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ORIGINAL PAPER

Teacher growth in exploiting mathematics competencies through STEAM projects

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Abstract

This article is aimed at educators concerned with curricular initiatives that foster STE(A)M projects in secondary education to promote mathematics competencies. Research has recently reported that these projects superficially address mathematics content, hampering the development of competencies the consensus deems necessary to prepare citizens for daily life. This study shows that learning goals may be achieved when teachers receive personalised training and sustained assistance in their project experiences. We examine how two Spanish teachers, with advisors' support, progress in exploiting mathematics competencies within the implementation of a single project each over a period of 3 years. Their evolution was not the result of minor recommendations but of continuous interactions with the advisors. These interactions intended to maintain a balance between teacher confidence and project enhancements, which required commitment and constancy. Four of the five competencies considered in the Spanish curriculum emerged powerfully after sustained refinement. The frequently mobilised competencies were intra-mathematics, representations, as well as collaborative work and positive identity, followed by modelling. The last of which was difficult to address, but when it was, the other three emerged more naturally. Computational thinking was poorly represented mainly because of the advisors' background and its recent incorporation into the reference curriculum. The teachers' progress was influenced by the advisor's academic background, pedagogical expertise, ability to transfer research outcomes into teaching, and experience supporting others.

Keywords Mathematics competencies · Secondary education · Project-based learning · STEM education

1 Introduction

In countries like Denmark, China, and Spain, mathematics curricula are defined not just in terms of attained knowledge but also of developed competencies. The Council of the European Union (2018) promotes several key competencies—literacy; multilingual; STEM (Science, Technology, Engineering, and Mathematics); digital; personal, social and learning-to-learn; citizenship; entrepreneurship; as well as cultural awareness and expression—to prepare citizens for an increasingly interconnected world. These competencies should provide students with the knowledge, attitudes, and skills required for a new society in which mathematics plays a significant role (Niss & Højgaard, 2019). Key competencies must be adapted to each context through specific competencies (Council of the European Union, 2018). The Spanish mathematics curriculum has split the STEM key competency into five—modelling, computational thinking, intra-mathematical connections, mathematical representations, and collaborative work and positive identity—for instructing students in solving contextualised or real projects (Ministry of Education and Vocational Training [MEVT], 2022). This instruction requires a shift in the teaching methodology and project-based learning is viewed as an effective approach for fostering integrated

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education in mathematics classrooms (Beswick & Fraser, 2019; Diego-Mantecón et al., 2021).

Several researchers claim that promoting school mathematics through STEM project-based learning is a chimaera because mathematics is often underrepresented (English, 2022; Fitzallen, 2015). Lasa et al. (2020) and Ubuz (2020), among others, advise on the difficulty of raising mathematics in contextualised or real situations and indicate that such situations neither foster abstract thinking nor cover content rigorously. Other authors criticise the suitability of the methodology; for instance, Godino et al. (2015) stress that its student-orientated characteristic denotes a lack of teacher guidance that does not stimulate feedback and thus mathematics learning. De Loof et al. (2022) add that this low mathematics engagement is because STEM activities are often implemented in short sessions without time for exploration. Diego-Mantecón et al. (2021) report that mathematics is overshadowed in many projects designed by out-of-field mathematics teachers who likely emphasise science, technology, and/or engineering. In this regard, Just and Siller (2022) stress that teacher specialisation is a crucial factor affecting how mathematics is addressed. Ubuz (2020) goes further, claiming that teacher experience is also essential for an effective STEAM project implementation.

In this study, we investigate how two Spanish mathematics teachers progressively refined their projects (Water Rocket and Mathematics Museum) over three academic years to exploit the national curriculum's mathematics competencies. We also examine how the sustained support from advisors contributed to such refinement. Both of the secondary education teachers have attended a STEAM professional development programme since 2017 and implemented their respective projects several times in their classrooms with various student groups. The Water Rocket project involves science, technology, engineering, and mathematics disciplines, whereas the Mathematics Museum project deals with science, technology, arts, and mathematics. To achieve our research objectives, we used a qualitative approach to cross data from observations, student reports, and teachers' and advisors' focus groups.

2 STE(A)M projects

Project-based learning is a student-centred methodology in which students take responsibility for their own learning and teachers act as facilitators of such process (Diego-Mantecón et al., 2021). Their goal is to use student understanding constructively, not transferring knowledge directly but connecting new and prior information and managing group and whole class discussions (Han et al., 2015). Executing STEAM projects implies attaining a final product, mobilising knowledge and skills from Science, Technology, Engineering, Arts and/or Mathematics (Diego-Mantecón et al., 2019b). From now on, we will use STE(A)M to refer to STEM or STEAM projects.

Several studies have analysed the disciplines emerging in STE(A)M projects (Martín-Páez et al., 2019) and in particular the mathematics appearing therein (Diego-Mantecón et al., 2021; Just & Siller, 2022; Lasa et al., 2020). In these studies, mathematics rarely plays a dominant role; Martín-Páez et al. (2019) established that this discipline was not central in any of the 19 experiences analysed. Diego-Mantecón et al. (2021) discovered that mathematical content did not appear in 16 of 41 scrutinised projects, and high cognitive demands were fostered in only 15 of them. Similarly, in most projects examined by Lasa et al. (2020), the mathematical content was basic and utilitarian, not matching curriculum standards. These outcomes explain why authors like Fitzallen (2015) claim mathematics is often underrepresented in STE(A)M education. According to Doig and Williams (2019), the scarcity of mathematics in integrated contexts stems from its initial popularity among technology and science teachers, while mathematics educators need to become more familiar with its application.

Researchers have proposed various ways to make mathematics more explicit in integrated education. Beswick and Fraser (2019) suggest increasing teacher abilities in identifying the STE(A)M knowledge and skills required in single projects. To raise mathematics in context, Bergsten and Frejd (2019) and Triantafillou et al. (2021) recommend getting in-depth information about the workplace. Others advocate for collective learning, establishing collaborations with teachers from various specialisations (Diego-Mantecón et al., 2021). Just and Siller (2022) claim that project modifications through teacher reflections could significantly promote mathematics in STE(A)M environments. Diego-Mantecón et al. (2022) and Han et al. (2015) add that such teacher reflections would benefit from the support of trainers.

