HARDWARE IMPAIRMENTS-AWARE DESIGN OF NONCOHERENT GRASSMANNIAN CONSTELLATIONS

Diego Cuevas[1], *Javier Álvarez-Vizoso*[1], *Mikel Gutiérrez*[1], *Ignacio Santamaria*[1], *Vít Tuček*[2] *and Gunnar Peters*[2]

[1]Universidad de Cantabria, Department of Communications Engineering, Santander, Spain
[2]Huawei Technologies, Department of Wireless Algorithms, Kista, Sweden

ABSTRACT

In this paper, we propose a robust algorithm for designing unstructured Grassmannian constellations for noncoherent MIMO communications that accounts for the effect of hardware impairments (HWIs) such as I/Q imbalance (IQI) and carrier frequency offset (CFO). The algorithm uses the minimum diversity product as a cost function to ensure full-diversity constellations. The constellation points in the Grassmannian are optimized to be robust against any value of the HWIs belonging to a given uncertainty set, the values of which are determined by the characteristics of the hardware used. The cost function is optimized by means of a gradient ascent algorithm on the Grassmann manifold. Simulation results suggest that the constellations designed with the robust algorithm show a significant improvement in symbol-error-rate (SER) performance over the HWI-unaware algorithm optimized for ideal devices.

Index Terms— Noncoherent MIMO communications, Grassmannian constellations, hardware impairments, I/Q imbalance, carrier frequency offset

1. INTRODUCTION

In multiple-input multiple-output (MIMO) wireless communications, the channel state information (CSI) is typically estimated at the receiver side by sending a few known pilots and then used for decoding at the receiver and/or for precoding at the transmitter. These are known as coherent schemes. The channel capacity for coherent MIMO systems increases linearly with the minimum number of transmit and receive antennas at high signal-to-noise ratio (SNR) [1,2] when the channel remains approximately constant over a long coherence time (slow-fading scenarios). However, in fast-fading scenarios, accurately estimating the channel would require pilots to occupy a disproportionate fraction of communication resources. CSI acquisition by orthogonal pilot-based schemes results in significant overheads in massive MIMO systems [3] even in slowly varying channels. These scenarios motivate the use of noncoherent MIMO communications schemes in which neither the transmitter nor the receiver have any knowledge about the instantaneous CSI.

In the single-user case and under additive Gaussian noise, it was proved in [4, 5] that the $T \times M$ transmit matrices $\mathbf{X}$ (where T is the number of time slots and M is the number of transmit antennas) that achieve the ergodic noncoherent capacity for the MIMO block-fading model can be factored as the product of an isotropically distributed $T \times M$ truncated unitary matrix and a diagonal $M \times M$ matrix with real nonnegative entries. Further, when $T >> M$ the nonzero entries of the diagonal matrix take the same value, showing that in this regime it is optimal to transmit unitary space-time codewords $\mathbf{X}^H\mathbf{X} = \mathbf{I}_M$. Using the same signal model, it was proved in [6] that at high SNR and when $T \geq 2M$, ergodic capacity can be achieved by transmitting isotropically distributed unitary matrices. Motivated by these results, numerous methods for the design of constellations formed by truncated unitary signal matrices, called unitary space-time modulations (USTM), have been investigated and proposed over the last decades [7–16]. In MIMO noncoherent constellations, information is carried by the column span (i.e., a subspace) of the transmitted $T \times M$ matrix, $\mathbf{X}$. The problem of designing noncoherent codebooks is thus closely related to finding optimal packings in Grassmann manifolds [6, 17], and the resulting constellations are referred to as Grassmannian constellations.

Wireless communication devices are never completely ideal in practice. Hardware impairments (HWIs) impose a major challenge on next-generation communication systems, which can degrade the overall system performance [18,19]. HWIs may occur due to imperfections such as quantization noise, phase noise, amplifier nonlinearities, I/Q imbalance (IQI), and frequency offset due to mismatched local oscillators [20]. In this paper, we focus on two types of HWIs: those caused by IQI and carrier frequency offset. These impairments are particularly important in noncoherent Grassmannian constellations because even small imbalances in the I/Q branches or moderate mismatches between the Tx and Rx oscillators can significantly modify the transmitted subspaces. We further assume that the values of IQI and frequency offset are not known and, therefore, cannot be compensated. Instead, we consider maximum values of IQI and frequency offset that determine an uncertainty set, and we seek to design noncoherent constellations that are robust to all possible values within the uncertainty set. This could be of interest, for instance, in an Internet of Things (IoT) scenario with hundreds of low-cost uncalibrated devices that communicate using noncoherent constellations.

