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Comparative analysis of flexural strength prediction in SFRC using frequentist, Bayesian, and Machine Learning approaches

Ángel De La Rosa [a,*], José Sáinz-Aja [b], Isaac Rivas [b], Gonzalo Ruiz [c], Diego Ferreño [b]

[a] *DIMME, Grupo de Durabilidad e Integridad Mecánica de Materiales Estructurales, Universidad Rey Juan Carlos, C. Tulipán s/n, Móstoles, Madrid 28933, Spain*
[b] *LADICIM (Laboratory of Materials Science and Engineering), ETSI Caminos, C. y P., Universidad de Cantabria, Av. Los Castros 44, Santander 39005, Spain*
[c] *ETSI Caminos, C. y P. de Ciudad Real, Universidad de Castilla-La Mancha, Av. Camilo José Cela 2, Ciudad Real 13071, Spain*

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ABSTRACT

Steel fiber reinforcement significantly enhances the flexural strength of concrete, which is vital for structural integrity. Annex L of the new Eurocode 2 classifies steel fiber-reinforced concrete by its flexural performance, aiding engineers in designing resilient structures. This study investigates the flexural behavior of steel fiber-reinforced concrete (SFRC) using three data-driven methodologies: Frequentist Inference (FI), Bayesian Inference (BI), and Machine Learning (ML). A comprehensive database was constructed from three-point bending tests on SFRC specimens, encompassing various compressive strengths, fiber quantities, and geometric parameters, to identify key factors influencing material properties.

The findings indicate that all three methodologies yield comparable predictive capabilities for flexural responses in SFRC. Notably, FI models emphasize the importance of compressive strength and fiber volume fraction, along with fiber properties such as non-dimensional length and tensile strength. BI models enhance predictive stability by integrating prior knowledge and quantifying uncertainty, demonstrating their advantage, particularly in data-scarce situations. Additionally, ML analysis reveals that linear regression (LR) models can achieve accuracy similar to or greater than that of more complex models. This research provides novel insights into the application of BI and ML in concrete technology, emphasizing their potential to enhance predictive modeling. Additionally, it offers practical guidelines for optimizing SFRC design through a case study that compares residual flexural strengths obtained via Bayesian analysis, classifying the material in accordance with Annex L of the new Eurocode 2.

1. Introduction

Recent advancements in steel-fiber reinforced concrete (SFRC) technology have resulted in specific provisions in structural design codes [1,2], including Annex L of Eurocode 2 [3], the Model Code 2020 [4], and the new Spanish Structural Code [5]. However, uncertainties in modeling and variability in results remain significant challenges, hindering wider adoption despite extensive scientific literature on the material's performance under various loads [6]. These challenges highlight the need to revisit existing models and variables. The development of new technological tools and access to comprehensive databases will facilitate a deeper investigation

* Corresponding author.
E-mail address: angel.delarosa@urjc.es (Á. De La Rosa).

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Nomenclature

BI	Bayesian inference
d_f	Fiber diameter
DT	Decision Tree
E_{cf}	Elastic modulus of SFRC in 150 × 300 mm^2 cylinders
f_{cf}	Compressive strength of SFRC in 150 × 300 mm^2 cylinders
f_{cuf}	Compressive strength of SFRC in 150 mm–edged cubes
$f_{L,\mathrm{k}}$	Characteristic flexural strength (or proportionality limit) of SFRC in 150 × 150 × 550 mm^3 prisms
$f_{L,\mathrm{m}}$	Average or mean flexural strength (or proportionality limit) of SFRC in 150 × 150 × 550 mm^3 prisms
$f_{R,1\mathrm{k}}$	Characteristic residual flexural strength for $w_M = 0.5$ mm of SFRC in 150 × 150 × 550 mm^3 prisms
$f_{R,1\ \mathrm{m}}$	Average or mean residual flexural strength for $w_M = 0.5$ mm of SFRC in 150 × 150 × 550 mm^3 prisms
$f_{R,3\mathrm{k}}$	Characteristic residual flexural strength for $w_M = 2.5$ mm of SFRC in 150 × 150 × 550 mm^3 prisms
$f_{R,3\ \mathrm{m}}$	Average or mean residual flexural strength for $w_M = 2.5$ mm of SFRC in 150 × 150 × 550 mm^3 prisms
f_{uf}	Ultimate tensile strength of the steel fiber
FI	Frequentist inference
GB	Gradient Boosting
i, j	Index assigned to factors and responses
KNN	k Nearest Neighbour
ℓ_f	Fiber length
$\ell*_f = \frac{\ell_f}{\ell_0}$	Non–dimensional fiber length
$\ell_0 = 30$ mm	Coefficient to keep non–dimensionality
LR	Multiple Linear Regression
ML	Machine Learning
n	Number of factors or independent variables
$N[\beta_{0,m}, \beta_{0,std}]$	Mean ($\beta_{0,m}$) and standard deviation ($\beta_{0,std}$) of β_0 parameter following a Normal distribution function of probability in the Bayesian paradigm
$N[\beta_{i,m}, \beta_{i,std}]$	Mean ($\beta_{i,m}$) and standard deviation ($\beta_{i,std}$) of β_i parameter following a Normal distribution function of probability in the Bayesian paradigm
$N[\sigma_m, \sigma_{std}]$	Mean (σ_m) and standard deviation (σstd) of σ parameter following a Normal distribution function of probability in the Bayesian paradigm
p-value	Statistical parameter to determine statistical significance
R^2	Determination coefficient / multiple correlation coefficient
R_a^2	Adjusted determination coefficient
RF	Random Forest
RMSE	Root mean square error
RSM	Response Surface Methodology
SFRC	Steel–fiber reinforced concrete
w_M	Crack mouth opening displacement (CMOD)
x_i	Independent factors or variables (significant factors)
X_i	Values from the dataset
$\overline{X_i}$	Mean values from the dataset
y	Response or dependent variable
z_i	Normalized variable
α	Non-dimensional coefficient
β_0	Term of constant value
β_i	Coefficients of linear adjustment
$\lambda = \frac{\ell_f}{d_f}$	Fiber aspect ratio
μ	Mean value
ξ	Random error observed in the response
σ	Standard deviation
ϕ_f	Volume fraction of fiber (steel–fiber volume per m^3)
$\phi_f\lambda$	Fiber factor
θ	Standard deviation from the dataset

into these issues.

Focusing on the flexural response, steel-fiber reinforced concrete (SFRC) demonstrates impressive energy absorption and ductility following cracking, along with residual strengths for various crack openings [7]. The post-cracking behavior of SFRC is influenced by several critical factors, including fiber content, geometric characteristics, concrete strength [8], fiber-matrix bonding strength, as well as fiber orientation and distribution [6]. To promote the broader application of this innovative material, the development of high-quality databases for scientific analysis is vital. Significant contributions to this effort include studies by Tiberti *et al.* [7] and Galeote *et al.* [6], alongside the ongoing initiatives of the International Federation for Structural Concrete (*fib*), which is establishing harmonized databases such as the Fiber Reinforced Concrete (FRC) Residual Strength Database [9]. These resources are crucial for advancing our understanding and utilization of SFRC in structural applications.

Statistics play a crucial role in this endeavor, aiming to make inferences about population parameters based on limited observations [10–12]. Frequentist inference (FI) and null hypothesis significance testing are commonly used in empirical data analysis [13]. FI treats population parameters as unknown constants, estimating characteristics through probability, which describes measurement outcomes [13,14]. While FI has its strengths, it opens the door to exploring the potential of Bayesian inference (BI), another statistical paradigm [15]. The scientific process, comprising observation, knowledge accumulation, and prediction—where the accuracy of predictions is contingent upon the quality of initial observations—is effectively illustrated by BI [10–12]. BI considers population parameters as quantifiable random variables, with uncertainty described through probability distribution functions. Subjective probability statements about these parameters—considered a strength of Bayesian analysis [13,16]—are based on experience and knowledge, representing a degree of belief in the parameter values being studied [13,17,18]. As data accumulates, it combines with prior information, resulting in a probability distribution that provides an updated assessment of parameter values [10–12].

Another approach involves the utilization of Machine Learning models (ML). These models discern patterns and relationships within extensive datasets, facilitating the prediction of outcomes [19–21]. In recent years, ML models have gained particular significance due to factors such as the exponential increase in the availability of digital data, advancements in computational capacity, and the development of more sophisticated algorithms [22,23]. The use of these techniques has widely expanded in civil engineering due to their positive results, allowing for more accurate predictions of the behavior of structures and materials, which increases the efficiency and safety of projects [24–26]. Moreover, their adoption has driven significant advancements in concrete manufacturing and the optimal design of concrete structures, improving process efficiency [27].

Data-driven methods can enhance predictions when ample experimental data is available. However, databases often suffer from high variability, missing values, and substantial noise, complicating modeling and interpretation. Bayesian Inference (BI) allows for quantifying uncertainty associated with models derived from limited experimental data, a task that is more challenging using frequentist approaches. Even with limited data, an ML-based strategy can provide valuable insights independent of specific models, thereby enhancing predictive capabilities. Currently, data-driven approaches are increasingly being applied to study the flexural behavior of steel fiber-reinforced concrete (SFRC). However, there is a notable lack of comprehensive studies that combine Frequentist Inference (FI), Bayesian Inference (BI), and ML. Most existing literature addresses mechanical responses in isolation, utilizing FI [6–8], FI and BI [28], or ML [29–32] separately.

This research makes significant contributions to concrete technology by exploring the applications of Bayesian Inference (BI) and Machine Learning (ML) in predicting the flexural strength of steel fiber-reinforced concrete (SFRC), with their combined analysis representing a novel approach that underscores the originality of the article. We begin with Frequentist Inference (FI) methods to establish baseline models and identify key factors influencing flexural response. Recognizing the limitations of traditional FI, which often relies on singular parameter estimates, we advocate for presenting parameters as probability density functions to better accommodate the variability of composite materials and enhance prediction reliability. The integration of Bayesian Statistics is particularly impactful, as it allows for the incorporation of expert knowledge and improves the interpretability of data analysis. Additionally, ML techniques are employed to uncover complex relationships within the dataset, though they require extensive and homogeneous datasets, which can be challenging in concrete technology.

This study's implication lies in the comparison of Bayesian statistics, Machine Learning techniques, and traditional methodologies, providing critical insights into their respective strengths and weaknesses in predicting the flexural strengths of steel fiber-reinforced concrete (SFRC). It clarifies the intricate relationship between material properties and flexural behavior while showcasing the innovative application of Bayesian inference (BI) and Machine Learning (ML), thereby advancing predictive modeling in the field. Additionally, our findings emphasize the significance of addressing uncertainty and variability in material behavior, aligning with recent advancements in design codes such as Eurocode 2 [3] and the Model Code 2020 [4]. By offering a valuable framework for engineers and researchers, this study facilitates informed design decisions and enhances the reliability of SFRC structural elements. Ultimately, it contributes to the broader discourse in the field and highlights the importance of diverse analytical methods in improving decision-making and fostering the development of more resilient and efficient structural designs.

The paper is organized as follows: the introduction outlines the study's motivation and objectives, providing a concise background on the methodologies used. The methodology section describes the experimental setup, the statistical techniques employed, and the Machine Learning models developed. Subsequently, the results and discussion section highlights the findings from both frequentist and Bayesian inference, incorporating a case study that demonstrates the effective application of the proposed Bayesian model in predicting the flexural behavior of steel fiber-reinforced concrete. This is followed by a comparative analysis of the Machine Learning results. The discussion section contextualizes these findings within the existing literature and explores their practical implications. Lastly, the conclusion encapsulates the key insights and proposes avenues for future research.

2. Methodology and materials

2.1. Methodology

2.1.1. Frequentist and Bayesian paradigm

The objective of this section is not to deliver an exhaustive introduction to frequentist and Bayesian inference (abundant scientific literature exists for that purpose), but rather to delineate the foundational aspects of both methodologies as applied in this paper's methodology section.

2.1.1.1. Frequentist linear regression model. The experimental data will be fitted to a first-degree polynomial regression model. This model selection is based on two key criteria: its capacity to accurately predict the response within the range of factor levels and, importantly, the absence of multicollinearity. The choice of linear models is particularly advantageous due to their simplicity, allowing for easier interpretation and communication of results.

Multicollinearity occurs when explanatory variables in a model exhibit high correlation, which can compromise the precision of estimating factor terms and obscure their statistical significance. By avoiding multicollinearity, we enhance the reliability of our findings, enabling clearer insights into the individual effects of each factor. Understanding the importance of these factors is crucial, as they directly influence the model's predictive power and the validity of our conclusions. Thus, the straightforward nature of linear models, combined with the careful consideration of multicollinearity, ensures a robust methodological framework for analyzing the experimental data.