3 Mathematics competencies through project-based learning: Spanish curriculum

The Spanish mathematics curriculum aims to address contextualised or real problems through project-based learning (MEVT, 2022), decomposing the STEM key competency into five specific competencies: modelling, computational thinking, intra-mathematical connections, mathematical representations, and collaborative work skills and positive identity. These competencies cannot be independently interpreted as they overlap like in the Danish (Niss & Højgaard, 2019) and Ontarian (OME, 2005) curricula from which the Spanish one draws.

3.1 Modelling

To solve contextualised problems, the Spanish curriculum fosters modelling by applying strategies and reasoning to explore different solutions (MEVT, 2022). This implies establishing extra-mathematical connections; that is, applying mathematical elements in non-mathematical contexts (De Gamboa et al., 2020). The mathematical modelling process is often described by three steps: (1) translating questions, objects, and relationships from an extra-mathematical world into the mathematical one; (2) executing investigations within the mathematical world to obtain mathematical solutions; and (3) interpreting these solutions to be transferred into the extra-mathematical world (Jankvist & Niss, 2020). This process implies activating sub-competencies like reading, communicating, and creativity (Cevikbas et al., 2022). Engaging in modelling problems, including science and engineering, demands a more profound interaction with the context than traditional mathematical problems (English, 2022). To assess mathematical modelling in STE(A)M projects, the Spanish standards encompass statements that resonate with Borromeo Ferri's (2007) actions: understanding the task, simplifying the problem, mathematising, working mathematically, and interpreting and validating the solution.

3.2 Computational thinking

The Spanish curriculum promotes organising data, recognising patterns, as well as interpreting, modifying, and creating algorithms to model realistic situations (MEVT, 2022). The main role of computational thinking in STE(A)M projects is to produce artefacts to satisfy a necessity (Ye et al., 2023), but not necessarily using electronic devices. The Swedish curriculum has introduced computational thinking to develop algebraic sense (Bråting et al., 2022). Its potential for cultivating geometric (Sinclair & Patterson, 2018) and probabilistic senses within projects (Diego-Mantecón et al., 2019a, b) has also been identified. Research shows that computational thinking influences mathematics competencies like modelling (Ng & Cui, 2021) and representation (Grizioti & Kynigos, 2021; Sinclair & Patterson, 2018). The Spanish standards of computational thinking are related to Weintrop et al.'s (2016) framework, highlighting the following dimensions: manipulating and analysing data, preparing a problem for computational solutions, constructing computational models, as well as using models for finding solutions and understanding concepts.

3.3 Intra-mathematical connections

For developing STEAM projects, the Spanish curriculum recognises the value of establishing intra-mathematical connections (i.e., linking mathematical elements) and acquiring a perception of mathematics as a whole (MEVT, 2022). This relates to the belief that mathematics is essential for solving integrated activities and tackling real problems (Fitzallen, 2015; Usiskin, 2015). Therefore, STEAM projects could help to perceive mathematics like an entity rather than as isolated facts, as students normally see it (Diego-Mantecón et al., 2019a). The official Spanish documents highlight the connection between concepts and procedures, as well as new and previous ideas, whereas Rodríguez-Nieto et al. (2023) also link theorems, arguments, and representations. Several intra-mathematical connections distinguished by Rodríguez-Nieto et al. concur with Spanish evaluation standards: procedural (applying algorithms), features (identifying and relating characteristics of concepts), meaning (attributing sense to a mathematical concept), part-whole (identifying inclusions or generalisations among objects or sets), and implication (identifying that a concept leads to another one through a logical relationship).

3.4 Mathematical representations

This competency, also identified in the Danish (Niss & Højgaard, 2019) and Ontarian (OME, 2005) curricula, entails the representation of concepts, procedures, and results to visualise ideas and organise mathematical processes (MEVT, 2022). It implies learning depiction techniques that expand the ability to interpret and solve real-life situations. The MEVT does not illustrate specific forms of representation, while Goldin (2020) identifies diagrams, graphs, tables, and equations, among others. Technology enriches representations as it allows creating automatic and dynamic productions (Gravemeijer et al., 2017; Jacinto & Carreira, 2023; MEVT, 2022). There is no consensus on what forms of representations should be taught first (e.g. manipulative materials before mathematical expressions; Usiskin, 2015), but researchers agree that STE(A)M projects have the potential to promote different ones (Greca et al., 2021; English et al., 2017), and doing so establishing further intra-mathematical connections. Mainali (2021) identifies four main categories of representation: verbal (written and spoken words), graphic (pictures and coordinate planes), algebraic (symbols and formulas) and numeric (displaying data in an organised way).

3.5 Collaborative work and positive identity

This competency seeks to develop collaborative work skills by actively and reflectively participating in projects within diverse teams with assigned roles, building a positive identity as a mathematics learner and promoting personal and group well-being (MEVT, 2022). This competency aligns with Schoenfeld's (2018) agency, ownership, and identity dimension, emphasising active participation. Project-based learning is a setting for this active participation because, as Hannula (2015) stated, open approaches provide opportunities to meet student needs of autonomy, competency, and belonging. Although STE(A)M projects involve different levels of complexity and can thus generate contradictory reactions, Diego-Mantecón et al. (2019a) found that, under this environment, low-achievers report more positive beliefs about the value of mathematics and its applicability, as well as a positive attitude as a learner of this discipline. Accordingly, evaluating collaborative work and positive identity competency involves judging the acquisition of interest, motivation, and a positive perception of mathematics usefulness when working within assigned team roles.

4 Objectives, research questions, and methods

As noted in the literature, many studies show that school mathematics does not receive the required attention in integrated education, suggesting that STE(A)M project-based learning is unsuitable for exploiting this subject. Just and Siller (2022) claim that small project modifications promote mathematics competencies, and Diego-Mantecón et al. (2022) and Han et al. (2015) state that such modifications can be more easily introduced when teachers receive advisors' assistance. However, there are no studies focused on systematically evaluating teachers' evolution across years when implementing a single STE(A)M project to exploit mathematics competencies. As far as we know, only Greca et al. (2021) amended a STEAM project over six months with the suggestions of primary education teachers. In this study, we evaluate whether secondary education mathematics teachers grow in developing a STE(A)M project to exploit mathematics competencies when they are provided with the necessary time and support. This objective is addressed through the following question: To what extent do teachers evolve to exploit mathematics competencies over time, and how do advisors contribute to such evolution? To examine teachers' successive project implementations and advisors' support, we employed a qualitative approach. Such an approach does not pursue generalising results but identifies the main factors leading teachers to effectively instruct STEAM projects for fostering mathematics competencies.

