

MIMO Capacity Maximization with Beyond-Diagonal RIS

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Abstract—This paper addresses the problem of maximizing the capacity of a multiple-input multiple-output (MIMO) link assisted by a beyond-diagonal reconfigurable intelligent surface (BD-RIS). We maximize the capacity by alternately optimizing the transmit covariance matrix, and the BD-RIS scattering matrix, which, according to network theory, should be unitary and symmetric. These constraints make the optimization of BD-RIS more challenging than that of diagonal RIS. To find a stationary point of the capacity we apply an iterative algorithm in the manifold of unitary matrices. This leads to an efficient algorithm that always improves the capacity obtained by a diagonal RIS. Through simulation examples, we study the capacity improvement provided by a passive BD-RIS architecture over the conventional RIS model in which the phase shift matrix is diagonal.

Index Terms—Beyond diagonal reconfigurable intelligent surface, multi-antenna communications, manifold optimization.

I. INTRODUCTION

Reconfigurable intelligent surfaces (RISs) are emerging as a new paradigm for the sixth generation (6G) of wireless communications. Composed of a large number of passive elements that can be tuned to modify their electromagnetic reflection properties, an RIS can modify the propagation channel to achieve favorable conditions for wireless communications [1]–[5]. Conventional RIS models, called single-connected RIS, lead to diagonal scattering matrices. Recently, the concept of passive beyond-diagonal RIS or BD-RIS has been proposed as a generalization in which all RIS elements can be connected by variable reactances to each other, thus leading to fully connected scattering matrices which have to be unitary and symmetric [6], [7], [8], [9]. These constraints introduce new challenges in the optimization of the BD-RIS matrix.

In [8], [10], the authors consider a multi-user multiple-input single-output (MU-MISO) downlink channel assisted by a multi-sector BD-RIS in which the RIS matrices of each cell are single-connected and therefore

diagonal. The optimization of multi-sector BD-RIS coefficients is performed on the complex sphere manifold. In [7], the same authors consider more general BD-RIS models including fully-connected architectures with hybrid (reflective + transmissive) BD-RIS, and perform the optimization of the BD-RIS matrix on the Stiefel manifold. Optimization techniques for BD-RIS with a group-connected architecture of group size two have been proposed in [11] which apply the convex-concave procedure to convexify the unitary constraint. Closed-form fully-connected BD-RIS solutions that maximize the equivalent channel gain exist for single-input single-output (SISO) and MISO/SIMO channels [9], [12]. The solutions in [9] and [12] are based on two different symmetric and unitary BD-RIS matrix factorizations, but they are otherwise equivalent. More recently, an approximate solution that maximizes the sum of the equivalent channel gains when the direct channel is not blocked in an MU-MISO scenario has been described in [13]. This suboptimal solution is based on solving a relaxed problem and then projecting the relaxed solution onto the set of symmetric and unitary matrices, for which a closed-form solution exists. The optimization of an BD-RIS to maximize the power received by users in an MU-MISO scenario is considered in [14]. This work considers a frequency-dependent model for the BD-RIS, but the optimization is performed at a certain frequency called priority frequency. Moreover, the restriction of a lossless BD-RIS is not imposed in [14] and therefore the BD-RIS matrix is not necessarily unitary, only symmetrical.

Most of the aforementioned works consider either SISO or MISO/SIMO systems, maximizing the equivalent channel gain or the sum rate in the downlink channel [6], [8]–[11]. In this work, we consider a multiple-input multiple-output (MIMO) link, possibly with an unblocked direct channel, and propose an algorithm to maximize its achievable rate by alternately optimizing the transmit covariance matrix and the fully-connected BD-RIS. The Takagi decomposition [15], [16] allows us to factorize

the symmetric and unitary BD-RIS matrix as the product of a unitary matrix and its transpose. This factorization is exploited in the paper to derive an optimization algorithm on the manifold of unitary matrices. The proposed algorithm provides rate improvements over a diagonal RIS that become more significant as the number of streams, the number of BD-RIS elements, or the transmitted power increases.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

We consider a MIMO link assisted by an BD-RIS in which the transmitter is equipped with N_T antennas, the receiver is equipped with N_R antennas, and the RIS has M elements. We assume perfect channel state information (CSI) at both the transmitter and the receiver. The equivalent $N_R \times N_T$ MIMO channel is

$$\mathbf{H}_{eq} = \mathbf{H} + \mathbf{F}\mathbf{\Theta}\mathbf{G}^H, \quad (1)$$

where $\mathbf{G} \in \mathbb{C}^{N_T \times M}$ is the channel from the transmitter to the RIS, $\mathbf{F} \in \mathbb{C}^{N_R \times M}$ is the channel from the RIS to the receiver, $\mathbf{H} \in \mathbb{C}^{N_R \times N_T}$ is the MIMO direct link, and $\mathbf{\Theta}$ is the $M \times M$ BD-RIS matrix.