The performance of coherent wireless communication systems under different types of HWIs has been extensively studied and some relevant solutions have been published in the literature [21, 22]. However, for noncoherent communication systems the impact of HWIs has been studied only for differential modulations [23], and we are not aware of any solution that overcomes the performance degradation due to HWIs for Grassmannian constellations. Thus, we propose in this paper a robust optimization algorithm that considers

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I/Q imbalance and carrier frequency offset in the design of noncoherent Grassmannian constellations. The proposed algorithm uses as a cost function the minimum diversity product of the constellation, which was shown to be more effective than the chordal distance while keeping a low computational complexity [14].

Notation: In this paper, matrices are denoted by bold-faced upper case letters, column vectors are denoted by bold-faced lower case letters, and scalars are denoted by light-faced lower case letters. The superscript $(\cdot)^H$ denotes Hermitian conjugate, and $(\cdot)^*$ denotes complex conjugate. The trace and determinant of a matrix $\mathbf{A}$ will be denoted, respectively, as $\mathrm{tr}(\mathbf{A})$ and $\det(\mathbf{A})$. The identity matrix of size n is denoted as $\mathbf{I}_n$ and $\mathrm{diag}(\mathbf{a})$ denotes a diagonal matrix whose diagonal is $\mathbf{a}$. A continuous uniform distribution between a and b is denoted as $U(a, b)$ and $\mathcal{CN}(0, 1)$ denotes a complex proper Gaussian distribution with zero mean and unit variance.

2. SYSTEM MODEL

We consider a transmitter with M antennas communicating in a noncoherent MIMO system with a receiver equipped with N antennas, over a frequency-flat block-fading channel with coherence time T symbol periods, such that $T \geq 2M$. Hence, the channel matrix $\mathbf{H} \in \mathbb{C}^{M\times N}$ remains constant during each coherence block of T symbols, and changes in the next block to an independent realization. The MIMO channel $\mathbf{H}$ is unknown to both the transmitter and the receiver and assumed to have a Rayleigh fading distribution with entries $h_{ij} \sim \mathcal{CN}(0, 1)$. Within a coherence block, the transmitter sends a unitary matrix $\mathbf{X} \in \mathbb{C}^{T\times M}$, $\mathbf{X}^H\mathbf{X} = \mathbf{I}_M$, that is an orthonormal basis for the linear subspace spanned by the columns of $\mathbf{X}$, denoted in this paper as $[\mathbf{X}]$, within $\mathbb{C}^T$. The signal at the receiver $\mathbf{Y} \in \mathbb{C}^{T\times N}$ is

$$\mathbf{Y} = \mathbf{XH} + \sqrt{\frac{M}{T\rho}}\mathbf{W}, \tag{1}$$

where $\mathbf{W} \in \mathbb{C}^{T\times N}$ represents the additive Gaussian noise, with entries modeled as $w_{ij} \sim \mathcal{CN}(0, 1)$, and ρ represents the signal-to-noise-ratio (SNR).

The optimal Maximum Likelihood (ML) detector that minimizes the probability of error, assuming equiprobable codewords, is given by

$$\hat{\mathbf{X}} = \arg\max_{\mathbf{X}\in\mathcal{C}} \mathrm{tr}\left(\mathbf{Y}^H\mathbf{P}_{\mathbf{X}}\mathbf{Y}\right), \tag{2}$$

where $\mathcal{C} = \{\mathbf{X}_1, \ldots, \mathbf{X}_K\}$ represents the codebook of K codewords and $\mathbf{P}_{\mathbf{X}} = \mathbf{XX}^H$ is the orthogonal projection matrix onto the subspace $[\mathbf{X}]$. Each codeword carries $\log_2(K)$ bits of information.