Linear regression (Eq. (1)) serves as the cornerstone of frequentist statistical modeling. This method focuses on establishing linear relationships between a dependent or response variable and one or more explanatory or independent variables. A thorough understanding of the key factors involved in this methodology is essential, as these factors directly influence the model's effectiveness and predictive accuracy. By identifying and analyzing these relationships, we can gain valuable insights into how changes in the independent variables impact the dependent variable, thereby enhancing the interpretability and relevance of our findings. This analysis is essential for clarifying the significance of each factor in relation to our research objectives.

$$y = \beta_0 + \sum_{i=1}^{n} \beta_i x_i + \xi \tag{1}$$

where,

x_i: independent factors or variables
y: response or dependent variable
β_0: a term of constant value
β_i: coefficients of linear adjustment
ξ: the random error observed in the response (attributed mainly to experimental errors and adjustments). The random error is independently and identically normally distributed, with a mean of zero and constant variance.

It is essential for the experimental data to conform to a normal distribution, as determined through residuals. Residuals represent the variations between the observed or experimental values and those adjusted by the model. If these residuals demonstrate a normal distribution, it indicates that the data also adheres to a normal distribution.

For parameter estimation and model fitting, the open-source software R has been utilized. To estimate the parameters, various methods such as Ordinary Least Squares (OLS) and Maximum Likelihood are utilized. Model fitting can be accomplished using the function to fit linear models (R – function lm), which can be employed for regression and single-stratum analysis of variance (ANOVA).

The data analysis will be conducted using ANOVA. The assessment of statistical significance relies on the parameter p-value within a 95 % confidence interval (considering statistical significance if p-value ≤ 0.05). In relation to the examined factors, a p-value less than or equal to 0.05 indicates a statistically significant association between the response variable and the respective term, demonstrating that the effect is not merely due to chance.

The model's accuracy is gauged using the coefficient of determination, R_a^2, which reflects the corrected goodness of fit of the model to the response. This parameter is more suitable than the coefficient R^2 when comparing models with varying numbers of predictors. While R^2 consistently increases with added effects, R_a^2 endeavors to rectify this overestimation, reducing its value if an effect fails to enhance the model.

2.1.1.2. Bayesian linear regression model. Bayesian methodologies have widespread applicability in diverse domains and practical scenarios [33,34], particularly in situations where making well-informed decisions heavily relies on thorough data analysis and interpretation [35–37]. These approaches are centered around families of parameterized distributions, wherein the parameters are regarded as random variables themselves [33,37]. Instead of sticking to a single distribution model, Bayesian frameworks encompass a range of possibilities within a chosen family, utilizing a linear combination of various models. This methodology is pivotal as it broadens the spectrum of available models, allowing the data to guide the selection of the most appropriate one [36,37]. The Bayesian approach to probabilistic modeling typically involves the following stages [38]:

- Selection of the appropriate likelihood model.
- Determination of the prior distribution for the parameters, a crucial step in the methodology as the outcomes for small sample sizes heavily rely on it. This prior information selection can be accomplished through various approaches:
  - Consulting with domain experts to gather a representative virtual sample based on their prior expertise.
  - Extracting information from existing, but not updated, specialized scientific literature.
  - Utilizing data from previous experiments conducted by our own research team (ensuring the quality of the information is paramount).
- Acquisition of data from the sample.
- Calculation of the posterior distribution.
- Combination of the posterior distribution with the likelihood to derive the predictive distribution, which serves as our primary tool.

In this study, we introduce Bayesian methodologies to identify influential factors within linear models that have previously been analyzed using frequentist approaches, utilizing probability density functions. We shift from deterministic parametric models to probabilistic ones by employing Bayesian analysis through the open-source software R [39]. By using the brm function from the brms [40] and rstanarm packages in R [39], powered by the probabilistic programming language Stan, we enable the fitting of complex models through techniques such as Hamiltonian Monte Carlo (HMC) and the No-U-Turn Sampler (NUTS). The model was run with four chains, each consisting of 10000 iterations, to thoroughly explore the posterior probability density functions. This approach allows us to gain a deeper understanding of the significant factors affecting our research objectives.

Our objective is to forecast flexural responses in SFRC based on composite and steel fiber reinforcement factors, necessitating the estimation of specific parameters. The probability density functions governing the mean flexural strength in the proportionality limit ($f_{L,\mathrm{m}}$) the mean residual flexural strength for a crack mouth opening displacement equal to 0.5 mm ($f_{R1,\mathrm{m}}$) and the mean residual flexural strength for a crack mouth opening displacement equal to 2.5 mm ($f_{R3,\mathrm{m}}$) adhere to a Normal distribution with mean μ and standard deviation σ. We estimate μ through a linear model $\mu = \beta_0 + b_i\, x_i$, where x_i represents the significant factors obtained in the frequentist analysis. Model parameters (β_0, β_i, σ) require prior probability density functions. Stan assumes that parameters β_0, β_i, and σ follow Gaussian (Normal) prior probability density functions, with mean, $_m$, and standard deviation, $_{std}$ (see the linear Bayesian model used in this study, defined by Eqs. (2), (3), (4), (5), (6)).

$$f_{L,\mathrm{m}}, f_{R1,\mathrm{m}}, f_{R3,\mathrm{m}} \sim N\,[\mu,\sigma] \tag{2}$$

$$\mu = \beta_0 + \beta_i\, x_i \tag{3}$$

$$\beta_0 \sim N[\beta_{0,m}, \beta_{0,std}] \tag{4}$$

$$\beta_i \sim N[\beta_{i,m}, \beta_{i,std}] \tag{5}$$

$$\sigma \sim N[\sigma_m, \sigma_{std}] \tag{6}$$

2.1.2. Machine learning models

Machine Learning (ML), primarily focused on predictive modeling, explores the potential of leveraging computational capabilities to enhance processes through training and experiential learning [41]. Recent advancements in ML encompass various categories, including classification, regression, ranking, clustering, dimensionality reduction, and manifold learning. Contemporary ML models incorporate diverse approaches, such as linear predictors, boosting, stochastic gradient descent, kernel methods, and nearest neighbors, among others. In comparison to traditional statistical methods, ML models offer notable advantages. These algorithms swiftly identify trends and patterns by analyzing extensive datasets. Furthermore, they continuously enhance their accuracy and efficiency through iterative training with new data. Additionally, ML algorithms effectively handle multi-dimensional and multi-variety data in dynamic or uncertain environments. Lastly, they provide compelling automation capabilities for a variety of decision-making tasks.

In the realm of ML, a diverse range of algorithms is available for crafting predictive models. No single model universally excels; rather, effectiveness depends on the specific dataset. To identify the most suitable model, various models are tested, and the one yielding optimal results is selected. The evaluation of a model's effectiveness commences with dividing the data into training (75 % of the total) and testing groups. Training data adjust model parameters, while the test data, previously untouched by the model, evaluate its performance. For model evaluation, a 3-fold cross-validation strategy was employed, ensuring that the dataset was split into three subsets to optimize the use of the available data and assess model performance across multiple iterations. Model quality is measured by comparing predicted values with actual values, utilizing Root Mean Square Error (RMSE) as the metric.

Given limited data, the bootstrapping resampling technique assesses model robustness. The selected predictive models include:

- Multiple Linear Regression (LR): predicts a dependent variable using two or more independent variables, under the assumption of a linear relationship between them. This method is effective for understanding how multiple factors influence an outcome.
- k Nearest Neighbour (KNN): classifies or predicts a sample based on the majority label or the average value of its k nearest neighbors in the feature space. This non-parametric method is simple yet powerful for classification and regression tasks.

- Decision Tree (DT): constructs a hierarchical tree where each internal node represents a feature, each branch a decision rule, and each leaf node represents the predicted outcome. This intuitive model is highly interpretable and can handle both categorical and numerical data.
- Random Forest (RF): an ensemble learning technique that builds multiple decision trees and aggregates their outputs for more robust and accurate predictions. By combining the wisdom of multiple trees, Random Forest reduces overfitting and improves generalization.
- Gradient Boosting (GB): a sequential ensemble method where models are built iteratively. Each new model focuses on correcting the errors of its predecessors, leading to progressively improved accuracy. Gradient Boosting is particularly effective in handling complex patterns in data.

These models, chosen for relevance and efficacy, aim to identify the most suitable for the specific dataset. The model selection process involves using the following combination of inputs. We not include the formulas for the models because detailed information on the Machine Learning models are in specialized parers as [42].

2.2. Materials

This study examines the flexural behavior of steel fiber-reinforced concrete (SFRC) through three-point bending tests, with the goal of advancing the understanding of its mechanical properties. Notably, the classification of this behavior by strength classes has recently been included in Annex L of the new Eurocode 2 [3], providing valuable insights for its application in structural engineering and construction. The concrete specimens used in this research contain steel fibers with hooked ends and a single fold, designed to enhance their performance under load-bearing conditions.

2.2.1. Database

Selecting the factors for analyzing various responses is crucial for achieving favorable outcomes with the implemented method. High-quality data must be collected within the desired intervals to observe the responses effectively. Prior knowledge of the relationships between dependent and independent variables is fundamental for identifying factors that genuinely influence outcomes. For this purpose, the database created for the research about the flexural post-cracking behavior of steel fiber reinforced concrete by Tiberti et al. [7] has been utilized. This database was generated through a series of experimental campaigns involving three-point bending tests on steel fiber-reinforced concrete (SFRC) specimens. These specimens, designed as prisms with dimensions of 150 × height × 550 mm^3 (width × height × length), featured steel fibers with hooked ends and a single fold. Conducted at the University of Brescia in Italy, this effort resulted in a custom database containing a total of 484 outcomes.

The sample size of the database consists of 74 observations, each representing the mean of 3–13 tests, yielding a comprehensive collection of 7 independent variables (Table 1) and 6 dependent variables ($f_{L,\mathrm{m}}$, $f_{R1,\mathrm{m}}$, $f_{R3,\mathrm{m}}$, $f_{L,\mathrm{k}}$, $f_{R1,\mathrm{k}}$, $f_{R3,\mathrm{k}}$ – Table 2). These independent variables encompass a range of factors influencing the flexural response of SFRC according to scientific literature [6–8]. For data preprocessing, each input variable was normalized using a standard scaler, which involved subtracting the mean and dividing by the standard deviation (Eq. (7)). This normalization creates a dimensionless ratio, facilitating comparison and analysis. It is particularly important for distance-based algorithms and beneficial for assessing variable importance, as all inputs typically fall within a range of ± 2.

$$z_i = \frac{X_i - \overline{X_i}}{\theta} \tag{7}$$

where z_i denotes the normalized variable X_i, with $\overline{X_i}$ and θ representing the mean and standard deviation values derived from the dataset, respectively.

Fig. 1 visually presents the distribution of the dataset, where the height of each bar indicates the relative frequency or probability density of each value, ensuring the total area of the histogram equals 1. Furthermore, a preliminary data analysis was conducted, and a correlation matrix was generated using the complete dataset, as depicted in Fig. 2. This matrix illustrates the relationships between input factors and the target variable, enhancing understanding of how various elements interact within the dataset (note that $\phi_f\lambda$ is the product of two independent variables).

Based on the observations in Fig. 2, a significant positive correlation is evident among certain variables. Particularly notable is the high correlation between the compressive strength in cubic specimens (f_{cuf}) and the compressive strength in cylindrical specimens (f_{cf})

Table 1
Statistics of the independent variables of the database.

	f_{cuf} [MPa]	ϕ_f [%]	ℓ^*_f	λ	$\phi_f\lambda$	E_{cf} [GPa]	f_{uf} [MPa]
Mean	59.0	0.6	1.00	60	0.361	35.9	1473
Median	53.7	0.6	1.10	60	0.320	36.5	1345
Mode	75.0	0.4	1.17	56	0.640	35.1	1500
Maximum	89.7	1.0	1.87	100	0.800	42.0	3100
Minimum	30.6	0.3	0.70	44	0.160	23.7	1100
Standard deviation	14.0	0.2	0.23	11	0.145	3.7	455

Table 2
Statistics of the dependent variables of the database.

	$f_{L,m}$ [MPa]	$f_{R1,m}$ [MPa]	$f_{R3,m}$ [MPa]	$f_{L,k}$ [MPa]	$f_{R1,k}$ [MPa]	$f_{R3,k}$ [MPa]
Mean	5.5	5.7	5.6	4.7	4.5	4.3
Median	5.4	5.6	5.5	4.7	4.5	4.3
Mode	4.5	4.6	5.2	4.8	1.7	5.4
Maximum	7.7	12.4	11.8	7.0	10.8	10.0
Minimum	3.0	1.7	1.2	2.7	1.1	0.7
Standard deviation	1.1	2.3	2.4	0.9	2.1	2.2

a well-known phenomenon. However, given the substantial correlation, it is crucial to consider excluding both variables simultaneously when calibrating predictive models. Fig. 2 also reveals that none of the inputs exhibit a strong direct correlation with any of the outputs. Consequently, it can be inferred that models constructed based on a single variable may not yield satisfactory results. Finally, it is noteworthy to mention the substantial correlation among the outputs, with a high correlation observed between $f_{L,m}$ and the characteristic flexural strength in the proportionality limit ($f_{L,k}$) as well as between $f_{R1,m}$, the characteristic residual flexural strength for a crack mouth opening displacement equal to 0.5 mm, $f_{R1,k}$, $f_{R3,m}$, and the characteristic residual flexural strength for a crack mouth opening displacement equal to 2.5 mm, $f_{R3,k}$, though not between the two families.