4.1 Context

This study is set in the context of the Open STEAM professional development programme at the University of Cantabria (Spain), where 19 secondary education teachers and 11 STE(A)M trainers have participated since 2015. Until June 2022, a total of 47 projects have been developed in the classroom. Over the years, the programme features were continuously improved according to teachers' needs. From 2015 to 2017, it delivered mainly theoretical sessions on STE(A)M education and workshops on digital tools and content integration. In 2017/18, sessions were refined to better conceptualise integrated education and learning-related methodologies within the existing curriculum. Teachers were asked to propose STE(A)M projects where the dominant discipline matched their specialisation. The programme currently preserves this idea, evolving towards a more sophisticated training where teachers receive advisors' support throughout their designs and implementations (Diego-Mantecón et al., 2022).

4.2 Sampling selection and description

We selected two secondary education teachers with their corresponding projects because both hold a mathematics degree and implemented a single project multiple times and for a longer period than their colleagues. Teachers in the Open STEAM programme develop several projects a year but rarely execute the same one over time. The two chosen individuals were a male and a female working at state schools. The female had taught mathematics for seven years in secondary education, whereas the male had fifteen years of experience as a high school teacher and six as an associate university lecturer. Until their participation in STEAM courses, they described themselves as traditional teachers who used textbooks extensively. In 2017/18, they attended a training course and were asked to design and implement a STE(A)M project. The male teacher planned to develop a water rocket and the female a mathematics museum. The former project sought to learn about rockets and trajectories, dealing with functions in context, and the latter entailed creating a museum decorated with mathematics-related pictures. The projects were refined with the aid of two advisors (a male and a female) after each implementation. The female holds an engineering degree and has been a mathematics, physics, and technology teacher in secondary education for over 10 years. The male is a mathematician with 10 years of experience as a mathematics education researcher. Both have been trainers of the Open STEAM programme for the last eight years.

4.3 Interventions, data collection, and analysis

The projects were designed and implemented by the teachers in 2017/18, and then refined and executed with the advisors' assistance in 2018/19 and 2021/22. During the three academic years, the Water Rocket was performed with five student groups aged 17 and the Mathematics Museum with four teams aged 15. In total, 123 subjects participated in the former project and 92 in the latter through 77 sessions. On average, each project took nine one-hour sessions for teachers to present the problem, monitor progress, and supervise student reports (Table 1).

To collect data, we employed observations, student reports, and focus groups. The advisors observed each session, avoiding direct intervention (Cohen et al., 2007), and analysed student reports to comprehend teachers' practices. After each implementation, the two advisors shared their data analyses to recommend project improvements. These recommendations were discussed with each teacher during the focus groups, which lasted approximately 90 min each (Table 1). A field notes template containing two checklists and a free text section was employed for the observations. The first checklist comprised statements that gauged the presence of mathematics competencies; for example, for analysing modelling, we used the evaluation standards defined in Sect. 3, including understanding the problem, working mathematically, interpreting solutions, and validating them. The second checklist appraised whether the modifications agreed upon in the focus groups were implemented; for instance, whether quadratic interpolations were incorporated for modelling and error comparison was introduced. The free text section was utilised to document descriptive and reflective information about aspects not considered beforehand. Three co-authors of this article audio-recorded and transcribed the focus groups; these data contributed to establishing whether teachers and advisors described their instructions in the same way and helped to register their requests and consensus for subsequent iterations. This methodological strategy differs from the one described by Greca et al. (2021), in which researchers devised the original project and refined it with teachers' suggestions.

Concerning data analysis, we employed a content analysis approach to interpret data systematically. We began by identifying patterns and trends within the data extracted from the field notes and student reports. After breaking down the data into specific units of analysis, we established ways of refining the projects and improving teachers' instruction. The improvements were driven by the advisors' pedagogical knowledge of STEAM and their experience integrating empirical findings into teaching. We matched the data categorised in the focus groups with the analysis of the observations to track teachers' evolution and the advisors' contribution to such progress. The teachers decided what modifications to introduce after attending the training courses suggested by the advisors.

5 Results

In the following, we present the primary outcomes of each project.

5.1 Water rocket

5.1.1 Implementation 1

In the focus group after Implementation 1, the teacher and the two advisors jointly concluded that modelling was not strongly promoted.

The teacher initially stated: "This competency randomly appeared when the students worked on an abstract idea [the notion of real function] by doing an experiment [throwing and recording the rocket] and digitally generated the model representing the movement".

One advisor intervened: "Do you think students' interpretations were sufficiently sound to formulate the maths

Table 1 Number of students, sessions, and focus groups per implementation

	Water Rocket		Maths Museum	
	Implementation 1 (17/18)			
	Group 1	Group 2	Group 1	Group 2
Students	23	24	18	25
Sessions	10	10	5	5
Focus groups	3		2	
	Implementation 2 (18/19)			
	Group 3	Group 4	Group 3	
Students	27	24	21	
Sessions	7	7	11	
Focus groups	2		4	
	Implementation 3 (21/22)			
	Group 5		Group 4	
Students	25		28	
Sessions	9		13	
Focus groups	1		1	

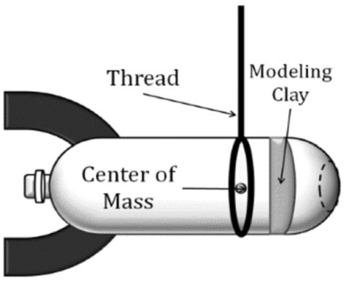

Fig. 1 Sketch of the rocket

Fig. 2 Rocket and mechanism construction

Fig. 3 Rocket launching

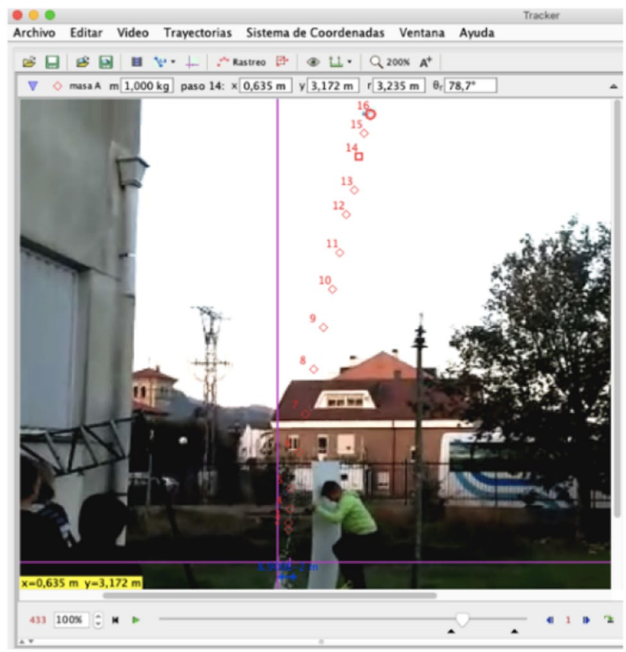

Fig. 4 Point set in tracker

problem and select strategies? Or did they follow direct instructions?"