A fully connected reflective BD-RIS can be modeled as an M -port reciprocal network where each port is connected to all other ports by a reconfigurable reactance. The feasibility set for the BD-RIS scattering matrix is [6], [7], [9]

$$\mathcal{T}_1 = \{\mathbf{\Theta} \mid \mathbf{\Theta}^H \mathbf{\Theta} = \mathbf{I}_M, \mathbf{\Theta} = \mathbf{\Theta}^T\}, \quad (2)$$

where the unitary constraint $\mathbf{\Theta}^H \mathbf{\Theta} = \mathbf{I}_M$ means that the BD-RIS is passive lossless, whereas the symmetry constraint $\mathbf{\Theta} = \mathbf{\Theta}^T$ comes from the fact that the BD-RIS is a reciprocal passive network for which the power losses are the same between any pair of ports regardless of the direction of propagation.

B. Problem Formulation

The transmitter sends proper Gaussian signals $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_{xx})$, with $\mathbf{R}_{xx} = \mathbb{E}[\mathbf{x}\mathbf{x}^H] \succeq \mathbf{0}$ and $\text{tr}(\mathbf{R}_{xx}) \leq P$. Under these assumptions, the maximization of the capacity of an BD-RIS-assisted MIMO link can be formulated as follows:

$$(\mathcal{P}_1) : \max_{\mathbf{\Theta}, \mathbf{R}_{xx}} \log \det \left(\mathbf{I}_{N_R} + \frac{1}{\sigma_n^2} \mathbf{H}_{eq} \mathbf{R}_{xx} \mathbf{H}_{eq}^H \right) \quad (3a)$$

$$\text{tr}(\mathbf{R}_{xx}) \leq P, \quad \mathbf{R}_{xx} \succeq \mathbf{0} \quad (3b)$$

$$\mathbf{\Theta} \in \mathcal{T}_1. \quad (3c)$$

where $\sigma_n^2 \mathbf{I}_{N_R}$ is the noise covariance matrix of the additive white Gaussian noise at the receiver, and \mathbf{H}_{eq} is the equivalent channel in (1). Besides, (3b) are the transmit covariance matrix constraints. In this paper, \log denotes the natural logarithm [nats].

III. CAPACITY MAXIMIZATION WITH PASSIVE BD-RIS

As it is usually done to solve this type of non-convex problems, we consider an alternating optimization procedure in which the covariance matrix \mathbf{R}_{xx} is first optimized for a fixed $\mathbf{\Theta}$ and, subsequently, $\mathbf{\Theta}$ is optimized for a fixed \mathbf{R}_{xx} , repeating the iterations until convergence.

A. Optimizing \mathbf{R}_{xx} for a given $\mathbf{\Theta}$

For a fixed $\mathbf{\Theta}$, finding the optimal \mathbf{R}_{xx} that maximizes capacity is a convex problem whose solution is $\mathbf{R}_{xx} = \mathbf{V}\mathbf{P}\mathbf{V}^H$ where \mathbf{V} is the left eigenspace of the equivalent MIMO channel matrix \mathbf{H}_{eq} and $\mathbf{P} = \text{diag}(P_1, \dots, P_d)$ is a diagonal matrix where P_i denotes the optimal power allocated to the i th stream given by the water-filling strategy to satisfy the power constraint $\sum_i P_i = P$.