3. Results and discussion

As mentioned earlier, to carry out the model fitting, we have utilized the open-source software R [39]. For the frequentist linear regression model, the lm function has been employed, and for the Bayesian linear regression model, the brm function from the brms [40] and rstanarm libraries has been used. Both functions require providing the formula and the data to be used, while keeping all subsequent arguments at their default values. For the analysis using ML techniques, Python [43], a high-level programming language with a wide range of libraries for data analysis and ML, was used. In particular, for the implementation of the models, the scikit-learn library was utilized, which collects the main algorithms of ML, as well as tools for data analysis.

3.1. Frequentist linear regression analysis

Three different models have been studied, in each of which a series of factors was selected to carry out frequentist linear regression analysis of the three primary responses to flexural strength in steel-fiber reinforced concrete: $f_{L,m}$, $f_{R1,m}$, and $f_{R3,m}$, until statistically significant factors (p-values ≤ 0.05) were obtained. In all cases, the response values are given in MPa.

3.1.1. Model no 1

Table 3 displays the calculated values of each factor for the response variables, while Table 4 shows the corresponding p-values. For each response, the first row presents the coefficients from the initial iteration conducted with all factors, while the last row exhibits the coefficients obtained by exclusively selecting the statistically significant factors from the previous row (or previous iteration).

The p-values obtained in Table 4 indicate statistically significant factors (that is, according to the analysis of variance, these factors have a real impact on the response): for $f_{L,m}$, f_{cuf} and ϕ_f; for $f_{R1,m}$, f_{cuf} and f_{uf}; for $f_{R3,m}$, f_{cuf}, ℓ^*_f and f_{uf}.

3.1.2. Model no 2

Similar to model no 1, Table 5 displays the calculated values of each factor for the response variables, while Table 6 shows the corresponding p-values.

The p-values obtained in Table 6 reflect statistically significant factors: for $f_{L,m}$, f_{cuf} and ϕ_f; for $f_{R1,m}$, f_{cuf}, ϕ_f and λ; for $f_{R3,m}$, f_{cuf}, ϕ_f, ℓ^*_f and λ.

3.1.3. Model no 3

Again, Table 7 displays the calculated values of each factor for the response variables, while Table 8 shows the corresponding p-values.

The p-values incluided in Table 8 show statistically significant factors: for $f_{L,m}$, f_{cuf} and ϕ_f; for $f_{R1,m}$, f_{cuf}, ϕ_f, λ and f_{uf}; for $f_{R3,m}$, f_{cuf}, ϕ_f, ℓ^*_f, λ and f_{uf}.

3.1.4. Discussion of the frequentist linear regression analysis

Three different models were studied with varying complexities: model no 1 included all potential factors affecting flexural response, model no 2 focused on a simplified set of key factors, and model no 3 expanded on model 2 by including fiber tensile strength.

Model no 1 includes those factors related to the composite material, such as compressive strength, f_{cuf}, and elastic modulus, E_{cf}, and those exclusively related to the fiber, such as volume fraction of fiber, ϕ_f, non-dimensional length, ℓ^*_f, aspect ratio, λ, fiber factor, $\phi_f\lambda$, and tensile strength, f_{uf}. It represents the most complex model among those proposed. The results obtained indicate, for the three analyzed responses, the importance of two factors above the rest: f_{cuf} and ϕ_f. This is reasonable since both clearly influence the flexural response of SFRC, as indicated by Ruiz et al. [8]. Furthermore, for residual flexural strengths, two factors related exclusively to the

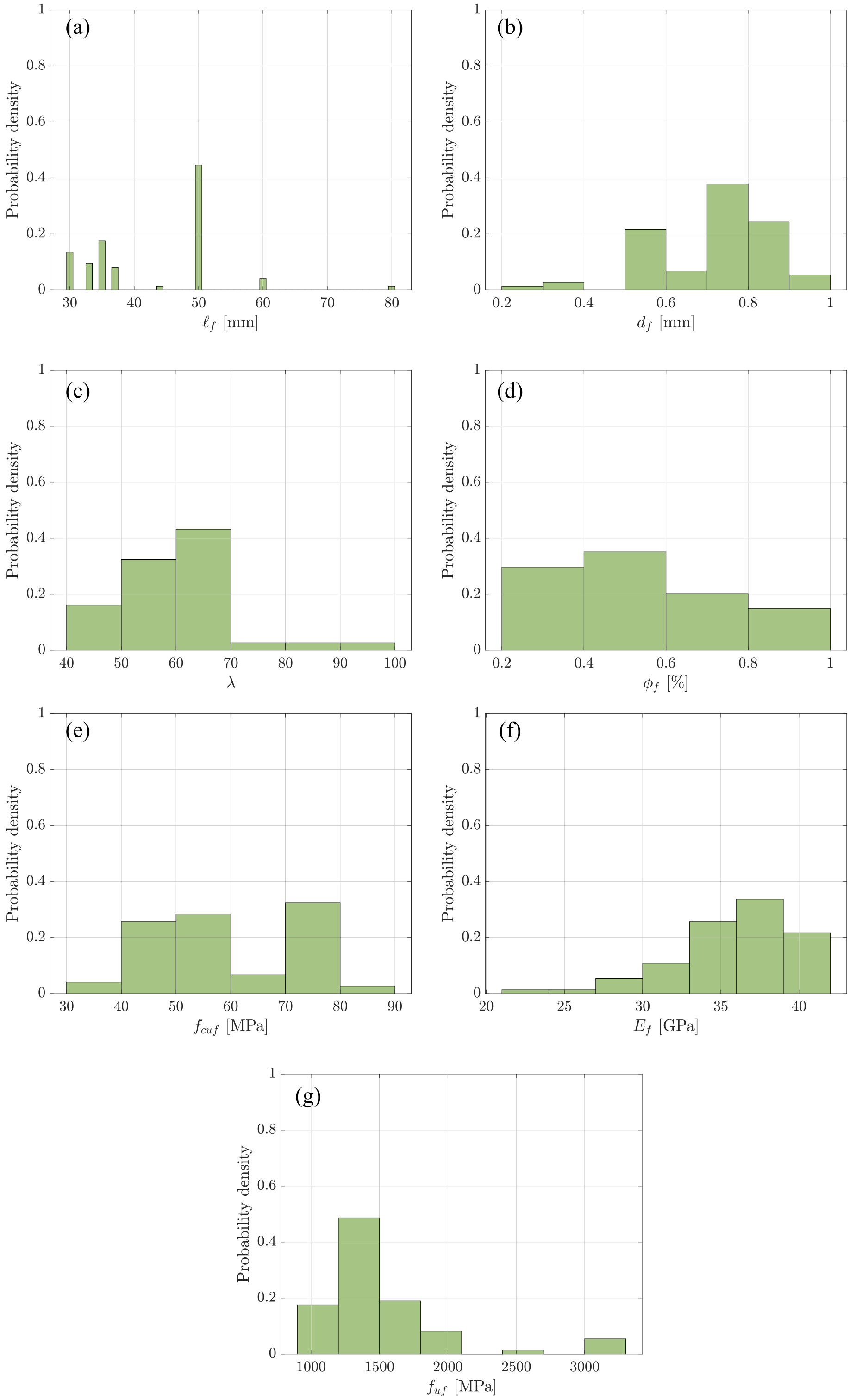

Fig. 1. Distribution of the data within the database: (a) non–dimensional fiber length (ℓ^*_f), (b) fiber diameter (d_f), (c) aspect ratio (λ), (d) volume fraction of fiber (ϕ_f), (e) compressive strength of SFRC (f_{cuf}), (f) elastic modulus of SFRC (E_{cf}), (g) ultimate tensile strength of the steel fiber (f_{uf}).

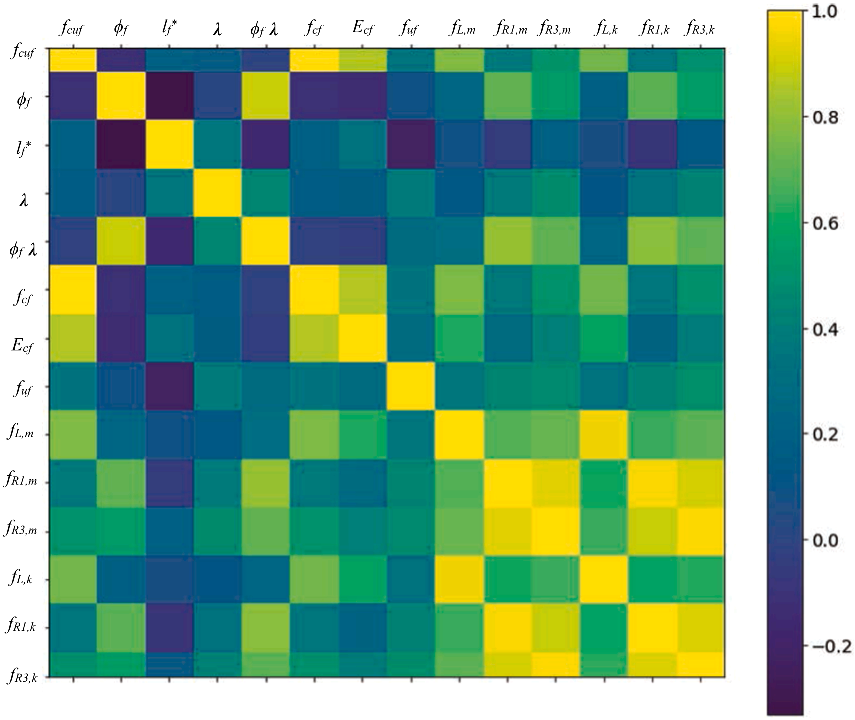

Fig. 2. Correlation matrix of dataset.

Table 3
Coefficients of the model 1.

	β_0	f_{cuf}	ϕ_f	ℓ^*_f	λ	$\phi_f\lambda$	E_{cf}	f_{uf}
$f_{L,\mathrm{m}}$	5.49	0.93630	0.91386	0.13488	0.17501	−0.58470	−0.13796	0.13620
$f_{L,\mathrm{m}}$	5.49	0.86435	0.37221	–	–	–	–	–
$f_{R1,\mathrm{m}}$	5.6866	1.1228	0.4311	0.3543	−0.1797	1.5006	−0.4630	0.4590
$f_{R1,\mathrm{m}}$	5.68662	0.59834	–	−0.04693	–	–	–	0.80103
$f_{R1,\mathrm{m}}$	5.6866	0.5835	–	–	–	–	–	0.8170
$f_{R3,\mathrm{m}}$	5.5970	1.2388	0.4977	0.8797	−0.1260	1.2636	−0.3822	0.6560
$f_{R3,\mathrm{m}}$	5.5970	0.7908	–	0.5149	–	–	–	0.9648

Table 4
p-values of the model 1.

	β_0	f_{cuf}	ϕ_f	ℓ^*_f	λ	$\phi_f\lambda$	E_{cf}	f_{uf}	Model	R_a^2 model
$f_{L,\mathrm{m}}$	$< 2 \times 10^{-16}$	3.04×10^{-9}	0.050	0.163	0.460	0.255	0.329	0.142	$< 2 \times 10^{-16}$	0.699
$f_{L,\mathrm{m}}$	$< 2 \times 10^{-16}$	$< 2 \times 10^{-16}$	9.16×10^{-7}	–	–	–	–	–	$< 2 \times 10^{-16}$	0.699
$f_{R1,\mathrm{m}}$	$< 2 \times 10^{-16}$	6.83×10^{-6}	0.57682	0.03073	0.65138	0.08426	0.05387	0.00398	$< 2.2 \times 10^{-16}$	0.818
$f_{R1,\mathrm{m}}$	$< 2 \times 10^{-16}$	0.02898	–	0.85655	–	–	–	0.00414	0.0001495	0.218
$f_{R1,\mathrm{m}}$	$< 2 \times 10^{-16}$	0.02445	–	–	–	–	–	0.00194	3.753×10^{-5}	0.228
$f_{R3,\mathrm{m}}$	$< 2.2 \times 10^{-16}$	4.86×10^{-7}	0.5053	3.39×10^{-7}	0.7429	0.1315	0.0983	3.85×10^{-5}	$< 2.2 \times 10^{-16}$	0.841
$f_{R3,\mathrm{m}}$	$< 2.2 \times 10^{-16}$	0.002188	–	0.035101	–	–	–	0.000254	8.295×10^{-8}	0.372

Table 5
Coefficients of the model 2.