3. HARDWARE IMPAIRMENTS

The performance of communication systems can be affected by the non-idealities of the radio frequency transceivers. This usually happens when low-cost/low-complexity radio front-ends are used, which impairs the quality of the received signal and thus impacts the overall system performance. In this paper, we consider two types of HWIs that particularly affect noncoherent Grassmannian constellations: carrier frequency offset and I/Q imbalance.

3.1. Carrier frequency offset

A typical HWI that affects noncoherent communication systems is the carrier frequency offset (CFO) since there are no pilots that can be used for CFO correction. CFO is caused either by a mismatch between the Tx and Rx oscillators or by a Doppler shift, provoking a slight shift between the transmitter and receiver carrier frequencies. There is always some difference between the device specifications and the CFO values observed in reality. Also, CFO values vary (slowly) with temperature, pressure, age, and some other factors, so it is difficult to accurately estimate and compensate the CFO, especially in noncoherent communications.

A CFO of value $\Delta\omega$ modifies the transmitted space-time codeword $\mathbf{X}$ as follows

$$\mathbf{F}(\Delta\omega) = \mathrm{diag}\left(1, e^{j\Delta\omega}, e^{j2\Delta\omega}, \ldots, e^{j(T-1)\Delta\omega}\right) \cdot \mathbf{X}. \tag{3}$$

Note that $\mathbf{F}(\Delta\omega)$ in (3) satisfies $\mathbf{F}(\Delta\omega)^H\mathbf{F}(\Delta\omega) = \mathbf{I}_M$ and therefore is a basis for a transformed subspace, i.e., $[\mathbf{F}(\Delta\omega)] \neq [\mathbf{X}]$.

We assume that the CFO $\Delta\omega$ is not known nor can it be estimated at the receiver, but we do know its maximum value from the hardware specifications. In the presence of CFO, the received signal is $\mathbf{Y} = \mathbf{F}(\Delta\omega)\mathbf{H} + \sqrt{M/(T\rho)}\mathbf{W}$.

3.2. I/Q imbalance

Another HWI that affects noncoherent communication systems is the I/Q imbalance (IQI). The nature of IQI consists of a phase difference of not exactly 90 degrees and an amplitude difference between the I and Q branches of the local oscillator. An IQI at the transmitter side[1] results in a perturbation of the transmitted codeword $\mathbf{X}$ that can be modeled as

$$\mathbf{Z}(G, \theta) = \left(\frac{1 + Ge^{j\theta}}{2}\right)\mathbf{X} + \left(\frac{1 - Ge^{-j\theta}}{2}\right)\mathbf{X}^*, \tag{4}$$

where G and θ capture the amplitude and rotational errors, which are determined by the hardware specifications. We assume that each transmitter branch has the same G and θ since the antennas share the same local oscillator, but their exact values are not known and, as in the CFO case, cannot be estimated through the use of pilots. Only the maximum values of these parameters are known, thus defining an uncertainty set $G \leq G_{max}$ and $0 \leq \theta \leq \theta_{max}$. In the presence of IQI at the Tx side, the received signal is $\mathbf{Y} = \mathbf{Z}(G, \theta)\mathbf{H} + \sqrt{M/(T\rho)}\mathbf{W}$.

4. ROBUST CONSTELLATION DESIGN

4.1. Cost function

In this section, we propose an algorithm for designing Grassmannian constellations that are robust against IQI and CFO impairments. It is based on the algorithm described in [14] for designing Grassmannian constellations for ideal devices. The optimization algorithm in [14] maximizes the minimum diversity product, which is defined as:

$$\underset{[\mathbf{X}_1],\ldots,[\mathbf{X}_K]}{\mathrm{argmax}} \quad \min_{k\neq j} \det(\mathbf{I}_M - \mathbf{X}_k^H\mathbf{X}_j\mathbf{X}_j^H\mathbf{X}_k), \tag{5}$$

which is equivalent to minimizing the maximum coherence between subspaces [24–27]. As shown in [16], this criterion is motivated by the high-SNR analysis of the pairwise error probability in noncoherent communications. The optimization of this so-called coherence

[1] As the channel is unknown in noncoherent communications, accounting for IQI at the receiver side requires more careful analysis. Therefore, in our model, we consider exclusively the IQI at the transmitter side.

criterion is done in [14] by means of a gradient ascent algorithm on the Grassmann manifold that uses an adaptive step size.