B. Optimizing $\mathbf{\Theta}$ for a given \mathbf{R}_{xx}

Once \mathbf{R}_{xx} is obtained we can define $\bar{\mathbf{H}} = \mathbf{H}\mathbf{V}\mathbf{P}^{1/2}$ and $\bar{\mathbf{G}} = \mathbf{P}^{1/2}\mathbf{V}^H\mathbf{G}$. The new equivalent channel, with the \mathbf{R}_{xx} absorbed in $\bar{\mathbf{H}}$ and $\bar{\mathbf{G}}$, is denoted as $\bar{\mathbf{H}}_{eq}(\mathbf{\Theta}) = \bar{\mathbf{H}} + \mathbf{F}\mathbf{\Theta}\bar{\mathbf{G}}^H$, where we now stress the dependency of the channel on the scattering matrix. The max-capacity problem involving only the BD-RIS is

$$(\mathcal{P}_2) : \max_{\mathbf{\Theta} \in \mathcal{T}_1} \log \det \left(\mathbf{I}_{N_R} + \frac{1}{\sigma_n^2} \bar{\mathbf{H}}_{eq}(\mathbf{\Theta}) \bar{\mathbf{H}}_{eq}(\mathbf{\Theta})^H \right).$$

Problem \mathcal{P}_2 is non-convex because the capacity function is non-concave over $\mathbf{\Theta}$. The standard technique to solve this type of problem consists of applying a minorize-majorization (MM) approach [17] based on the concave lower bound shown in the following lemma. This lower bound is proved in [18, Appendix B] and has successfully been used in rate optimization problems [5], [19], [20].

Lemma 1 ([18]). *Let us denote*

$$C(\mathbf{\Theta}) = \log \det \left(\mathbf{I}_{N_R} + \frac{1}{\sigma_n^2} \bar{\mathbf{H}}_{eq}(\mathbf{\Theta}) \bar{\mathbf{H}}_{eq}(\mathbf{\Theta})^H \right),$$

given a feasible solution at iteration t , $\mathbf{\Theta}_t$, the following concave-lower bound of the capacity exists.

$$C(\mathbf{\Theta}) \geq C_t + \frac{2}{\sigma_n^2} \text{Re} \{ \text{tr}(\mathbf{H}_t(\bar{\mathbf{H}}_{eq}(\mathbf{\Theta}) - \mathbf{H}_t)^H) \} \\ - \text{tr}(\mathbf{R}_t^H(\bar{\mathbf{H}}_{eq}(\mathbf{\Theta})\bar{\mathbf{H}}_{eq}(\mathbf{\Theta})^H - \mathbf{H}_t\mathbf{H}_t^H))$$

where $C_t = C(\mathbf{\Theta}_t)$, $\mathbf{H}_t = \bar{\mathbf{H}}_{eq}(\mathbf{\Theta}_t)$, and $\mathbf{R}_t = \frac{1}{\sigma_n^2} \mathbf{I}_{N_R} - (\sigma_n^2 \mathbf{I}_{N_R} + \mathbf{H}_t\mathbf{H}_t^H)^{-1}$.

Let us define $\mathbf{F}_t = \mathbf{R}_t^{1/2}\mathbf{F}$ and $\mathbf{Z}_t = \mathbf{H}_t^H/\sigma_n^2 - \bar{\mathbf{H}}^H\mathbf{R}_t$. After some straightforward calculations, the concave lower bound can be expressed more compactly as

$$C(\mathbf{\Theta}) \geq A_t + J(\mathbf{\Theta}), \quad (5)$$

where A_t is a constant and we have defined

$$J(\Theta) = 2 \operatorname{Re} \left\{ \operatorname{tr}(\mathbf{Z}_t \mathbf{F} \Theta \bar{\mathbf{G}}^H) \right\} - \|\mathbf{F}_t \Theta \bar{\mathbf{G}}^H\|_F^2,$$

which is a quadratic function of the BD-RIS matrix Θ . Therefore, the optimization problem to be solved is the maximization of $J(\Theta)$ subject to the unitary and symmetric matrix constraints. To this end, we apply Tagaki's factorization which allows us to write the optimization problem only under the constraint of unitarity. The Takagi factorization theorem [12], [15], [16] proves that any unitary and symmetric complex matrix can be factored as $\Theta = \mathbf{Q}\mathbf{Q}^T$, where \mathbf{Q} belongs to the manifold of complex unitary matrices denoted here as $\mathcal{U}(M)$. We use this fact to formulate the following optimization problem

$$(\mathcal{P}_3) : \max_{\mathbf{Q} \in \mathcal{U}(M)} J(\mathbf{Q}\mathbf{Q}^T). \quad (6)$$