	β_0	f_{cuf}	ϕ_f	ℓ^*_f	λ
$f_{L,\mathrm{m}}$	5.49	0.856057	0.389661	0.055698	−0.004836
$f_{L,\mathrm{m}}$	5.49	0.86435	0.37221	–	–
$f_{R1,\mathrm{m}}$	5.68662	0.94079	1.74160	0.02357	0.71150
$f_{R1,\mathrm{m}}$	5.6866	0.9433	1.7341	–	0.7195
$f_{R3,\mathrm{m}}$	5.5970	1.1877	1.5935	0.4708	0.7604

Table 6
p-values of the model 2.

	β_0	f_{cuf}	ϕ_f	ℓ^*_f	λ	Model	R_a^2 model
$f_{L,m}$	$< 2 \times 10^{-16}$	$< 2 \times 10^{-16}$	1.72×10^{-6}	0.489	0.949	$< 2 \times 10^{-16}$	0.693
$f_{L,m}$	$< 2 \times 10^{-16}$	$< 2 \times 10^{-16}$	9.16×10^{-7}	–	–	$< 2 \times 10^{-16}$	0.699
$f_{R1,m}$	$< 2 \times 10^{-16}$	3.44×10^{-10}	$< 2 \times 10^{-16}$	0.87	1.65×10^{-6}	$< 2 \times 10^{-16}$	0.788
$f_{R1,m}$	$< 2 \times 10^{-16}$	1.90×10^{-10}	$< 2 \times 10^{-16}$	–	2.46×10^{-7}	$< 2 \times 10^{-16}$	0.791
$f_{R3,m}$	$< 2 \times 10^{-16}$	2.18×10^{-13}	$< 2 \times 10^{-16}$	0.002	6.04×10^{-7}	$< 2 \times 10^{-16}$	0.794

Table 7
Coefficients of the model 3.

	β_0	f_{cuf}	ϕ_f	ℓ^*_f	λ	f_{uf}
$f_{L,m}$	5.49	0.81217	0.39322	0.11602	−0.06721	0.12341
$f_{L,m}$	5.49	0.86435	0.37221	–	–	–
$f_{R1,m}$	5.6866	0.7821	1.7545	0.2417	0.4860	0.4462
$f_{R1,m}$	5.6866	0.8438	1.6923	–	0.6081	0.3274
$f_{R3,m}$	5.5970	0.9581	1.6121	0.7864	0.4341	0.6457

Table 8
p-values of the model 3.

	β_0	f_{cuf}	ϕ_f	ℓ^*_f	λ	f_{uf}	Model	R_a^2 model
$f_{L,m}$	$< 2 \times 10^{-16}$	1.11×10^{-15}	1.32×10^{-6}	0.208	0.449	0.182	$< 2.2 \times 10^{-16}$	0.697
$f_{L,m}$	$< 2 \times 10^{-16}$	$< 2 \times 10^{-16}$	9.16×10^{-7}	–	–	–	$< 2.2 \times 10^{-16}$	0.699
$f_{R1,m}$	$< 2 \times 10^{-16}$	1.83×10^{-7}	$< 2 \times 10^{-16}$	0.12894	0.00210	0.00613	$< 2 \times 10^{-16}$	0.807
$f_{R1,m}$	$< 2 \times 10^{-16}$	1.05×10^{-8}	$< 2 \times 10^{-16}$	–	1.55×10^{-5}	0.0212	$< 2.2 \times 10^{-16}$	0.803
$f_{R3,m}$	$< 2 \times 10^{-16}$	2.24×10^{-10}	$< 2 \times 10^{-16}$	1.76×10^{-6}	0.00389	5.95×10^{-5}	$< 2.2 \times 10^{-16}$	0.835

fiber, such as ℓ^*_f and f_{uf}, appear statistically significant. This fact is particularly interesting and will be discussed further based on the analysis of model number 2.

Model no 2 exclusively considers the factors proposed by Ruiz et al. [8]: compressive strength, f_{cuf}, volume fraction of fiber, ϕ_f, non-dimensional length, ℓ^*_f, and aspect ratio, λ. It is the simplest model as it has the fewest number of factors, yet its application in experimental campaigns has yielded positive results. Obviously, these results are similar to those already shown by Ruiz et al. [8]: f_{cuf} and ϕ_f are the most important factors in the flexural response of SFRC, and as the composite undergoes substantial crack openings, the factors associated with fiber dimensions (λ and ℓ^*_f) become relevant.

In this study, we specifically focused on steel fibers with a single bend at the ends. This choice is due to the observation that straight steel fibers tend to show a decrease in residual strength with wider cracks, as they lack an enhanced anchorage mechanism compared to hooked-end steel fibers [6]. Tiberti et al. [7] concluded that the residual flexural strengths of concrete are primarily affected by concrete strength when the fiber type and amount are specified. They noted a direct correlation between fracture characteristics and fiber content within a specific range of volume fraction, contingent upon fiber aspect ratio and concrete strength. Beyond this range, fibers exhibit diminished influence. Fracture characteristics exhibit an upward trend with increasing λ for a fixed dosage of fibers and concrete strength. Moreover, the residual flexural behavior of SFRC is enhanced when utilizing fibers with greater tensile strength, particularly in high strength concrete where the tensile strength of the fibers is fully utilized.

Galeote et al. [6] emphasize the significance of ϕ_f, λ, f_{uf}, and the modulus of elasticity of fibers in residual flexural strengths. Wang et al. [28] suggest that the primary factors influencing the strengths and elastic modulus of plain concrete and SFRC with a fiber volume fraction below 1.5 % are the water-to-cement ratio (related to f_{cuf}) and the shape of the steel fibers.

It's worth noting that model no 1, due to its greater number of factors, masks the emergence of these factors from model 2. However, once again, a factor related to fiber geometry (ℓ^*_f) and another parameter related to its strength (f_{uf}) appear in model number 1. Nevertheless, if we consider that fiber failure will be due to pullout, it makes sense to use model 2 (with fewer factors) that does not consider fiber strength.

Model no 3 is identical to model no 2, except that it includes the fiber tensile strength, f_{uf}, as it has been observed in model no 1 to potentially have significant impact on the responses. For all three responses, the results match those of model 2, with the inclusion of f_{uf} for residual flexural strength responses. This highlights the importance and relevance of considering f_{uf} in the analysis of $f_{R1,m}$ and $f_{R3,m}$.

Across all models, f_{uf} and ϕ_f consistently emerged as significant predictors of flexural strength. Additionally, fiber-related properties such as ℓ^*_f and tf_{uf} showed statistical significance in predicting residual flexural strengths.

3.2. Bayesian linear regression analysis

Subsequently, for each of the 3 models conducted in the frequentist linear regression analysis, a Bayesian linear regression analysis was performed with the statistically significant factors obtained in the three primary responses to flexural strength, $f_{L,\mathrm{m}}$, $f_{R1,\mathrm{m}}$, $f_{R3,\mathrm{m}}$. Thus, instead of obtaining single values for them, they will be expressed as probability density functions. Again, in all cases, the response values are in MPa.

3.2.1. Model no 2

Table 9 includes the statistics of the parameters for each of the SFRC flexural responses in the model no 2, including only significant factors. Figs. 3, 4, and 5 display the pairs plot of parameters and their distributions with medians and 80 % intervals.

3.2.1.1. Model no 3. Table 10 presents the parameter statistics for each flexural response of SFRC in the model no 3, considering only significant factors. Figs. 6, 7, 8, and 9 illustrate the pairs plot of parameters along with their distributions, showing medians and 80 % intervals.

3.2.1.1.1. Discussion of the Bayesian linear regression analysis. Bayesian linear regression analysis is one of the main contributions and novelties of the research presented in this article. Expressing statistically significant factors as density functions instead of single values provides added value since one can selectively choose values to use based on probability. This approach represents an improvement in engineering and science models since it takes into account not only evidence but also expert knowledge in the Bayesian paradigm. Moreover, reality has a high random component, making it highly beneficial to consider this paradigm. Besides to the probability density functions of the parameters, pair plot graphs are provided. In this manner, Bayesian model refinement could result in models demonstrating improved predictive stability, specifically regarding the flexural and residual flexural responses of SFRC [28].

Pair plots (Figs. 3(a), 4(a), 5(a), 6, 8) are a visual tool used in Bayesian analysis to examine relationships between variables. In these plots, each pair of variables is depicted in a panel, with one variable on the x-axis and the other on the y-axis. Additionally, the main diagonal of the plot displays a histogram or probability density function for each individual variable. Pair plots offer valuable information for understanding the joint distribution of variables and potential relationships between them. By observing these plots, patterns, trends, and potential correlations between variables can be identified. Note that is important to consider the shapes of the variable distributions and any emerging patterns. Additionally, paying attention to the dispersion of points in panels off the main diagonal can reveal information about the strength and direction of associations between variables.

In general, the results indicate that the pair plots exhibit circular or circular-elliptical shapes, suggesting that the variables represented on the axes are not strongly correlated. This implies that there is no systematic relationship between the two variables.

3.3. Case study

This case study demonstrates the practical application and effectiveness of the proposed Bayesian model in predicting the flexural behavior of steel fiber-reinforced concrete. By integrating theoretical foundations with experimental data, this study critically assesses the model's predictive performance. The analysis provides key insights into how the Bayesian framework can be leveraged to optimize decision-making in engineering, offering improved reliability in structural performance evaluations.

The following dataset outlines the parameters and experimental results utilized for model validation:

- $f_{cuf} = 67.4$ MPa
- Nominal fiber length, $\ell_f = 40$ mm, where $\ell *_f = \frac{\ell_f}{\ell_0 = 30 \text{ mm}} = 1.3$
- Nominal fiber diameter, $d_f = 0.85$ mm

Table 9
Bayesian coefficients of the model 2.

Model	Parameter	Estimate	Lower 95 % C.I.	Upper 95 % C.I.
$f_{L,\mathrm{m}} = \beta_0 + f_{cu} + \phi_f$	β_0	5.490	5.351	5.625
	f_{cuf}	0.864	0.726	1.001
	ϕ_f	0.373	0.235	0.510
	σ	0.601	0.511	0.711
$f_{R1,\mathrm{m}} = \beta_0 + f_{cu} + \phi_f + \lambda$	β_0	5.689	5.439	5.938
	f_{cuf}	0.944	0.697	1.193
	ϕ_f	1.735	1.486	1.989
	λ	0.719	0.461	0.976
	σ	1.085	0.461	0.976
$f_{R1,\mathrm{m}} = \beta_0 + f_{cu} + \phi_f + \lambda + \ell *_f$	β_0	5.596	5.340	5.857
	f_{cuf}	1.188	0.925	1.452
	ϕ_f	1.595	1.319	1.871
	λ	0.759	0.480	1.036
	$\ell *_f$	0.472	0.175	0.767
	σ	1.113	0.943	1.320