The teacher thought this over: "Oh, I see… Students generated the model, but it's true that they didn't engage in reasoning."

The three individuals recognised that connections between the extra- and intra-mathematical worlds had been established, but these were too mediated by how the activity was guided and technology used. The field notes reflected a convincing presentation of the problem context through videos of actual rockets and trajectory predictions; nevertheless, the students built the rockets and launching mechanisms by assembling material through direct instructions (Figs. 1, 2) and the point set depicting the recorded movement of the launches was automatically adjusted using Tracker (Figs. 3, 4, 5) without room for working mathematically.

The teacher claimed that the collaborative work and positive identity competency emerged powerfully: "Students worked well in teams of 3 to 5; they were motivated during the rocket construction and its launches."

The advisors partly supported this assertion, leading the teacher to contemplate the source of motivation: "Were students excited by the construction and launches or by their understanding of the maths employed?"

The teacher then realised students' positive emotions stemmed from the hands-on nature of the activity rather than from comprehending the mathematics' applicability: "Yes, the students interpreted the curves generated by the program, but I agree they were excited because of the rocket construction and launches." He also stressed that the construction process was time-consuming and planned its elimination: "The time invested in this part was not worth the maths emerging. It over-prolongs the project and could be instructed by non-maths teachers." In particular, the teacher expressed his worry about how the intra-mathematics and representation competencies emerged: "Geometry and measurement concepts were considered for the artefacts' construction, but connections were not established between analysis and algebra because the software simplified the managing of coordinate axes, points, and curves." In the same focus group, he recognised that the way in which the

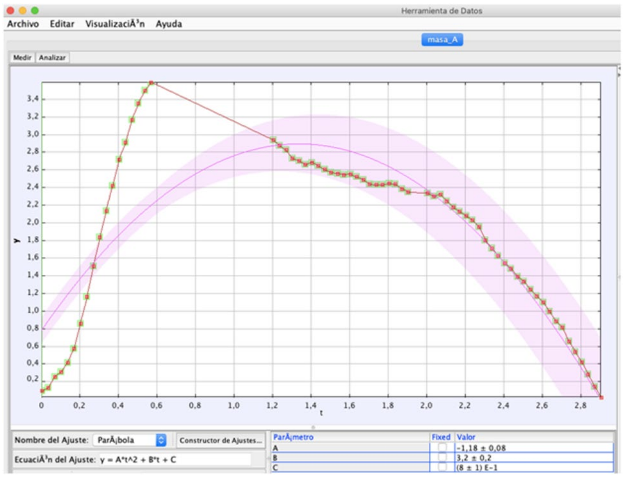

Fig. 5 Automatic tracker adjustment

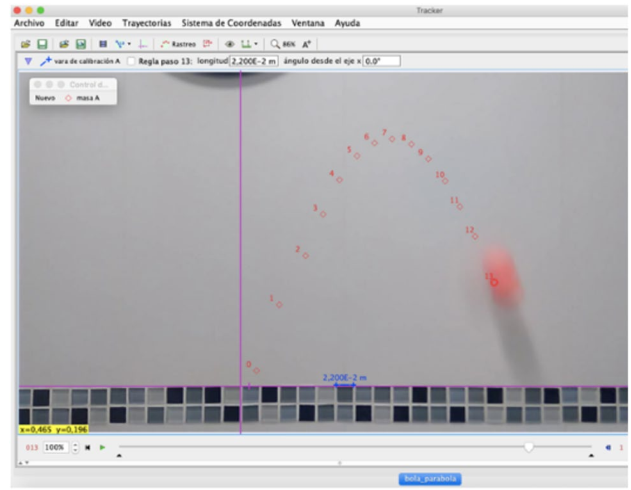

Fig. 6 Ball trajectory points in Tracker

project was approached and technology used hindered the enhancement of these two competencies. Thus, he sought assistance: "I need help to formulate the problem better and further exploit maths."

The advisors agreed on removing the rocket construction and invited the teacher to attend Tracker and GeoGebra courses to learn about their didactical use. They also scheduled successive meetings to discuss improvements for the project and practice; several recommendations were integrated into the next run.

5.1.2 Implementation 2

The analysis of field notes and student reports revealed that in the second run, the teacher put a greater emphasis on all evaluated competencies, except computational thinking. Concerning modelling, the teacher encouraged not only to interpret the problem situation but also to formulate the mathematics problem and understand solutions in context. To avoid a routine modelling procedure, he (assisted by the advisors) proposed launching the rockets constructed in Intervention 1 and comparing their trajectories with those of other easily recorded objects thrown manually without using a launcher. For example, Fig. 6 shows the points identified using Tracker when a ball was thrown. The students' reports demonstrated that launching various objects facilitates model creation, constraint identification, and assumption formulation. Due to the didactical Tracker and GeoGebra training, the teacher prevented automatic calculations and stimulated different solution strategies. He requested exporting the Tracker data set of the launches to GeoGebra for analysis. Unlike in the previous run, time was given to conclude that the set of points could be represented by a parabola ($f(t) = at^2 + bt + c | a, b, c \in \mathbb{R}$). The field notes disclosed that the teacher suggested two strategies to adjust the points: one based on creating three sliders corresponding with the function coefficients (Fig. 7), and the other focused on a quadratic interpolation from three points (Fig. 8). During a focus group, the teacher highly valued the latter: "I often do this with paper and pencil but without a context. I didn't know I could manage it with GeoGebra."