Taking the algorithm proposed in [14] as a starting point, some modifications are needed to make the designed constellations robust against CFO and IQI. The basic idea of the algorithm at each iteration is to perturb the constellation points using random values of the CFO and IQI uniformly distributed within the corresponding uncertainty sets. The modified algorithm performs a number ($N_{iter_{CFO}}$) of CFO perturbations which are followed by a number ($N_{iter_{IQI}}$) of gradient ascent optimization procedures which are IQI perturbed every q iterations. The perturbation procedures for the CFO and for the IQI are described below. A summary of the proposed method is shown in Algorithm 1.

Algorithm 1: Robust constellation design

Input: Initial non-HWI-aware codebook $\mathcal{C}_{ini} = \{\mathbf{X}_k\}_{k=1}^{K}$, HW specifications $|\Delta\omega|_{max}$, G_{max}, θ_{max}, step size μ, optimization parameters $N_{iter_{CFO}}$, N_{CFO}, $N_{iter_{IQI}}$, q

1 **for** $n = 1 : N_{iter_{CFO}}$ **do**
2   **for** $k = 1 : K$ **do**
3   Draw i.i.d. $\Delta\omega_p \sim U(-\Delta\omega_{max}, \Delta\omega_{max})$, $p = 1, \ldots, N_{CFO}$
4   Compute CFO-perturbations $\mathbf{F}_k(\Delta\omega_p)$, $p = 1, \ldots, N_{CFO}$ as in (3)
5   Compute CFO-perturbed mean projection matrix $\mathbf{P}_{\mathbf{F}_k}$ as in (6)
6   Substitute $\mathbf{X}_k \mapsto \mathbf{U}(:, 1 : M)$, where $\mathbf{U}\mathbf{\Sigma}\mathbf{V}^H = \mathbf{P}_{\mathbf{F}_k}$
7   **end for**
8   **for** $n = 1 : N_{iter_{IQI}}$ **do**
9   Draw $G_n \sim U(0, G_{max})$ and $\theta_n \sim U(0, \theta_{max})$
10   **for** $k = 1 : K$ **do**
11   **if** n *mod* q **then**
12   Substitute $\mathbf{X}_k \mapsto QR\left(\mathbf{Z}_k(G, \theta)\right)$, where $\mathbf{Z}_k(G, \theta)$ is the IQI-perturbed codeword from $\mathbf{X}_k$ given in (4)
13   **end if**
14   **else**
15   Find the element $\mathbf{X}_j$ with the smallest diversity product with $\mathbf{X}_k$
16   Construct the matrix $\mathbf{\Delta}_{kj}$ that yields the best direction to maximize the diversity product between $\mathbf{X}_k$ and $\mathbf{X}_j$ using the corresponding gradient
17   Move $\mathbf{X}_k$ in the direction defined by $\tilde{\mathbf{X}}_k = \mathbf{X}_k + \mu\mathbf{\Delta}_{kj}$
18   Retract $\tilde{\mathbf{X}}_k$ to the manifold by computing the Q factor in its reduced QR decomposition
19   Evaluate cost function and repeat steps 15-18 with smaller μ until cost function does not improve its value or μ is below a threshold
20   Update $\mathbf{X}_k \mapsto \tilde{\mathbf{X}}_k$
21   **end if**
22   **end for**
23   **end for**
24 **end for**