To solve (\mathcal{P}_3) we apply a gradient ascent optimization on the unitary group [21]. The tangent plane at a point $\mathbf{Q} \in \mathcal{U}(M)$ is obtained by differentiating $\mathbf{Q}^H \mathbf{Q} = \mathbf{I}_M$, which yields

$$\mathbf{Q}^H \Delta \mathbf{Q} + \Delta \mathbf{Q}^H \mathbf{Q} = 0,$$

so the tangent plane, $\Delta \mathbf{Q}$, is composed of all $M \times M$ matrices such that $\mathbf{Q}^H \Delta \mathbf{Q}$ is skew-Hermitian. This is a space of real dimension M^2 . The cost function in (6) is not analytic, therefore, we use Wirtinger calculus taking derivatives with respect to \mathbf{Q}^* assuming \mathbf{Q} constant. Furthermore, Theorem 3.4 in [22] proves that the direction where the real function J (for notational simplicity, from now on we drop the dependence of J on the matrix \mathbf{Q}) has the maximum rate of change is given by the complex matrix derivative with respect to \mathbf{Q}^*

$$\nabla_{\mathbf{Q}^*} J = \left[\mathbf{A}^H - (\mathbf{F}_t^H \mathbf{F}_t)(\mathbf{Q}\mathbf{Q}^T)(\bar{\mathbf{G}}^H \bar{\mathbf{G}}) \right] \mathbf{Q}^* \quad (7)$$

where $\mathbf{A} = \mathbf{G}_q^H \mathbf{Z}_t \mathbf{F}$. The projection of the unconstrained gradient onto the tangent space at \mathbf{Q} is $\pi_T(\nabla_{\mathbf{Q}^*} J) = \mathbf{Q} \mathbf{S}_{skew}$, where

$$\mathbf{S}_{skew} = ((\nabla_{\mathbf{Q}^*} J)^H \mathbf{Q} - \mathbf{Q}^H (\nabla_{\mathbf{Q}^*} J)) / 2 \quad (8)$$

is skew-Hermitian. Finally, to update the unitary matrix we move along the geodesic starting at \mathbf{Q} (the value at the current iteration) with direction $\nabla_{\mathbf{Q}^*} J = \mathbf{Q} \mathbf{S}_{skew}$ as

$$\mathbf{Q} = \mathbf{Q} e^{\mu \mathbf{S}_{skew}}, \quad (9)$$

where $e^{\mathbf{A}}$ denotes here the matrix exponential. The learning step size, $\mu > 0$, can be conveniently chosen and adapted using a line search procedure. A summary of the proposed method is shown in Algorithm 1¹.

¹Matlab code can be downloaded from <https://github.com/IgnacioSantamaria/Code-BD-RIS-SPAWC2024>.

Algorithm 1: Max. Capacity BD-RIS

Input: Initial $\mathbf{Q} \in \mathcal{U}(M)$; $\mathbf{H}, \mathbf{F}, \mathbf{G}, \sigma_n^2, P, \mu$, convergence thresholds: ϵ_C, ϵ_J

Output: Final BD-RIS $\Theta = \mathbf{Q}\mathbf{Q}^T$

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1 while Cap. improvement larger than  $\epsilon_C$  do
    /* Update  $\mathbf{R}_{xx}$  for a given  $\Theta$  */
2   Obtain  $\mathbf{R}_{xx} = \mathbf{V}\mathbf{P}\mathbf{V}^H$  via SVD+waterfilling
    /* Update  $\Theta$  for a given  $\mathbf{R}_{xx}$  */
3   Obtain  $\bar{\mathbf{H}} = \mathbf{H}\mathbf{V}\mathbf{P}^{1/2}$  and  $\bar{\mathbf{G}} = \mathbf{P}^{1/2}\mathbf{V}^H \mathbf{G}$ 
4   while Cap. improvement larger than  $\epsilon_C$  do
        /* Minorizer:  $J(\mathbf{Q}\mathbf{Q}^T)$  */
5       Obtain  $\mathbf{H}_t, \mathbf{R}_t, \mathbf{F}_t, \mathbf{Z}_t$ , and  $\mathbf{A}$  as
           indicated in Sec. III-B
        /*  $\mathcal{P}_3$ : Maximize  $J(\mathbf{Q}\mathbf{Q}^T)$  */
6       while  $\|J(\Theta_{k+1}) - J(\Theta_k)\| < \epsilon_J$  do
7           Calculate  $\nabla_{\mathbf{Q}^*} J$  as (7)
8           Find  $\mathbf{S}_{skew}$  as (8)
9           Update  $\mathbf{Q}$  as (9)

