makes it difficult to determine what caused the increase in the students' performance. Additionally, the number of problems presented during the baseline probes was only two, whereas the intervention probes presented four problems, making comparisons across conditions challenging. Despite these issues, the data was disaggregated and analyzed to depict the effects of instruction for each type of problem and participant. This allowed for a better understanding of the impact of the intervention on the different problem types and individual participants.

This study focused on three students with ASD who struggled with mathematical problem-solving. As ASD is a heterogeneous disorder, further similar studies should include students with ASD of different cognitive profiles. Additionally, our study identified difficulties with inconsistent language problems, which is in line with previous MSBI research with students with ASD (Polo-Blanco, González et al., 2024). Future research should therefore focus on providing instruction on how to address this type of problem.

Implications for Practice

The students displayed a noticeable increase in motivation during the intervention when compared to the baseline sessions. This was evident in their eagerness to engage right from the beginning of the session, as well as their expressions, such as: "I don't want you to leave, let's do more shark problems" (Alex). This may be attributed to their awareness of their progress in comprehending the problem statements and effectively solving them, which was made possible by the support of the MSBI methodology. Moreover, our study provides valuable information on conducting instruction on problems with two types of unknowns (DAU and CAU), extending the work of Root et al. (2017) and Kasap and Ergenekon (2017). Our findings shed light on a possible sequence of learning and the use of aids to facilitate student learning, making it easier to replicate our study and apply the methodology to different types of comparison problems. Finally, this research was conducted in an inclusive school that aims to integrate students with ASD. The professionals expressed keen interest in the methodology, believing it can be easily adapted for use in their classrooms, which helps promote ASD students' inclusion in general education settings.

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Declaration of Generative AI in the writing process. During the preparation of this work the authors used AI in order to improve the English wording. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

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