4.2. Robustness against CFO

To achieve robustness against CFO, for each constellation point $\mathbf{X}_k$ we draw N_{CFO} random values of $\Delta\omega$ uniformly distributed in $(-\Delta\omega_{max}, \Delta\omega_{max})$, $\Delta\omega_p$, $p = 1, \ldots, N_{CFO}$. Then, we generate a set of perturbed samples, $\mathbf{F}_k(\Delta\omega_p)$, $p = 1, \ldots, N_{CFO}$, as described in (3). We have generated in this way a cluster of perturbed subspaces around each initial codeword. The adaptation step to achieve robustness against the CFO consists of taking a new representative (centroid) for each cluster. This is done by computing the subspace average as described in [28]. Specifically, we first compute the average projection matrix for all perturbed subspaces associated to codeword $\mathbf{X}_k$ as

$$\mathbf{P}_{\mathbf{F}_k} = \frac{1}{N_{CFO}} \sum_{p=1}^{N_{CFO}} \mathbf{F}_k(\Delta\omega_p)\mathbf{F}_k(\Delta\omega_p)^H, \tag{6}$$

where $\mathbf{F}_k(\Delta\omega_p)\mathbf{F}_k(\Delta\omega_p)^H$ is the orthogonal projection matrix for the perturbed subspace generated with CFO $\Delta\omega_p$ from codeword $\mathbf{X}_k$. The updated codeword $\mathbf{X}_k$ is extracted by computing the M eigenvectors corresponding to the largest eigenvalues of $\mathbf{P}_{\mathbf{F}_k}$.

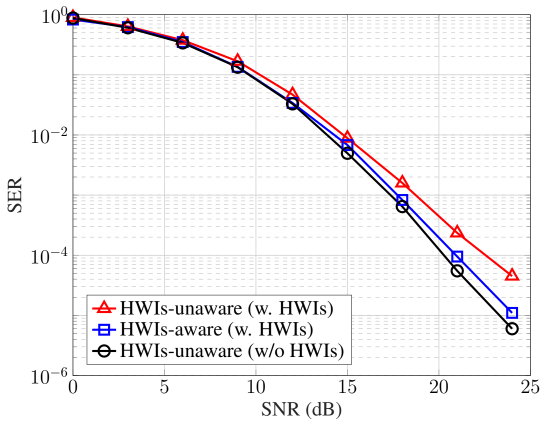
4.3. Robustness against IQI

After the CFO step, we have a new Grassmannian constellation that must again be perturbed, this time to achieve robustness against IQI. Since the $\mathbf{Z}(G, \theta)$ matrices with IQI generated according to (4) no longer belong to the Grassmann manifold, it is necessary to apply a different procedure than the one used to achieve robustness against CFO. First, we draw a random sample of G uniformly distributed in $(0, G_{max})$ and a random sample of θ uniformly distributed in $(0, \theta_{max})$, where G_{max} and θ_{max} are the maximum amplitude and phase imbalance values characterizing the devices. Then, we perform a QR decomposition of each $\mathbf{Z}_k(G, \theta)$ as a retraction step that brings back the transformed codeword to the Grassmann manifold. In this way, we build a new Grassmannian constellation with constellation points affected by IQI: $\mathcal{C} = \{QR(\mathbf{Z}_1(G, \theta)), \ldots, QR(\mathbf{Z}_K(G, \theta))\}$. Notice that all codewords are affected by the same amplitude and phase imbalances. The IQI-perturbed constellation is now optimized to maximize the minimum diversity product using the algorithm in [14].

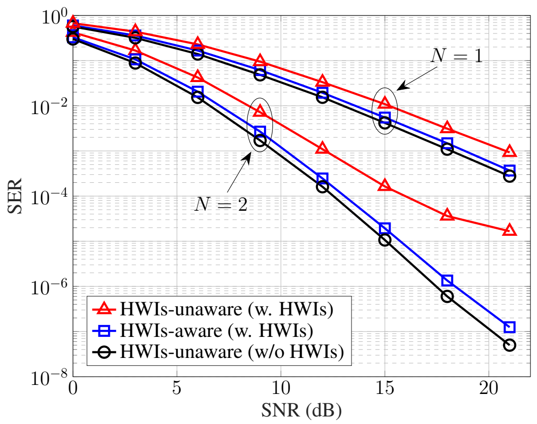
5. SIMULATION RESULTS

To assess the symbol-error-rate (SER) performance of the proposed robust HWIs-aware Grassmannian constellations, we conducted several computer simulations. All robust constellations used in these experiments have been designed with the parameters shown in Table 1.