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IV. SIMULATION RESULTS

In this section, we evaluate the results of the max-capacity BD-RIS algorithm. We compare the results with a diagonal RIS whose phase shifts have been optimized to maximize capacity using the algorithm in [23] (labeled as RIS in the figures), and the solution for a BD-RIS recently proposed in [13] (labeled as BD-RIS Low-complexity). The latter algorithm finds a suboptimal (but closed-form) solution for the maximization of the Frobenius norm of the MIMO channel and subsequently performs the projection of this solution onto the set of unitary and symmetric matrices.

a) Scenario description: The transmitter with N_T antennas and the receiver with N_R antennas are located at (0,0,1.5) and (50,0,1.5), respectively, where all coordinates are in meters. The RIS location is $(d, 5, 5)$ where d is varied along the x-axis from $d = 10[m]$ to $d = 100[m]$. The bandwidth is 20 MHz and the system operates at 2.4 GHz. The path loss is $PL = PL_0 - \alpha 10 \log_{10} d$ where $PL_0 = -28$ dB is the path loss at a reference distance of $d_0 = 1$ meter and α is the path loss exponent. The power spectral density for the additive noise is $\sigma_n^2 = -174 + 10 \log_{10} B$ dBm. The transmit power is $P = 100$ mW. The small-scale fading for the direct MIMO link is modeled as a Rayleigh channel, and for the large-scale fading we use a path-loss exponent $\alpha_d = 3.75$. The path loss exponent for the channels \mathbf{G} and \mathbf{F} through the RIS is $\alpha = 2$ and the small-scale fading is assumed to be Rician with a Rice factor $\gamma = 3$. Other parameters are taken from [24].

b) Results varying the BD-RIS position: The first scenario considers a 4×4 MIMO channel and an BD-RIS with $M = 100$ elements. Fig. 1 shows the spectral efficiency in (b/s/Hz) as the BD-RIS varies its location along the x-axis from $x_{RIS} = 10$ m. to $x_{RIS} = 100$ m. For each RIS position, we average the result of 100 independent channel realizations. The highest achievable rate is provided by the proposed BD-RIS algorithm, which always improves the results of a diagonal RIS with optimized phases, both being far superior to a diagonal RIS with random phases. As a baseline, Fig. 1 also shows the rate without RIS. Finally, the results of the closed-form BD-RIS solution proposed in [13] can be even worse than those of a diagonal RIS. This result is expected because the low-complexity BD-RIS solution performs a projection onto the set of unitary and symmetric matrices, which is clearly suboptimal in terms of capacity.

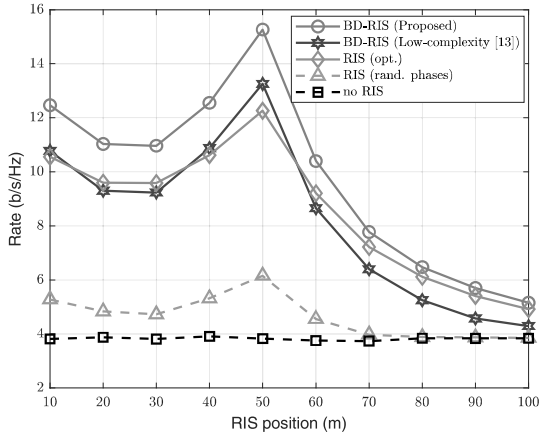


Fig. 1: Achievable rate for a 4×4 MIMO link assisted by an BD-RIS with $M = 100$ elements.

c) Results varying the number of BD-RIS elements: Fig. 2 shows the rates when the number of BD-RIS elements ranges from $M = 10$ to $M = 100$ when the RIS coordinates are (50,5,5) for 2×2 and 4×4 MIMO channels. For a 2×2 channel, the solution in [13] is quite competitive, especially for higher M values. As the number of streams of the MIMO system grows the differences between the proposed BD-RIS solution and the method in [13] become more significant.