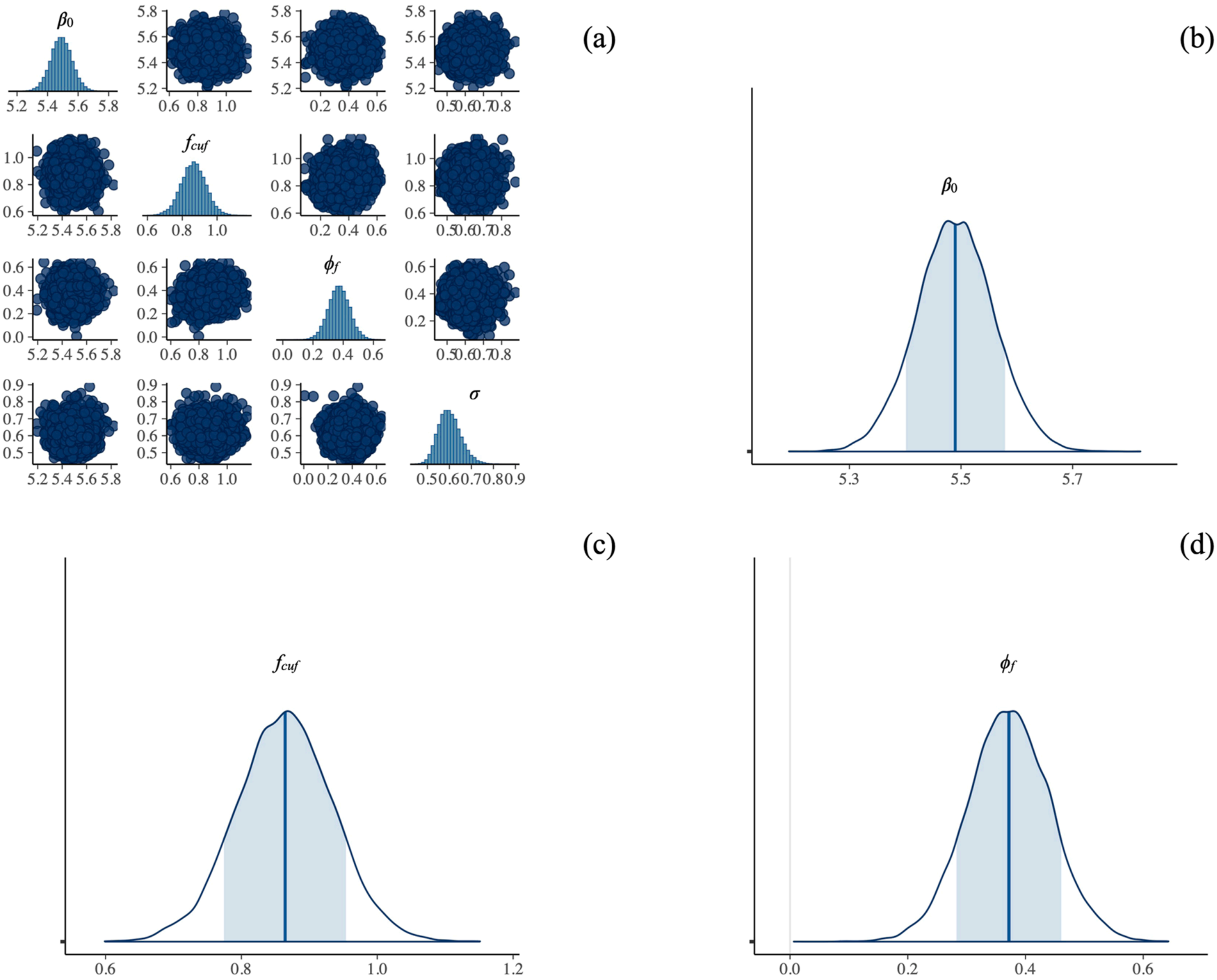

Fig. 3. (a) Pairs plot of parameters; (b), (c), (d), posterior distributions of parameters with medians and 80 % intervals for $f_{L,\mathrm{m}}$.

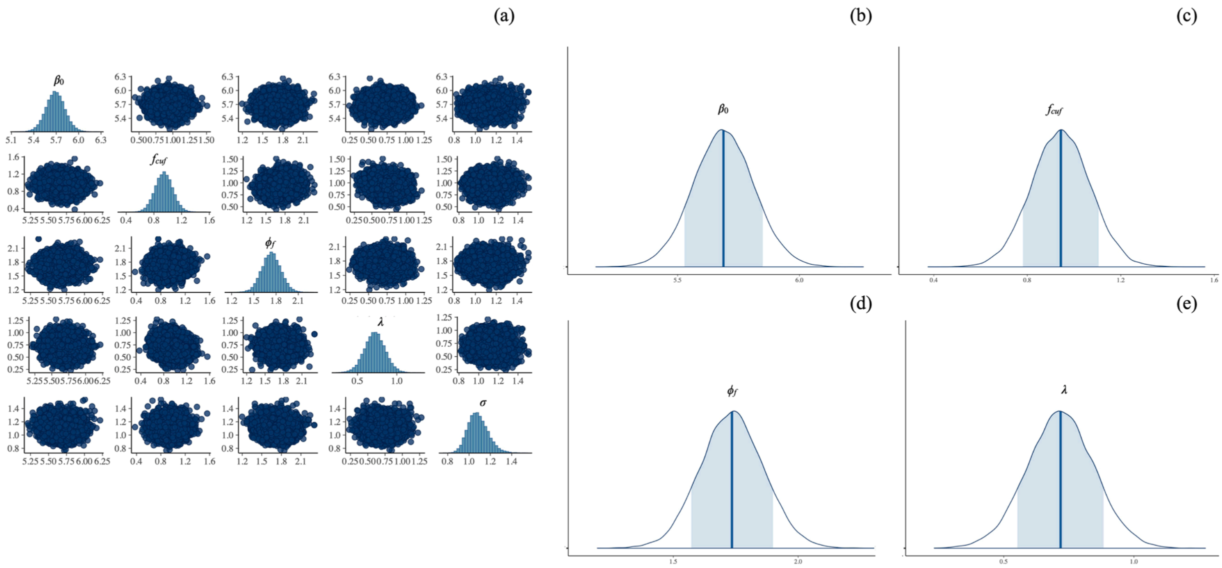

Fig. 4. (a) Pairs plot of parameters; (b), (c), (d), (e), posterior distributions of parameters with medians and 80 % intervals for $f_{R1,\mathrm{m}}$.

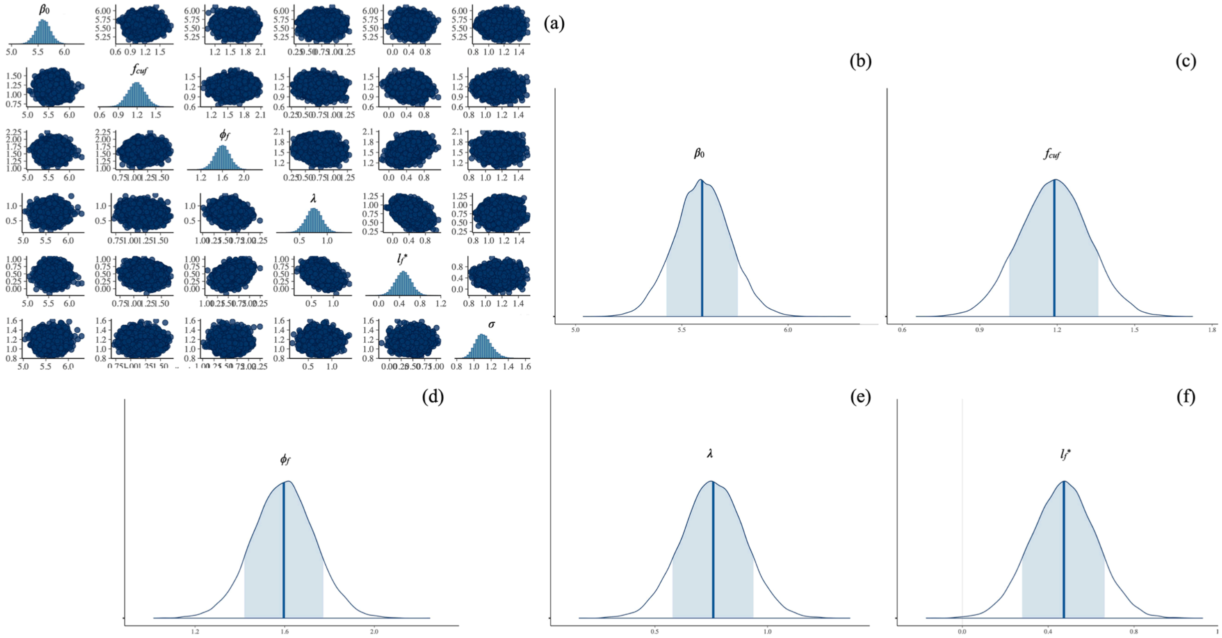

Fig. 5. (a) Pairs plot of parameters; (b), (c), (d), (e), (f), posterior distributions of parameters with medians and 80 % intervals for $f_{R3,m}$.

Table 10
Bayesian coefficients of the model 3.

Model	Parameter	Estimate	Lower 95 % C.I.	Upper 95 % C.I.
$f_{R1,m} = \beta_0 + f_{cu} + \phi_f + \lambda + f_{uf}$	β_0	5.686	5.445	5.926
	f_{cuf}	0.846	0.582	1.105
	ϕ_f	1.693	1.446	1.942
	λ	0.608	0.345	0.872
	f_{uf}	0.327	0.045	0.608
	σ	1.052	0.893	1.247
$f_{R3,m} = \beta_0 + f_{cu} + \phi_f + \lambda + \ell^*{}_f + f_{uf}$	β_0	5.598	5.365	5.828
	f_{cuf}	0.958	0.669	1.214
	ϕ_f	1.612	1.373	1.850
	λ	0.435	0.148	0.723
	$\ell^*{}_f$	0.786	0.489	1.082
	f_{uf}	0.646	0.344	0.940
	σ	0.995	0.843	1.179

- Fiber aspect ratio, $\lambda = 47$
- Volume fraction of fiber, $\phi_f = 0.6\ \% = 0.006$ (47.1 kg/m^3)

Fig. 10 illustrates the variability in the experimental stress-crack mouth opening displacement (σ_N-w_M) results obtained from the three-point bending tests. This observation underscores the necessity of employing Bayesian models to estimate the flexural response variable values based on the selected probability. Such an approach enables users to maintain greater control over the uncertainty associated with the variability of data inherent in the incorporation of fiber reinforcement into concrete.

For this case study, we will utilize the mean values of both flexural strength and residual flexural strength. Based on the dataset, the mean and standard deviation values for the various factors are in Table 1, providing the statistical basis for the analysis. The calculations for the flexural and residual flexural strengths of the SFRC in Bayesian model 2 are presented in Table 11.

According to the performance classes for steel fiber-reinforced concrete (SFRC) defined in Table L.2 of Eurocode 2 [3] (see Fig. 11, extracted from[44]), the classification of SFRC is as follows: note that we are working with average values, whereas the table presents characteristic values; however, it serves to illustrate the example for the case study.

- Lower 95 % C.I.: 5d
- Upper 95 % C.I.: 5e

In other words, we have two possible flexural strength classes for the same steel fiber-reinforced concrete (SFRC) based on the

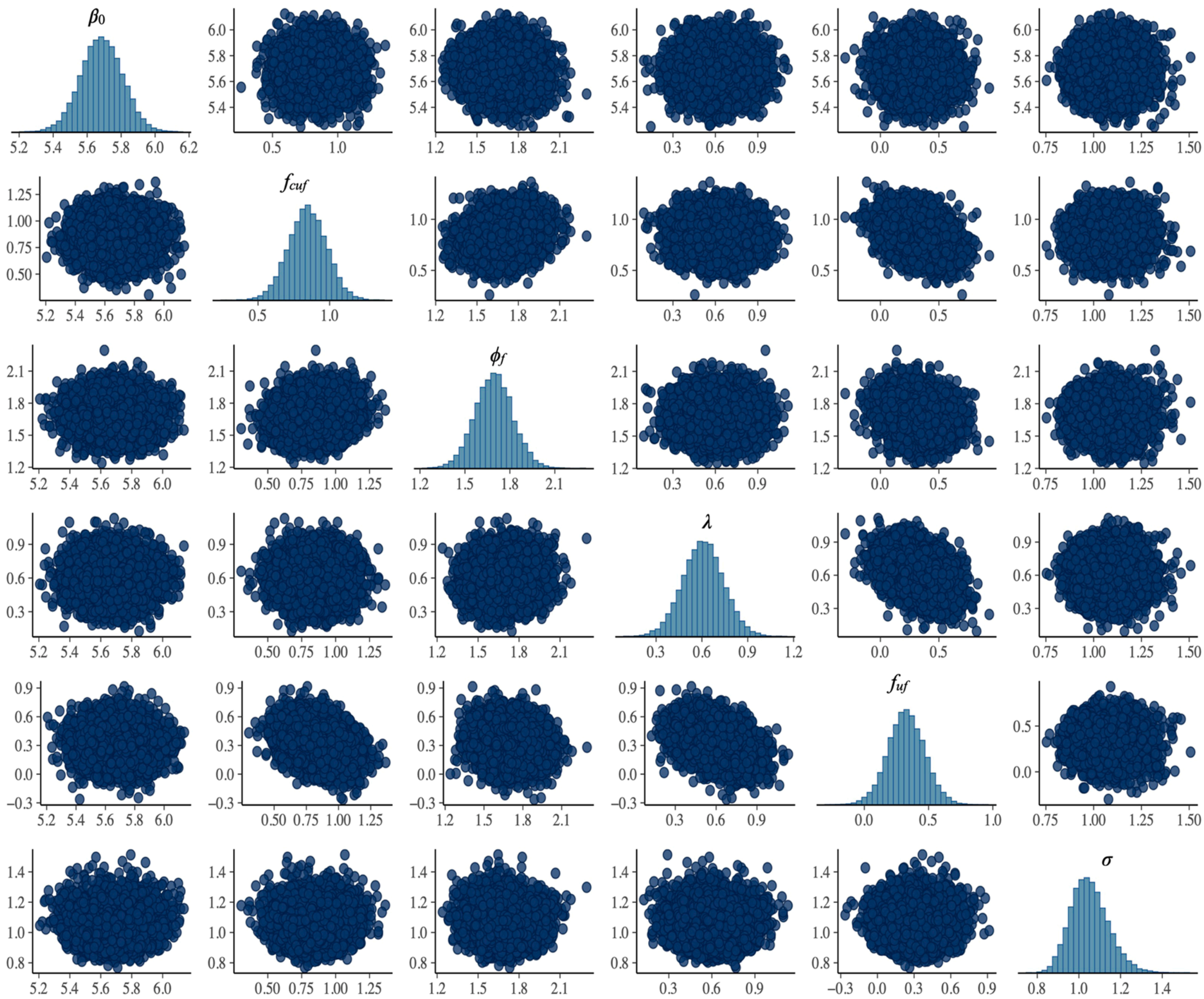

Fig. 6. Pairs plot of parameters for $f_{R1,m}$.

degree of probability considered. This situation becomes even more pronounced and sensitive to class changes within the lower range of values for $f_{R1,k}$ (between 1.0 and 5.0 MPa), as the transitions between classes occur with differences of 0.5 MPa instead of 1 MPa.