Figs. 1a and 1b show the SER performance of the HWIs-aware constellations designed for $|\Delta\omega|_{max} = 10$ degrees, $G_{max} = 3$ dB and $\theta_{max} = 15$ degrees, and different MIMO scenarios with coherence time $T \in \{4, 8\}$, $M = 2$ antennas at the transmitter, $N \in \{1, 2\}$ antennas at the receiver and $K = 64$ codewords. The performance of the designed constellation is compared in these two figures against the original constellation designed assuming ideal devices, which is obtained using the algorithm in [14]. As we can observe, the proposed HWIs-aware constellations outperform in both cases the original HWIs-unaware constellation designs. In fact, they



(a) $T = 4$, $N = 2$



(b) $T = 8$, $N \in \{1, 2\}$

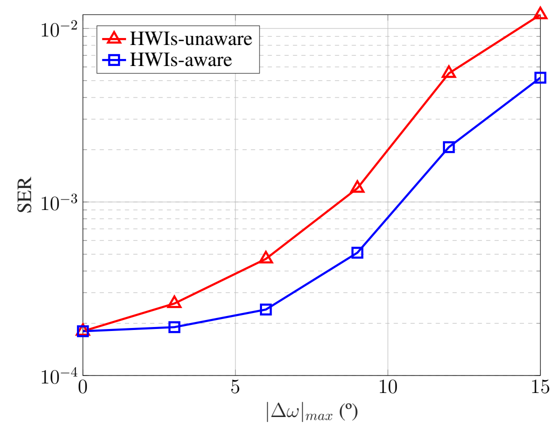
Fig. 1: SER curves for $|\Delta\omega|_{max} = 10°$, $G_{max} = 3$ dB, $\theta_{max} = 15°$, $K = 64$ codewords, $T \in \{4, 8\}$, $M = 2$ and $N \in \{1, 2\}$.

Parameter	Symbol	Value
Step size	μ	0.1
# iterations CFO loop	$N_{iter_{CFO}}$	50
# random values of $\Delta\omega$ for each CFO iteration	N_{CFO}	20
# iterations IQI loop	$N_{iter_{IQI}}$	20
# IQI iterations after which values of G and θ are changed	q	5

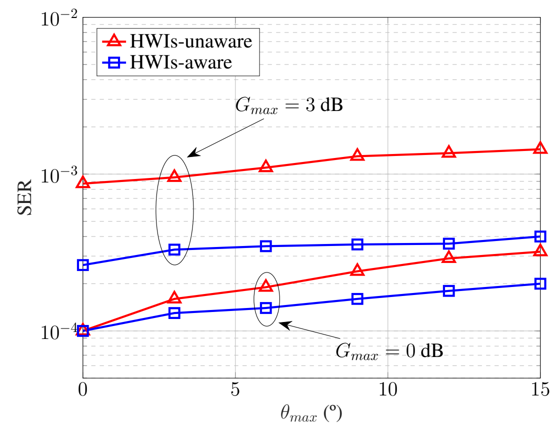
Table 1: Optimization parameters used to design the robust constellations.

almost reach the error rates achieved without HWIs. The performance gap seems to grow significantly when we increase the coherence time to $T = 8$.

For the scenario $T = 4$, $M = N = 2$ and $K = 64$, we study the variation of the SER for a fixed SNR of 20 dB as a function of $\Delta\omega_{max} \in (0, 15)$ degrees when there is only CFO (Fig. 2a), and as a function of $G_{max} \in \{0, 3\}$ dB and $\theta_{max} \in (0, 15)$ degrees when there is only IQI (Fig. 2b). In these figures, the leftmost points corresponding to $\Delta\omega = 0$ degrees, and ($G = 0$ dB, $\theta = 0$ degrees) correspond to the performance of the original constellation without



(a) $G_{max} = 0$ dB, $\theta_{max} = 0°$



(b) $|\Delta\omega|_{max} = 0°$

Fig. 2: SER variation with parameters $|\Delta\omega|_{max}$, G_{max} and θ_{max} for SNR = 20 dB, $K = 64$ codewords, $T = 4$, $M = 2$ and $N = 2$.