Regarding the intra-mathematics and representation competencies, the analyses showed that, for instance, the feature and part-whole connections were exhibited when the teacher himself requested choosing various triads of points to obtain distinct quadratic interpolations and to find the one that best fits the data. Graphic (coordinate plane) and numeric (table of points) representations were used in this task (Fig. 8). When he challenged students to compare their outcomes with the automatic adjustment of GeoGebra (Fig. 9), the feature connection appeared again. As described by a student in her report, "the quadratic coefficient closely approaches the ideal value in a

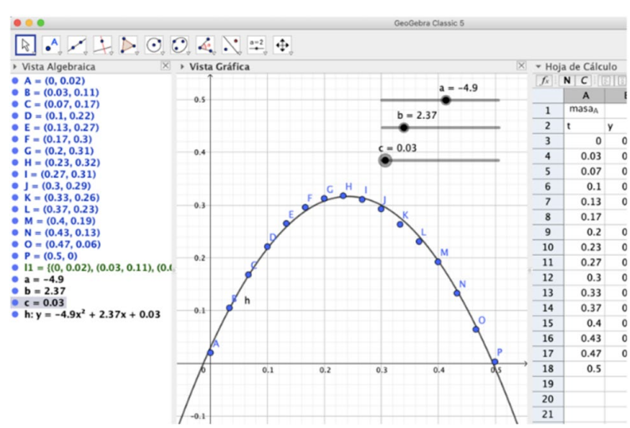

Fig. 7 Adjusting using sliders

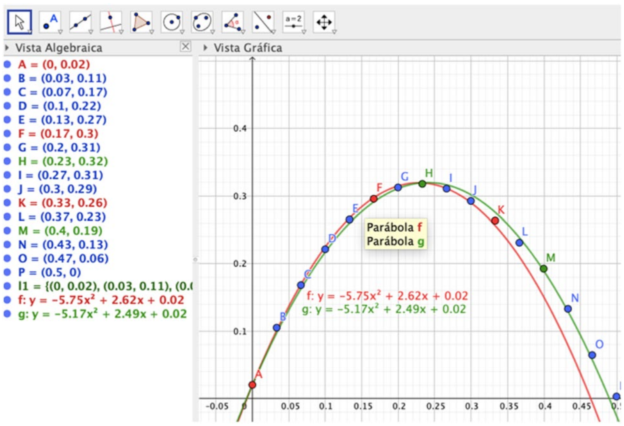

Fig. 8 Two quadratic interpolations

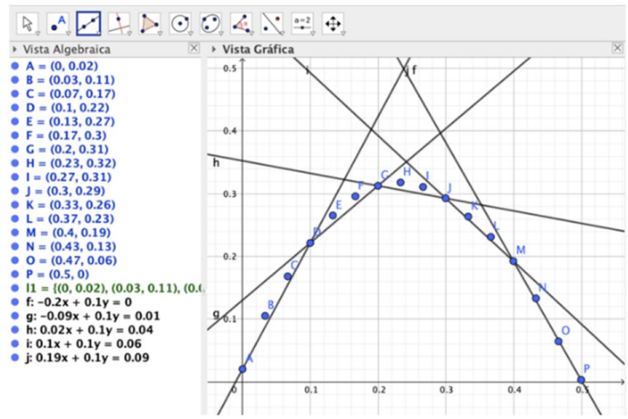

Fig. 10 Linear interpolation

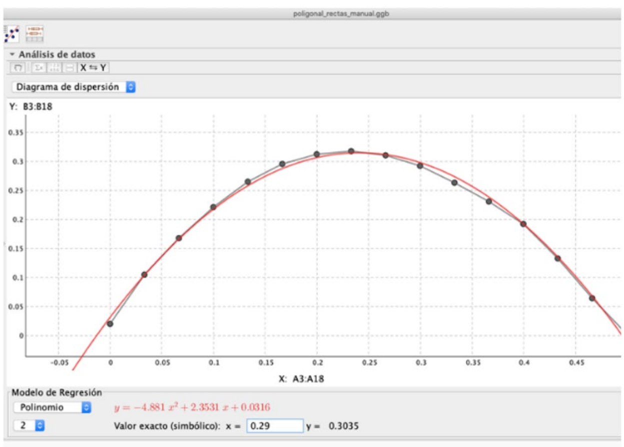

Fig. 9 Automatic GeoGebra adjustment

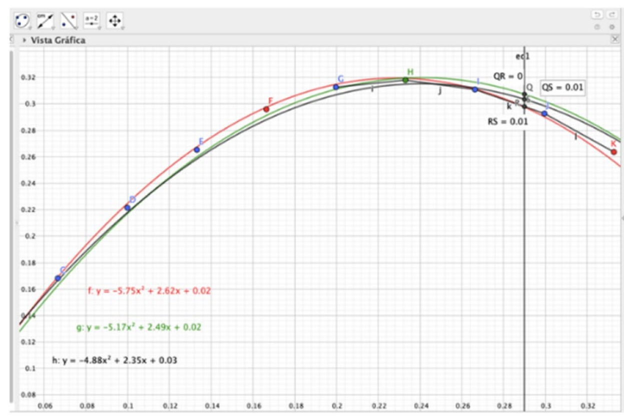

Fig. 11 Curve fitting

parabolic launch ($-g/2 \approx -4.9$ m/s^2) since objects falling solely under gravity have an approximate acceleration value of 9.81 m/s^2."

There was a greater emphasis on positive identity than in the first iteration. In the focus groups, the teacher and advisors agreed that this second iteration accentuated the value of school mathematics as students explored its applicability in verifying a physical phenomenon. However, as reflected in the field notes, collaborative work skills did not strongly manifest. This was because tasks requiring mathematics were often solved individually, while collaborative tasks lacked mathematics. In this implementation, the teacher felt he had become more confident through the recommendations and training, and his practice was more effective than the first one: "I feel excited; I'd like to incorporate new tasks." With this in mind, the advisors guided him into new ideas.

5.1.3 Implementation 3

In this third run, the teacher proposed modelling the trajectory by calculating piecewise linear interpolations with pairs of points to get polynomial expressions of degree 1 ($y = mx + n | m, n \in \mathbb{R}$). This strategy further mobilised the intra-mathematics and representation competencies as required by the advisors; verbal and graphic representations were employed, and features and part-whole connections were established when defining intervals for each algebraic expression. The screenshot in Fig. 10, taken from a student, shows an intermediate step for obtaining the algebraic expression of the ball trajectory as a piecewise function. Procedural connections were observed when introducing the notion of error to compare the linear and quadratic interpolations at specific times in relation to the GeoGebra automatic fit; Fig. 11 represents the graphs for calculating their values at t = 0.29 s. Following training with the advisors, the teacher suggested analysing the accuracy of the GeoGebra automatic fit to highlight the intra-mathematics competency. Feature and implication connections were encouraged when establishing an analogy between the adjustment and the space formula, whereas implication and procedural connections appeared when calculating the second derivative of the fit function concerning time to get the acceleration value.