In summary, the analysis of classical multiparametric linear models provides a straightforward interpretation of coefficients but assumes constant relationships among variables across the data range. Incorporating coefficients as density functions introduces greater flexibility, allowing coefficients to vary based on the data distribution. The density-based approach offers a more nuanced interpretation of how the relationship between variables can change across different parts of the distribution, which is particularly useful when dealing with heterogeneous data or non-linear relationships.

3.4. Machine Learning analysis

Table 12 and Fig. 12 present the outcomes for selecting the optimal model. Fig. 12 exhibits five graphs, one for each tested model, displaying experimental values on the x-axis and the corresponding predictions on the ordinates. Notably, these results exclusively originate from test data, untouched during the algorithm's calibration. Table 12 details the root mean square error (RMSE) results for each model and variable on the test data.

Based on the results presented in Table 12 and Fig. 12, the decision has been made to proceed with the LR model, described by Eq. (1). In all instances, it consistently delivers optimal results and is the simplest model among those tested. Subsequently, only this algorithm will be utilized in the subsequent phases.

3.4.1. Analysis using LR model

All possible combinations of variables were systematically assessed. Initially, the process began with the eight defined inputs. However, due to a significant correlation among three variables (f_{cuf}, f_{cf}, and E_{cf}), each of these variables was extracted, leading to the

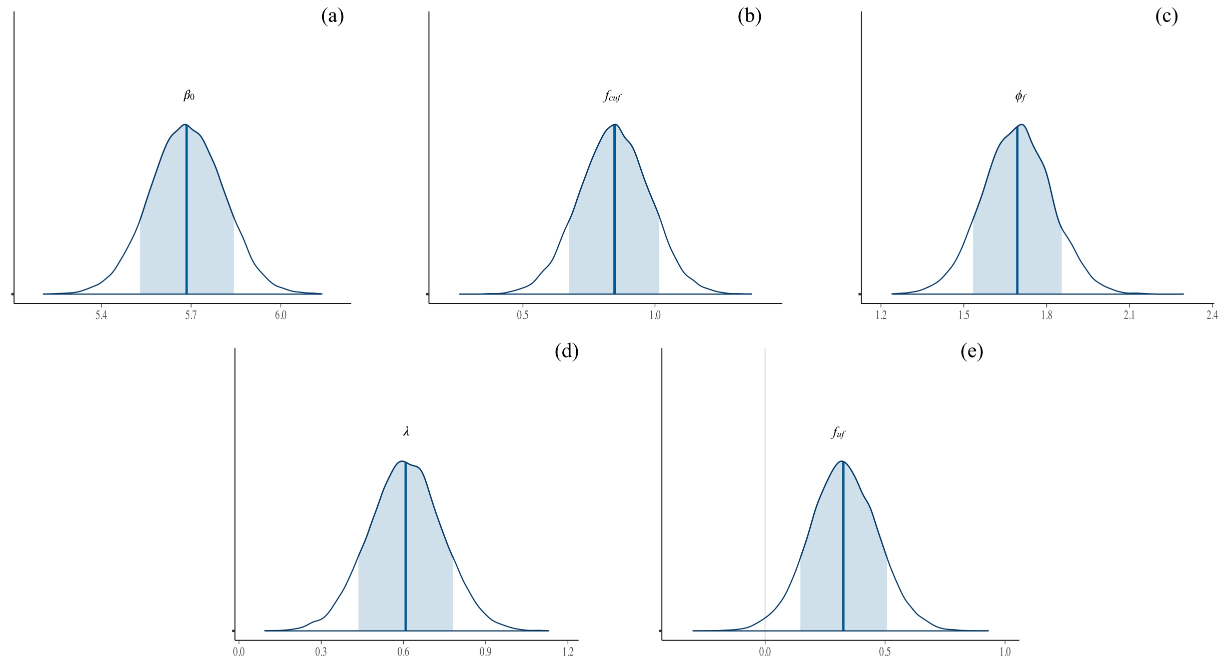

Fig. 7. (a), (b), (c), (d), (e) Posterior distributions of parameters with medians and 80 % intervals for $f_{R1,\mathrm{m}}$.

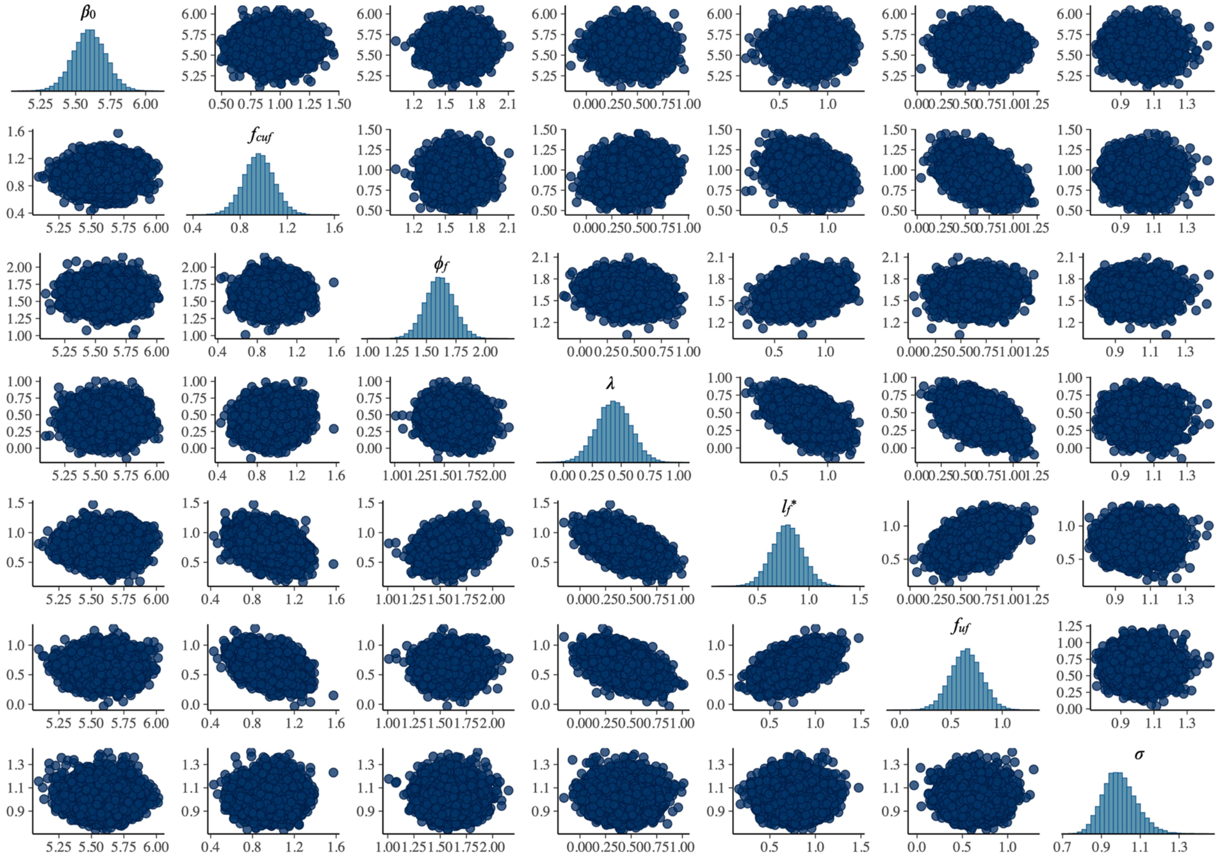

Fig. 8. Pairs plot of parameters for $f_{R3,\mathrm{m}}$.

creation of three distinct datasets. Each dataset consisted of one of the mentioned variables and the remaining five. Within these datasets, a thorough examination of every possible subgroup of variables occurred, starting from individual variables, then in pairs, followed by groups of three, four, and finally, all five together. This approach resulted in a total of 190 combinations, a number not divisible by three due to including a scenario where all eight variables were simultaneously used. As previously outlined, six potential outputs for the model were identified. Linear regression models were trained using the specified inputs against each output. To assess the robustness of these models, for every combination of inputs and outputs, the data were randomly divided into training and testing sets a thousand times. This process provided an average value and standard deviation for the RMSE of the model for each combination. Consequently, in this phase, a comprehensive evaluation of 1140000 regression models was conducted.

3.4.2. Identification of optimal variables for predictive models

Based on the obtained results, it can be concluded that the quality of the models is independent of whether f_{cuf} or f_{cf} is used. However, the use of E_{cf} reduces the quality of the models, increasing the RMSE from approximately 0.66 to approximately 0.85 (see Fig. 13).

In Fig. 14, the comparison of the model results using all variables as inputs is presented in a Taylor graph. The points in the graph illustrate the relationship between RMSE test media and R^2 Test media for each evaluated case, allowing for a visual assessment of the relative performance of each model. The results presented in Fig. 14 indicate that the models corresponding to the blue and orange points ($f_{L,\mathrm{m}}$ and $f_{L,\mathrm{k}}$, respectively) exhibit lower R^2 values and smaller RMSE values compared to the other models. This suggests that these models explain less variance in the data, yet produce lower errors. A potential reason for this behavior is the small dataset available, which may lead to difficulties in accurately capturing the underlying patterns. With fewer data points, models tend to generalize poorly, resulting in lower R^2. Additionally, the low RMSE could be a consequence of models fitting well to the limited data they are trained on, but without providing reliable generalization for unseen data. In contrast, the remaining four models show relatively similar performance, with higher R^2 and RMSE values, indicating that they better capture the variance in the data but with slightly higher prediction errors. This highlights the trade-off between capturing data complexity and avoiding overfitting when working with limited datasets.

3.4.3. Identification of relevant variables for predictive models

In Fig. 15, the correlation matrix is depicted, illustrating the relationship between each input with the mean value and the standard

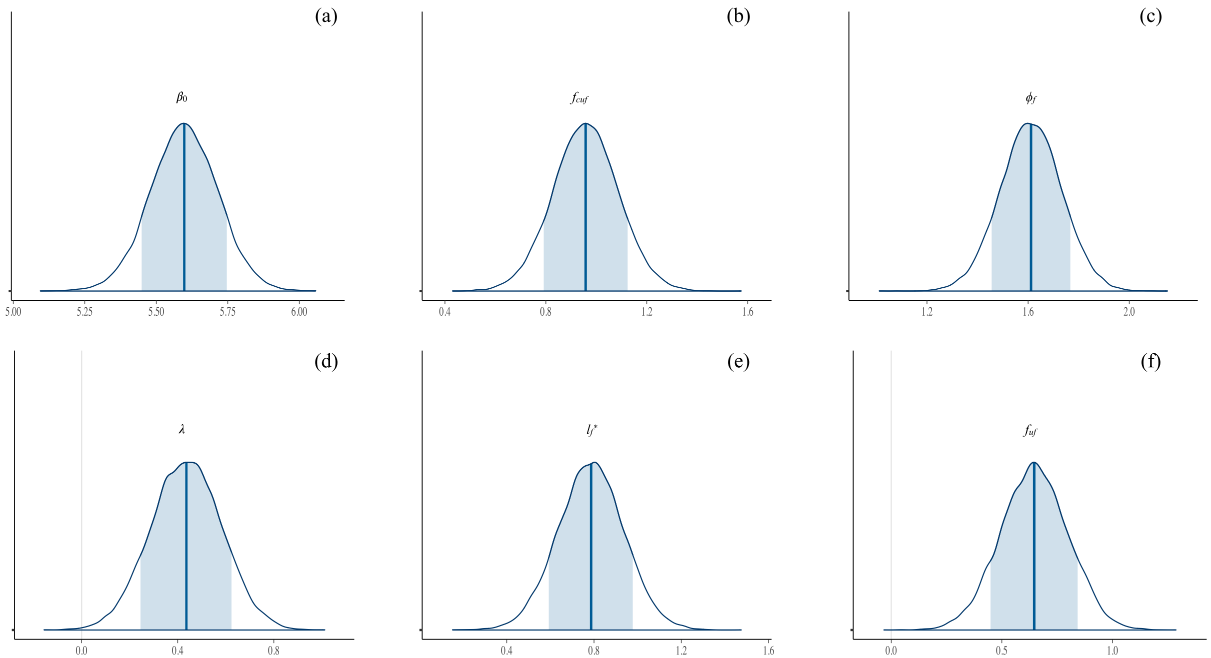

Fig. 9. (a), (b), (c), (d), (e), (f) Posterior distributions of parameters with medians and 80 % intervals for $f_{R3,m}$.