d) Results varying the Tx power: Finally, Fig. 3 shows the achievable rates obtained by varying the input power between 4 and 30 dBms (which corresponds approximately to SNRs at the receiver between 15 and 35 dB) for 4×4 and 8×8 MIMO links assisted by an BD-RIS with $M = 100$ elements. The improvements of the proposed algorithm are more appreciable for higher transmitting powers. Finally, Fig. 4 shows the number of

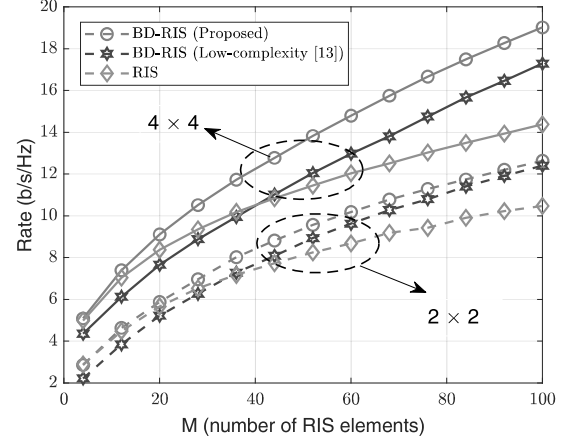


Fig. 2: Achievable rates for an increasing number of BD-RIS elements M for 2×2 and 4×4 MIMO channels.

active streams vs. the Tx power. To obtain the number of active streams we average the rank of the transmit covariance matrix after convergence over 100 simulations. This figure suggests that the capacity improvements of BD-RIS over RIS can be attributed to the fact that BD-RIS creates channels that are better conditioned to support the transmission of a larger number of streams (for lower SNR).

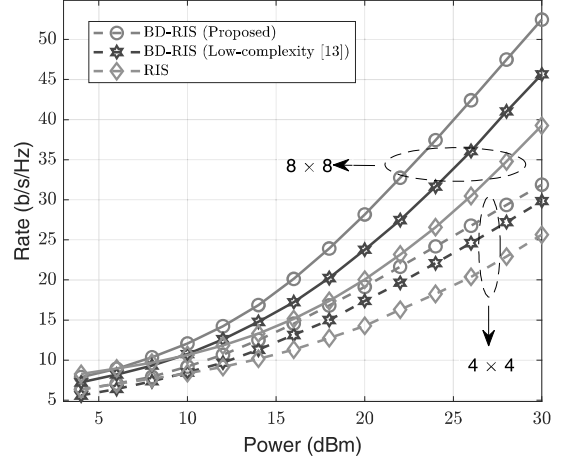


Fig. 3: Achievable rate vs. transmit power for BD-RIS and RIS.

V. CONCLUSIONS

We have proposed an algorithm for maximizing the capacity of a MIMO link assisted by a fully-connected BD-RIS. The main difficulty of the resulting optimization problem arises from the unitarity and symmetry constraints that the BD-RIS scattering matrix must satisfy.

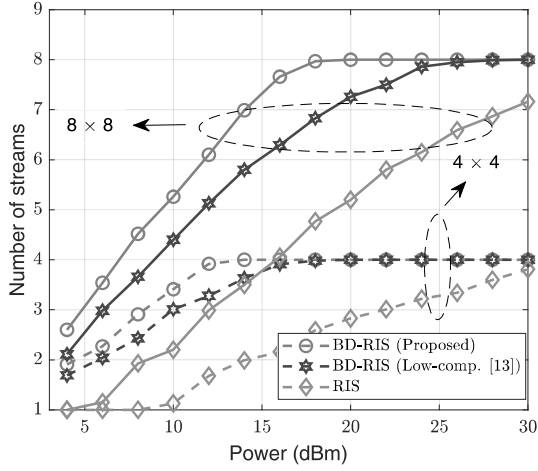


Fig. 4: Number of active streams vs. transmit power for BD-RIS and RIS architectures.

We have applied Takagi’s factorization to overcome this difficulty, which allows us to derive an optimization algorithm in the manifold of unitary matrices. Compared to a diagonal RIS, the improvements in the achievable rates are more significant as the number of streams, the number of reflector elements, or the transmitted power increases. The proposed algorithm can be extended to group-connected BD-RIS architectures, as well as to multi-user MIMO networks. The development and analysis of these extensions remain as future work.

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