HWIs. As we can observe, the performance degradation is much worse when there is only CFO than when there is only IQI. Fig. 2a shows an error rate loss of almost 2 orders of magnitude. This indicates that noncoherent Grassmannian constellations are very sensitive to carrier frequency offsets due to either mismatches between the Tx and Rx local oscillators or Doppler shifts. On the other hand, for IQI the parameter that affects the most is the amplitude imbalance G, while the effect of θ is minor, being practically negligible for values smaller than 2 degrees. These conclusions might be of interest for the design of noncoherent Grassmannian constellations.

6. CONCLUSION

We have proposed an optimization method for designing noncoherent MIMO constellations that are robust against hardware impairments such as I/Q imbalance or carrier frequency offset. To do so, we perform a gradient ascent algorithm on the Grassmann manifold based on the minimum diversity product that accounts for the effect of these HW impairments. Finally, we have shown via simulations that taking into account these HW impairments in the design process yields a significant improvement in SER performance, almost achieving the same error rates obtained with ideal hardware devices.

7. REFERENCES

[1] E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Transactions on Telecommunications*, vol. 10, no. 6, pp. 585–595, 1999.

[2] G. Foschini and M. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, pp. 311–335, 1998.

[3] L. Jing, E. Carvalho, P. Popovski, and A. Oliveras Martinez, "Design and performance analysis of noncoherent detection systems with massive receiver arrays," *IEEE Trans. Signal Proc.*, vol. 64, no. 19, pp. 5000–5010, 2016.

[4] T. L. Marzetta and B. M. Hochwald, "Capacity of a mobile multiple-antenna communication link in Rayleigh flat fading," *IEEE Trans. Inf. Theory*, vol. 45, no. 1, pp. 139–157, 1999.

[5] B. M. Hochwald and T. L. Marzetta, "Unitary space-time modulation for multiple-antenna communication in Rayleigh flat-fading," *IEEE Trans. Inf. Theory*, vol. 46, no. 6, pp. 543–564, 2000.

[6] L. Zheng and D.N.C. Tse, "Communication on the Grassmann manifold: a geometric approach to the noncoherent multiple-antenna channel," *IEEE Trans. Inf. Theory*, vol. 48, no. 2, pp. 359–383, 2002.

[7] B. M. Hochwald, T. L. Marzetta, T. J Richardson, W. Sweldens, and R. Urbanke, "Systematic design of unitary space-time constellations," *IEEE Trans. Inf. Theory*, vol. 48, no. 6, pp. 1962–1973, 2000.

[8] M. Beko, J. Xavier, and V. A. N. Barroso, "Noncoherent communications in multiple-antenna systems: receiver design and codebook construction," *IEEE Trans. Inf. Theory*, vol. 55, no. 12, pp. 5703–5715, 2007.

[9] R. H. Gohary and T. N. Davidson, "Noncoherent MIMO communication: Grassmannian constellations and efficient detection," *IEEE Trans. Inf. Theory*, vol. 55, no. 3, pp. 1176–1205, 2009.

[10] I. Kammoun, A. M. Cipriano, and J. Belfiore, "Non-coherent codes over the Grassmannian," *IEEE Trans. Wireless Comm.*, vol. 6, no. 10, pp. 3657–3667, 2007.

[11] G. Han and J. Rosenthal, "Geometrical and numerical design of structured unitary space-time constellations," *IEEE Trans. Inf. Theory*, vol. 52, no. 8, pp. 3722–3735, 2006.

[12] K. H. Ngo, A. Decurninge, M. Guillaud, and S. Yang, "Cube-split: A structured Grassmannian constellation for non-coherent SIMO communications," *IEEE Trans. Wireless Comm.*, vol. 19, no. 3, pp. 1948–1964, 2020.

[13] D. Cuevas, C. Beltrán, I. Santamaria, V. Tuček, and G. Peters, "A fast algorithm for designing Grassmannian constellations," in *25th International ITG Workshop on Smart Antennas (WSA 2021)*, (French Riviera, France), Nov. 2021.