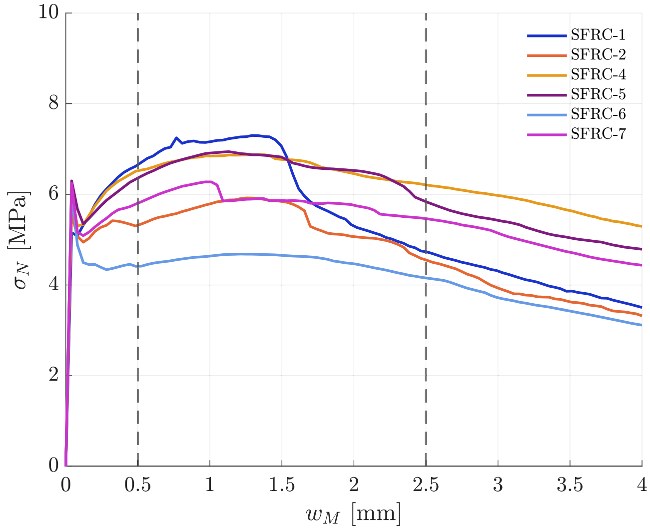

Fig. 10. Experimental three-point bending tests corresponding to the case study (σ_N–w_M relationship).

Table 11
Flexural and residual flexural strengths of Bayesian model 2 for the case study.

Model	Parameter	Estimate	$f_{L,m}$ [MPa]	Lower 95 % C.I.	$f_{L,m}$ [MPa]	Upper 95 % C.I.	$f_{L,m}$ [MPa]
$f_{L,m} = \beta_0 + f_{cu} + \phi_f$	β_0	5.490	6.0	5.351	5.8	5.625	6.2
	f_{cu}	0.864		0.726		1.001	
	ϕ_f	0.373		0.235		0.510	
	σ	0.601		0.511		0.711	
Model	Parameter	Estimate	$f_{R1,m}$ [MPa]	Lower 95 % C.I.	$f_{R1,m}$ [MPa]	Upper 95 % C.I.	$f_{R1,m}$ [MPa]
$f_{R1,m} = \beta_0 + f_{cu} + \phi_f + \lambda$	β_0	5.689	5.4	5.439	5.3	5.938	5.5
	f_{cu}	0.944		0.697		1.193	
	ϕ_f	1.735		1.486		1.989	
	λ	0.719		0.461		0.976	
	σ	1.085		0.877		1.345	
Model	Parameter	Estimate	$f_{R3,m}$ [MPa]	Lower 95 % C.I.	$f_{R3,m}$ [MPa]	Upper 95 % C.I.	$f_{R3,m}$ [MPa]
$f_{R3,m} = \beta_0 + f_{cu} + \phi_f + \lambda + \ell^*_f$	β_0	5.596	6.0	5.340	5.5	5.857	6.5
	f_{cu}	1.188		0.925		1.452	
	ϕ_f	1.595		1.319		1.871	
	λ	0.759		0.480		1.036	
	ℓ^*_f	0.472		0.175		0.767	
	σ	1.113		0.943		1.320	

Ductility classes	Strength classes SC ($f_{R,1k} \geq SC$)												Analytical formulae
	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	6.0	7.0	8.0	
a	0.5	0.8	1.0	1.3	1.5	1.8	2.0	2.3	2.5	3.0	3.5	4.0	$f_{R,3k} \geq 0.5\,SC$
b	0.7	1.1	1.4	1.8	2.1	2.5	2.8	3.2	3.5	4.2	4.9	5.6	$f_{R,3k} \geq 0.7\,SC$
c	0.9	1.4	1.8	2.3	2.7	3.2	3.6	4.1	4.5	5.4	6.3	7.2	$f_{R,3k} \geq 0.9\,SC$
d	1.1	1.7	2.2	2.8	3.3	3.9	4.4	5.0	5.5	6.6	7.7	8.8	$f_{R,3k} \geq 1.1\,SC$
e	1.3	2.0	2.6	3.3	3.9	4.6	5.2	5.9	6.5	7.8	9.1	10.4	$f_{R,3k} \geq 1.3\,SC$

Fig. 11. Performance classes for SFRC as defined in Table L.2 of Eurocode 2 (in MPa) [3,44].

deviation of the RMSE for each variable. This allows for the analysis of the influence of each input on the RMSE value and its associated uncertainty.

Fig. 16 presents boxplots comparing the RMSE for models that have or have not utilized each variable. This provides insights into the influence of each variable on the model error.

Table 12
RMSE of the optimized models for the test data ($f_{L,m}$, $f_{R1,m}$, $f_{R3,m}$, $f_{L,k}$, $f_{R1,k}$, $f_{R3,k}$, in MPa).

	Algorithm				
Variable	LR	KNN	DT	RF	GB
$f_{L,m}$	0.66	0.75	0.66	0.55	0.93
$f_{R1,m}$	1.04	1.04	1.48	0.93	1.35
$f_{R3,m}$	0.97	0.85	1.38	0.83	1.01
$f_{L,k}$	0.61	0.61	0.59	0.58	0.57
$f_{R1,k}$	1.10	1.44	1.76	1.39	1.79
$f_{R3,k}$	1.09	1.33	1.26	1.20	1.24

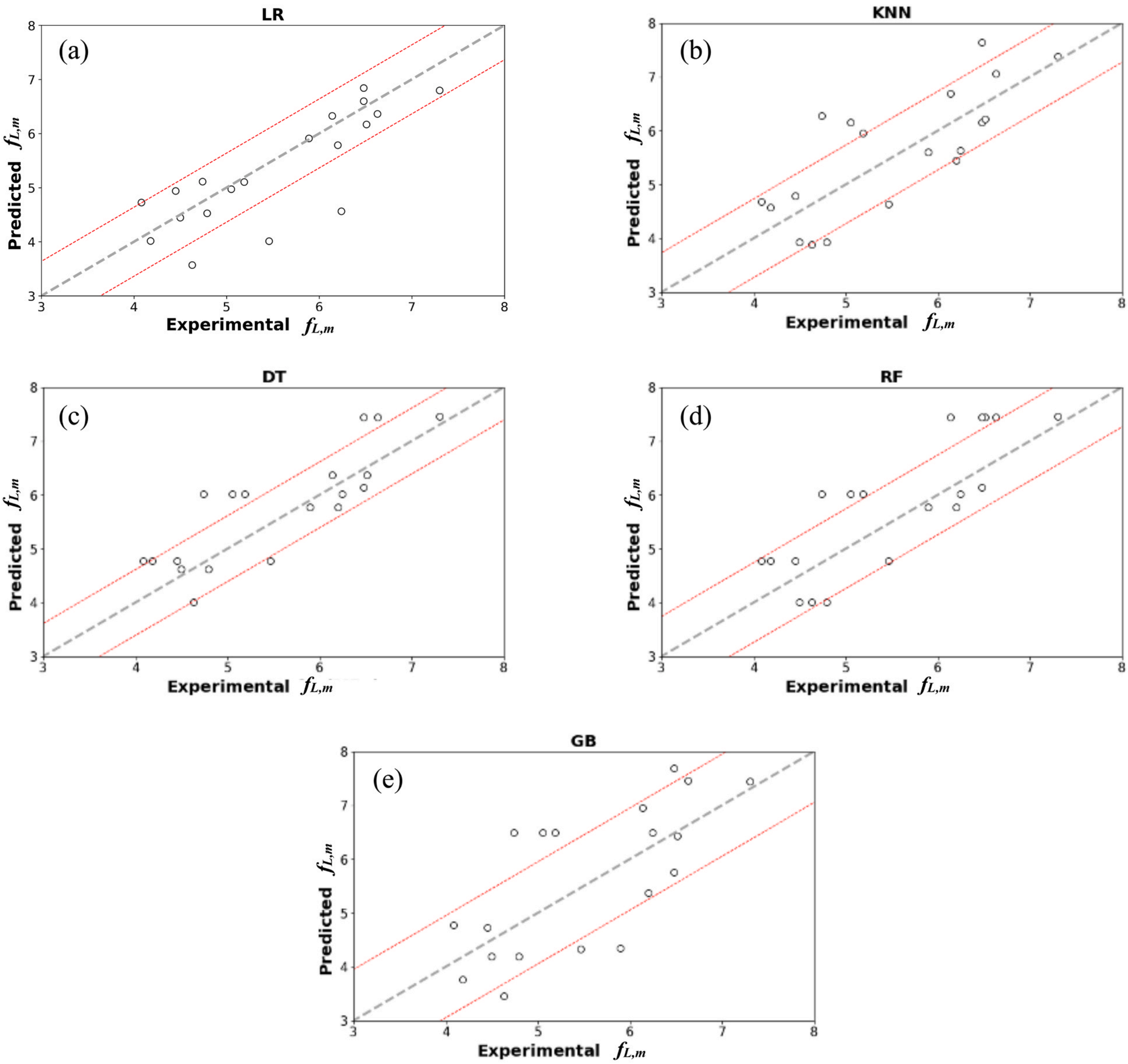

Fig. 12. Model selection by comparing the experimental results with the prediction of the different models: (a) LR, (b) KNN, (c) DT, (d) RF, (e) GB, in MPa.

3.4.4. Discussion of the Machine Learning analysis

The implementation of the LR model represents a robust method for forecasting the flexural and residual flexural strength of SFRC. Through these comprehensive examinations, the significant potential of Machine Learning methodologies is emphasized using

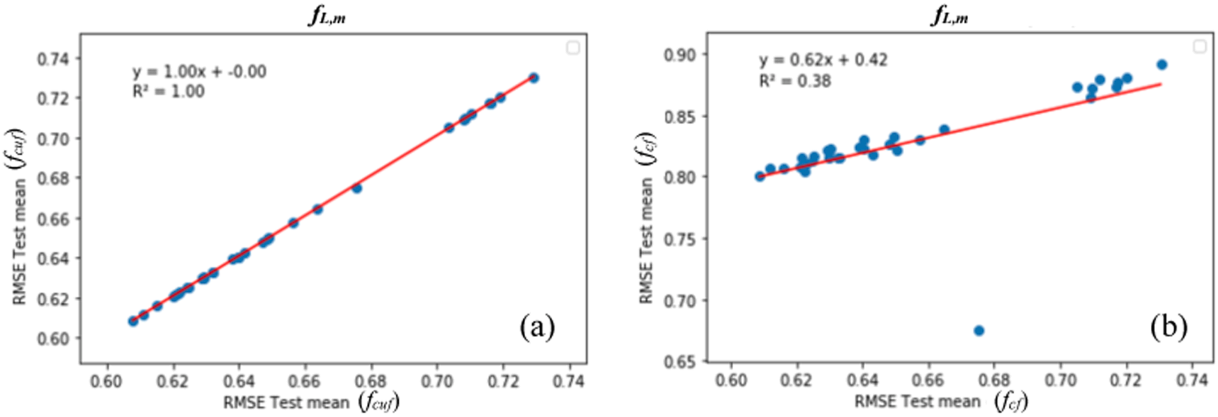

Fig. 13. Scatter plot for $f_{L,m}$, (in MPa).

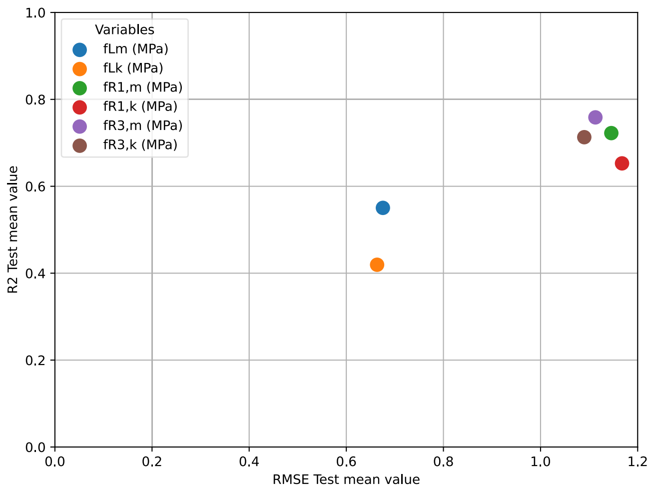

Fig. 14. Taylor graph comparing model performance using all input variables.