As documented in the field notes, to promote collaborative skills, the teacher shared his plan to circulate among the working groups and ask students individually to describe the problem-solving approach adopted by the team. While this adjustment facilitated students in developing a sense of belonging, the teacher and advisors believed that the promotion of a positive identity was because students established a closer relationship between school mathematics and its applicability in contextualised situations. The teacher stated: “I realised, probably for the first time, that my students really experienced the utility of maths. I believe this improved their interest in the discipline.”

He recognised that in Implementation 1, students were excited about collaborating to construct the Water Rocket, but they engaged with less mathematics than in Implementations 2 and 3. The advisors also corroborated this observation. Neither the teacher nor the advisors identified any dimension of computational thinking.

5.2 Mathematics museum

5.2.1 Implementation 1

During the focus groups, the advisors commented positively on the teacher’s idea of producing a mathematics museum based on art and nature themes. They also remarked that the context was well placed to raise curiosity and motivation, but the modelling competency needed to be emphasised. The teacher asked students to construct two cubic rooms of random dimensions using straws and tape and decorate them with images from the Internet (Figs. 12, 13, 14). To promote modelling in future runs, an advisor proposed creating a scaled cardboard replica of an actual museum: “If you ask students to reproduce a building, you may foster modelling

Fig. 12 Materials

Fig. 13 Construction process

as students should interpret the problem, apply strategies to get a maths solution, and validate it in context.”

The teacher reacted sceptically; to maintain her interest, the other advisor valued the teacher’s idea of group-based work with two assigned roles (architect or designer).

The teacher proposed tasks using nature and art images to illustrate concepts like the Golden ratio and Fibonacci spiral, as well as encourage calculations applying the Euler formula. With these tasks, she thought to promote intra-mathematics and representation competencies. The advisors agreed to some extent: “Don’t you think that the artistic or nature elements afforded forced task setting?”

The teacher stated: “Well… This setting was an excuse for calculating and understanding maths concepts”. One advisor further asserted: “So, can we say these are decontextualised tasks similar to the ones proposed in regular lessons?”

Based on Diego-Mantecón et al.’s (2019b) research, this advisor illustrated his argument with a photo of the Louvre Museum used in a textbook task to classify the polygons that form the pyramidal roof and to calculate its height from given measurements: “This photo is utilised to exemplify a geometric shape in real life, but the task doesn’t

Fig. 14 Museum prototype

foster an art-context to analyse mathematical techniques, for example."

At that point, the teacher realised that it was possible to exploit art and nature contexts in better ways, proposing less procedural and more demanding tasks. Drawing on the curricular initiatives, the advisors also suggested using technology and attending workshops to foster mathematical analyses.

5.2.2 Implementation 2

In this second iteration, the teacher did not feel confident fostering modelling by replicating a construction-to-scale: "I need more time practising in the Open STEAM programme to instruct this type of activity."

The advisors took a step back and let her maintain the original idea of constructing a structure with random dimensions. Unlike modelling, the representation and intra-mathematics competencies were more pronounced than in Implementation 1. As documented in the field notes, the teacher reduced procedural tasks and incorporated others in which art and nature were used for analysing and verifying mathematical concepts through digital and non-digital tools. For instance, implication and feature intra-mathematical connections, as well as verbal and graphic representations were promoted while exploring the *Alhambra* tiling through an open GeoGebra applet that comprises symmetry axes, translation vectors, and rotation centres (Fig. 15). The teacher discovered this applet during her training. Procedural and feature connections were exhibited when the teacher offered artworks to analyse their construction by verifying conjectures and using mathematical techniques. As observed in one student report, a drawing of the Parthenon was provided to measure segments and establish relationships to the golden ratio (Fig. 16). In this activity, suggested by the researcher advisor, the drawing was not used to illustrate concepts or properties but as an object of inquiry where lengths are measured to verify hypotheses.

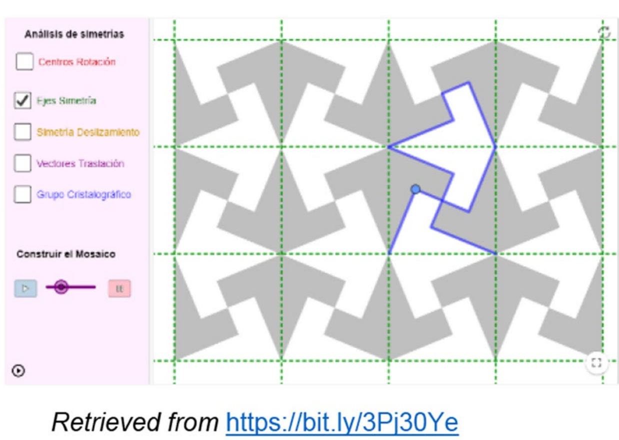

Fig. 15 GeoGebra applet—Alhambra tiling

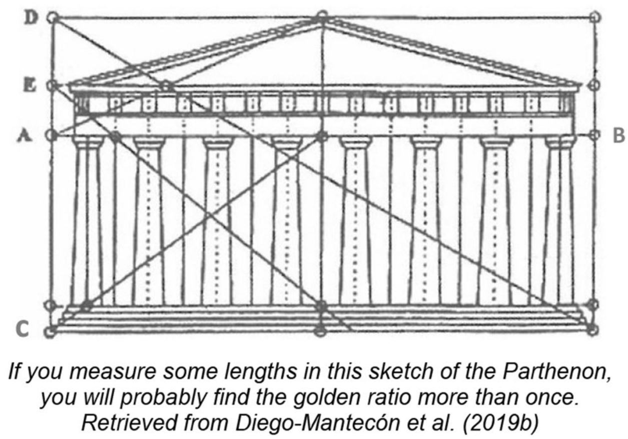

Fig. 16 Analysing Parthenon

In the focus groups, the advisors congratulated the teacher for her amendments and effort. The teacher enthusiastically expressed her aim of further refining the project: "I'm feeling more confident now; the tasks were more encouraging than in the first run. I'm thinking of a more contextualised problem […] for the next practice."