[14] J. Álvarez-Vizoso, D. Cuevas, C. Beltrán, I. Santamaria, V. Tuček, and G. Peters, "Coherence-based subspace packings for MIMO noncoherent communications," *30th Eur. Sig. Proc. Conf. (EUSIPCO 2022),* (Belgrade, Serbia), Sept. 2022.

[15] D. Cuevas, J. Álvarez-Vizoso, C. Beltrán, I. Santamaria, V. Tuček, and G. Peters, "Constellations on the sphere with efficient encoding-decoding for noncoherent communications," *IEEE Trans. on Wireless Comm.* (Early Access), 2023, doi: 10.1109/TWC.2023.3292935.

[16] D. Cuevas, J. Álvarez-Vizoso, C. Beltrán, I. Santamaria, V. Tuček, and G. Peters, "Union bound minimization approach for designing Grassmannian constellations," *IEEE Trans. on Comm.*, vol. 71, no. 4, pp. 1940–1952, 2023.

[17] J. H. Conway, R. H. Hardin, and N. J. A. Sloane, "Packing lines, planes, etc.: Packings in Grassmannian spaces," *Experimental Mathematics*, vol. 5, no. 2, pp. 139–159, 1996.

[18] S. Buzzi, I. Chih-Lin, T. E. Klein, H. V. Poor, C. Yang, and A. Zappone, "A survey of energy-efficient techniques for 5G networks and challenges ahead," *IEEE Journal on Selected Areas in Communications*, vol. 34, no. 4, pp. 697–709, 2016.

[19] E. Hossain and M. Hasan, "5G cellular: key enabling technologies and research challenges," *IEEE Instrumentation & Measurement Magazine*, vol. 18, no. 3, pp. 11–21, 2015.

[20] S. Javed, O. Amin, S. S. Ikki, and M.-S. Alouini, "Multiple antenna systems with hardware impairments: New performance limits," *IEEE Trans. on Vehicular Technology*, vol. 68, no. 2, pp. 1593–1606, 2019.

[21] A.-A. A. Boulogeorgos, V. M. Kapinas, R. Schober, and G. K. Karagiannidis, "I/Q-imbalance self-interference coordination," *IEEE Trans. on Wireless Comm.*, vol. 15, no. 6, pp. 4157–4170, 2016.

[22] M. Mokhtar, A. Gomaa, and N. Al-Dhahir, "OFDM AF relaying under I/Q imbalance: Performance analysis and baseband compensation," *IEEE Trans. on Comm.*, vol. 61, no. 4, pp. 1304–1313, 2013.

[23] S. Bucher, G. Yammine, R. F. H. Fischer, and C. Waldschmidt, "On the impact of hardware impairments in noncoherent massive MIMO systems," in *WSA 2020; 24th International ITG Workshop on Smart Antennas*, (Hamburg, Germany), 2020.

[24] D. Ramírez, I. Santamaría, and L. Scharf, *Coherence: In Signal Processing and Machine Learning*, Springer Nature, 2023.

[25] I. Santamaria, L. L. Scharf, and D. Ramírez, "Scale-invariant subspace detectors based on first- and second-order statistical models," *IEEE Trans. Signal Process.*, vol. 68, pp. 6432–6443, 2020.

[26] P. Pakrooh, L. L. Scharf, M. Cheney, A. Homan, and M. Ferrara, "Multipulse adaptive coherence for detection in wind turbine clutter," *IEEE Trans. Aerospace and Electronic Systems.*, vol. 53, no. 6, pp. 3091–3103, Dec. 2017.

[27] D. Ramírez, J. Vía, I. Santamaria, and L. L. Scharf, "Locally most powerful invariant tests for correlation and sphericity of Gaussian vectors," *IEEE Trans. Inf. Theory*, vol. 59, no. 4, pp. 2128–2141, Apr. 2013.

[28] V. Garg, I. Santamaria, D. Ramírez, and L. L. Scharf, "Subspace averaging and order determination for source enumeration," *IEEE Trans. on Signal Proc.*, vol. 67, no. 11, pp. 3028–3041, 2019.