Artificial Neural Network models [29]. Consequently, Machine Learning methodologies emerge as effective and beneficial tools, offering satisfactory precision while reducing the time and resources required compared to conventional laboratory experiments. This, in turn, enhances the effectiveness of design and quality control procedures [29].

Nonetheless, the use of Machine Learning requires the development of databases with abundant information, as this is where one of its potentials lies. However, this is also one of the main challenges: creating databases with coherent and comparable information between experiments. This is a task that needs to be developed in concrete technology, although there are proposals and initiatives for this, such as those from Fédération internationale du béton, *fib* [9].

4. Conclusions

This research explored the flexural behavior of steel fiber-reinforced concrete (SFRC) through three data-driven paradigms: Frequentist Inference (FI), Bayesian Inference (BI), and Machine Learning (ML). Utilizing a dataset of homogeneous experimental results from three-point bending tests on SFRC specimens with varied compressive strengths, fiber quantities, and geometric parameters, the primary objective was to identify the critical parameters governing material properties. The outcomes from each methodology were compared to ensure robustness.

Key findings from the study are summarized as follows:

- Frequentist Inference (FI) Models:
  - Model 1, with the highest number of variables, revealed the significance of compressive strength (f_{cuf}) and fiber volume fraction (ϕ_f) across all responses, while fiber properties like non-dimensional length (ℓ^*_f) and tensile strength (f_{uf}) were crucial for residual flexural strengths.
  - Model 2 confirmed the relevance of f_{cuf} and ϕ_f, highlighting the importance of fiber geometry (ℓ^*_f) as crack openings increased.

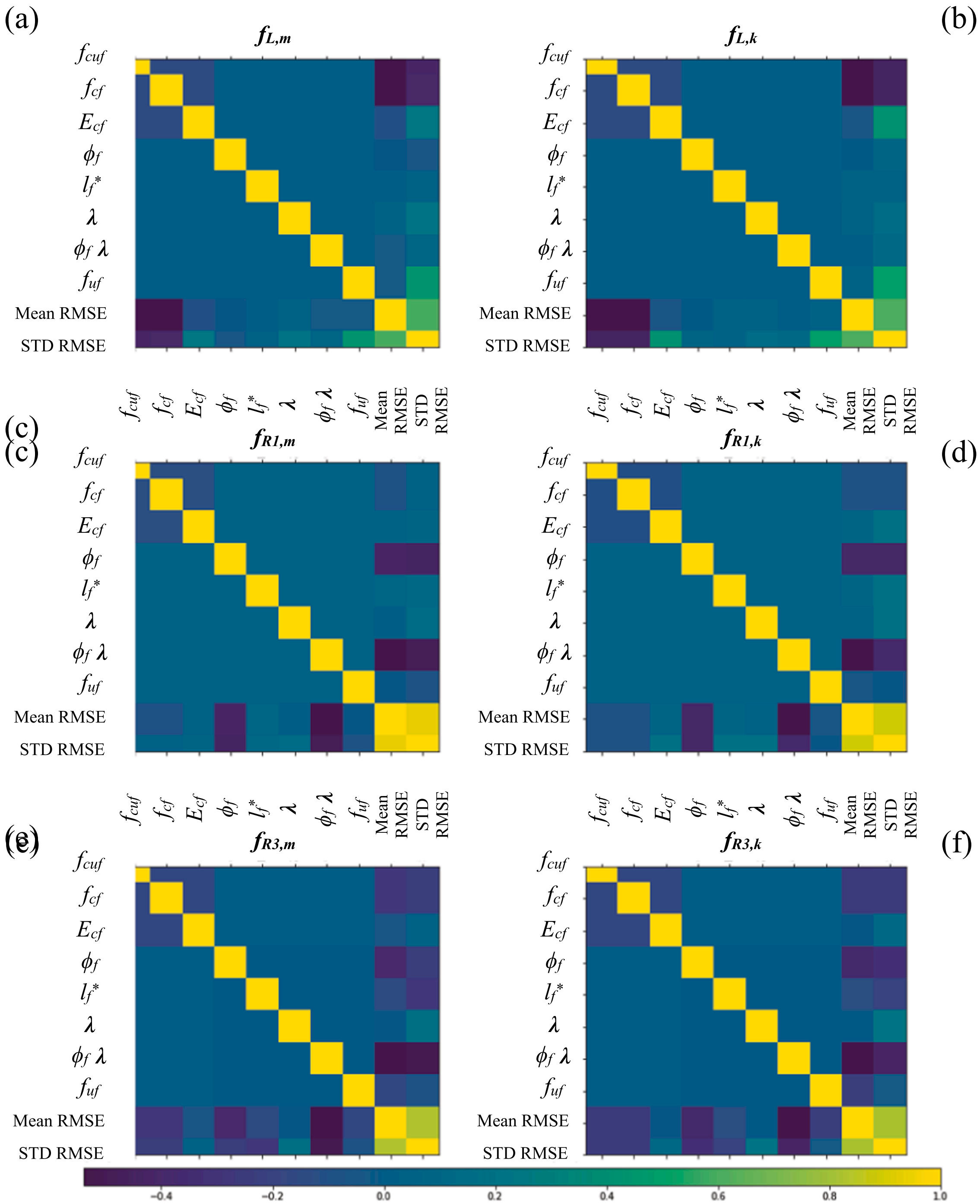

Fig. 15. Correlation matrix between each input and the average value, as well as the standard deviation of each output ($f_{L,m}$, $f_{R1,m}$, $f_{R3,m}$, $f_{L,k}$, $f_{R1,k}$, $f_{R3,k}$, in MPa).

  - Model 3 reiterated the findings of Model 2 while emphasizing the inclusion of fiber tensile strength (f_{uf}) in residual flexural strength analyses.
- Bayesian Inference (BI) Models:
  - BI models exhibited enhanced predictive stability, effectively capturing the flexural and residual flexural responses of SFRC.
  - The probabilistic nature of BI allowed for the integration of prior knowledge and uncertainty quantification, enriching the analysis.
  - Visual pair plots indicated systematic relationships among variables, reinforcing the robustness of the Bayesian approach.

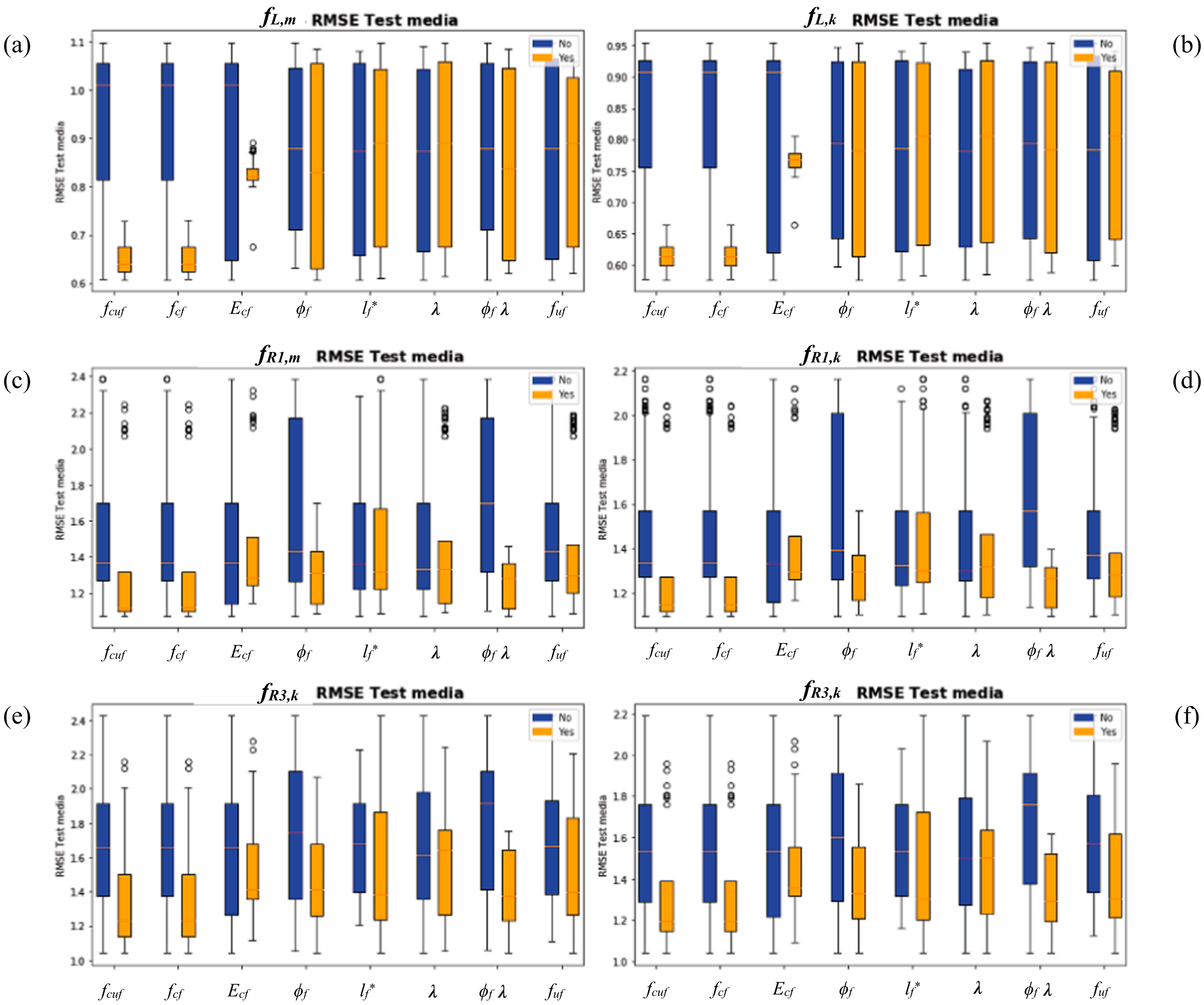

Fig. 16. Boxplots comparing the RMSE for models ($f_{L,m}$, $f_{R1,m}$, $f_{R3,m}$, $f_{L,k}$, $f_{R1,k}$, $f_{R3,k}$, in MPa).

- Machine Learning (ML) Analysis:
  - Linear regression (LR) models demonstrated comparable or superior accuracy compared to more complex models in predicting SFRC behavior.
  - The analysis showed that using f_{cf} instead of f_{cuf} yielded similar model quality, while the inclusion of elastic modulus (E_f) increased RMSE.
  - Key predictive factors included the strength and elastic modulus of the concrete, with diminishing significance of concrete properties as cracking progressed, indicating a shift in governing parameters.

The results across all methodologies indicated similar predictive capabilities regarding the relationships in flexural responses of concrete and key parameters related to fiber reinforcement. Notably, the case study demonstrated that utilizing probability density functions derived from Bayesian analysis enables engineers to effectively manage the uncertainty associated with the flexural response of SFRC. This method provides greater flexibility and insights for design purposes, allowing for the same concrete mix to be classified variably as 5d or 5e according to Annex L of the new Eurocode 2.

In summary, this study highlights the strengths of data-driven methodologies for predicting the flexural behavior of SFRC. However, several limitations should be addressed in future research:

- Data Dependency: The predictive accuracy relies heavily on the quantity and quality of available data. Access to larger and more detailed datasets could significantly improve model performance.
- Specificity to SFRC: Although the models demonstrated effectiveness in predicting the flexural responses of SFRC, their applicability to other concrete types remains untested. Future research should aim to extend this approach to analyze and improve the understanding of various concrete formulations.

Addressing these limitations will facilitate the development of more robust predictive models and expand the applicability of the findings.

Institutional Review Board Statement

Not applicable.

Informed Consent Statement

Not applicable.

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CRediT authorship contribution statement

Ángel De La Rosa Velasco: Writing – review & editing, Writing – original draft, Validation, Supervision, Software, Methodology, Investigation, Conceptualization. **José Sáinz-Aja:** Writing – review & editing, Validation, Software, Methodology, Conceptualization. **Diego Ferreño:** Writing – review & editing, Validation, Methodology. **Gonzalo Ruiz:** Writing – review & editing, Validation, Resources, Methodology, Funding acquisition, Conceptualization. **Isaac Rivas:** Writing – review & editing, Validation, Software, Methodology.

Declaration of generative AI and AI-assisted technologies in the writing process

The authors of this work would like to declare the use of ChatGPT 3.5 – a generative AI and AI-assisted technology – during the writing process to enhance the language and readability of the content. However, please note that the authors reviewed and edited the content as necessary and take full responsibility for the final publication.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability

Data will be made available on request.

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