Despite her positive feelings, there was no improvement in the collaborative work and positive identity competency. After being asked, the teacher stated: "Honestly, I didn't notice that the students had gained more confidence in maths or felt it was more useful. I am not sure whether they worked well in groups." One advisor intervened, suggesting grouping students according to their topic interest.

5.2.3 Implementation 3

Considering the advisors' suggestions, the teacher embarked on a construction to scale, setting the project context in the Ukraine and Russia conflict to attract student attention. Particularly, she requested building a cardboard mathematics museum with a prism proportional to the classroom's dimensions and a roof reproducing the Louvre pyramid (Figs. 17, 18). The student groups created the elements of the museum (e.g. floor, walls, roof, artworks, and security system) independently. Groups were formed according to student's interests, adopting roles such as architects, painters, or content creators. The advisor with school teaching experience suggested this strategy. Unlike previous implementations, the modifications led to the simultaneous enhancement of the five analysed competencies.

This project iteration largely emphasised the competencies appearing in previous runs. For example, modelling was accentuated when students reproduced their classroom and the Louvre pyramid at different scales, as well as

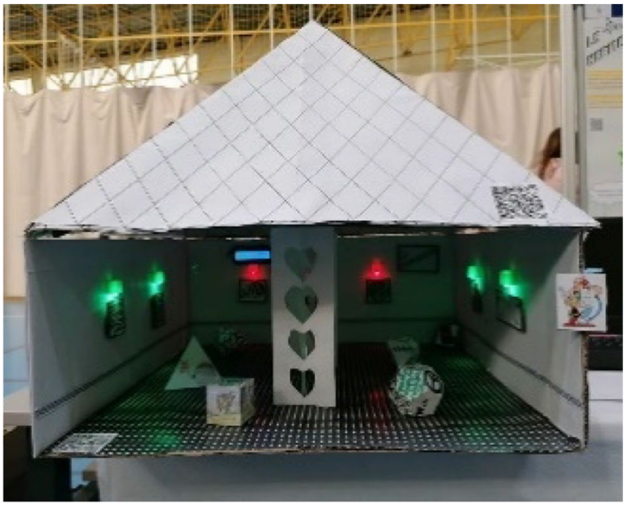

Fig. 17 Museum structure

Fig. 18 Museum inside

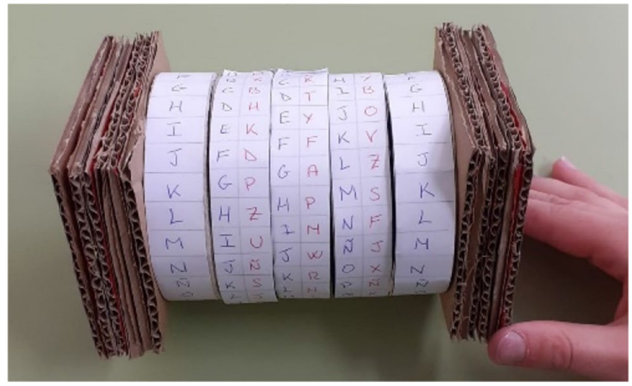

Fig. 19 Enigma machine prototype

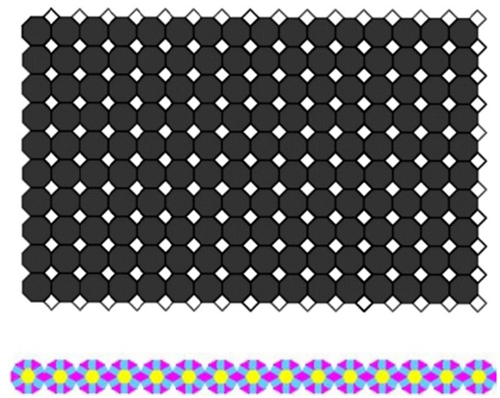

Fig. 20 Flooring and wall border

when they constructed a simulation of the Enigma Machine (Fig. 19). The student reports showed that the real problems were transformed into mathematical ones, and mathematical solutions were reached and turned into real solutions. For instance, for the museum structure, the teacher asked to measure classroom lengths to determine the 2D dimensions of cardboard pieces and assemble the 3D quadrangular pyramid. As suggested by the advisor, inspired by Gravemeijer et al.'s (2017) research, this practice aimed to simulate real-life conditions. In such a scenario, students must not only identify the mathematical variables and structures underlying real-world problems but also grasp contextual intricacies to approach the mathematical problem.

Representation and intra-mathematics competencies were mobilised during the design of the flooring (mosaic) and wall border (frieze) (Fig. 20). The student reports showed that to generate these elements with both manipulative materials and GeoGebra, properties of plane geometry, the idea of a generator element (tile) and movements in the plane were addressed, promoting implication and feature connections. This task also activated graphic and verbal representations. The aforementioned representations, as well as implication and procedural connections, were also displayed when creating a painting representing yin and yang; the circumference diameter was divided into four equal parts with a compass for drawing the *taijitu* symbol (Fig. 21). The feature and part-whole connections, together with representation competency, emerged when the teacher proposed replicating two peace doves made with the GeoGebra Tangram, which required decomposing each dove as the sum of Tangram pieces and drawing polygons on paper (Fig. 22). With the reproduction of the Enigma machine (Fig. 19), the procedural connections and representation competencies were again jointly raised when sketches and formulas emerged.

The collaborative work and positive identity competency was accentuated much more than in previous practices. The collaborative dimension was promoted due to the group distribution, which considered student professional

Fig. 21 Yin and yang

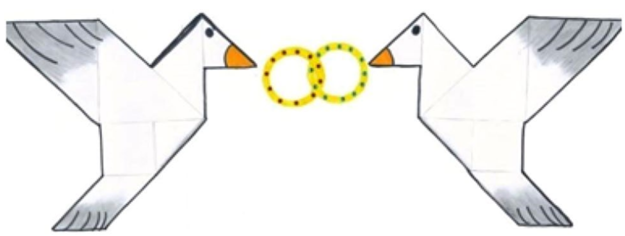

Fig. 22 Doves

preferences; for example, some students worked as content creators to design the frieze and mosaic, while others took on the painter role, producing art through mathematics. In the focus group, the teacher stated: "I noticed students were more engaged in activities that captured their interest. Working on different project parts was beneficial because they felt their skills and contributions were valuable."

The positive identity was promoted during the constructions, especially when students recognised mathematics applicability to generate mock-ups of actual buildings and their usefulness in a war context. The teacher indicated: "Students really valued the relevance of maths in World War II for transmitting coded info with the Enigma machine."

The computational thinking competency, absent in previous attempts, appeared when loops and conditionals were used for developing the security system. As outlined in the field notes, a block-based programming environment connected to physical input sensors and output devices was set up. If a painting is removed, increased light detected by the sensor triggers a warning on LED screens, accompanied by buzzing alarm activation.

6 Discussion

Our findings demonstrate that mathematics teachers, aided by advisors over an extended period, can enhance their ability to foster mathematics competencies through STEAM projects. With training and recommendations, the two teachers mobilised competencies that did not appear in their initial attempts and emphasised others at a higher level than in previous iterations. Contrary to Just and Siller's (2022) belief that small modifications can significantly enhance STE(A)M projects, our teachers required substantial assistance and personalised training to cultivate mathematics competencies. That is, the competencies did not emerge from simple suggestions but through intensive teacher-advisor interaction. This aligns with Triantafillou et al.'s (2021) findings, highlighting the difficulty in consensus-building with teachers and the essential role of reflective processes. In the following paragraphs, we discuss how teachers evolved to exploit each competency and the way in which the advisors contributed to such progress.

Modelling was a significant challenge for the teachers. In the first Water Rocket practice, this competency emerged due to the nature of the project, albeit routinely without meaningful mathematical involvement. The teacher offered excessive guidance, similar to what was observed in the pre-service teachers of Bergsten and Frejd (2019). This excessive supervision led to a procedural usage of technology. Only in the last two implementations, the teacher (guided by the advisors) explicitly required to transform real-world problems into mathematical ones and provide contextualised solutions. This is a significant achievement because, as Gravemeijer et al. (2017) report, extensive technology use overshadows mathematical processes. In the Mathematics Museum project, modelling was only fostered clearly in the third run when the teacher requested that students build a mock-up. The advisors were crucial in negotiating the context of this project, confirming that, as Diego-Mantecón et al. (2021) reported earlier, mathematics teachers face difficulties in applying school mathematics in unfamiliar contexts. The involvement of an advisor with a teaching background in engineering was essential in convincing the teacher to blend engineering and mathematics through a modelling scenario. When modelling arose in both projects, it enhanced not just mathematical content but competencies such as intra-mathematics and representation, in line with the expectations of Cevikbas et al. (2022).

Both projects showed concurrent progress of the intra-mathematics and representation competencies, resonating with Rodríguez-Nieto et al.'s (2023) approach that merges these competencies. The progressions were driven by the refinement of the tasks' focus. In the Museum, the teacher transitioned from a broad to a specific focus, emphasising the creation of artworks through mathematics. This shift avoided disconnected procedural tasks, which Lasa et al. (2020) and Martín-Páez et al. (2019) describe as common in STE(A)M projects. In the Water Rocket, the teacher incorporated linear and quadratic interpolations with GeoGebra to adjust data instead of relying on automatic fits of the software, bringing more explicit representations and intra-mathematical connections. Various representations and connections identified in this project were consistent with those found by Rodríguez-Nieto et al. (2023) in a similar context but without using technology. This suggests that, as Jacinto and Carreira (2023) indicate, pedagogical knowledge of technology helps express mathematics reasoning. The improvements stem not solely from advisors' mathematical knowledge and pedagogical expertise with digital tools but also from their experience integrating research outcomes into collaborative task design; an approach previously utilised by Triantafillou et al. (2021).

The collaborative work and positive identity competency manifested differently in the two projects. In the Water Rocket, motivational aspects were high in Implementation 1 due to the construction of artefacts. However, this motivation was primarily attributed to the hands-on nature of the activity, where mathematics engagement was not clearly displayed. Prior research has indicated that activities leading to a final product boost student motivation (Diego-Mantecón et al., 2019a), but as shown in this case, it is important to investigate the reasons behind such motivation. A balance between motivation and mathematical engagement was only achieved in Implementations 2 and 3. The establishment of a positive identity became evident as students recognised the practical utility of mathematics in contextualised situations. In the Mathematics Museum, beyond the enthusiasm generated by constructing a mock-up, the teacher fostered more positive interactions among students by assigning roles aligned with their career interests. This idea of allocating roles connected to professional paths was supported by one of the advisors, drawing from the work of Maass and Engeln (2019). This finding highlights the importance of teachers being mindful of their students' preferences when defining roles within STEAM projects.

Computational thinking was the least promoted competency in both projects, appearing only in the last iteration of the Mathematics Museum. Two main reasons hindered its presence; firstly, the incorporation of this competency into the Spanish mathematics curriculum had occurred in recent years, leading teachers to prioritise the other four competencies. Secondly, the academic background of the two advisors, with a lack of foundation in computational thinking, did not facilitate its development. As suggested by Bråting et al. (2022), trainers' specialisation is essential for developing computational thinking. In line with Han et al. (2015), this study indicates that guiding less-experienced teachers in developing mathematics competencies requires knowledge, time, and dedication. This differs from studies with pre-service teachers, who seem to find it more straightforward to integrate mathematics into STE(A)M contexts through programming activities (Bergsten & Frejd, 2019). Neither the teachers nor the advisors established connections between computational thinking and other competencies like modelling; a relationship authors such as Sinclair and Patterson (2018) frequently recognise.

7 Conclusion

The tendency of secondary education curricula to emphasise mathematics competencies through STE(A)M projects often assumes teacher capability; however, research has recently revealed difficulties in exploiting mathematics and the necessity for assistance when executing projects. This article explores these two dimensions by evaluating how two teachers, taking a long-term professional development programme, evolved their projects to enhance mathematics competencies with the support of two advisors and how this contributed to such progress.

Concerning teachers' growth, we argue that inexperienced teachers can cultivate mathematics competencies to a certain extent, but their progress is only enhanced after several iterations complemented with personalised training and support from advisors. The teachers' evolution was not due to minor recommendations but intensive interaction with the advisors, which required continuous reflection. This interaction helped teachers gain confidence and apply their own contributions. Still, some competencies were challenging to address. For instance, it was difficult to tackle modelling, but when it was, the intra-mathematics, representations, as well as the collaborative work and positive identity competencies emerged more naturally. Computational thinking was not very present because of the advisors' lack of expertise in this competency and their focus on promoting the other four. We, however, believe that with further refinements, there would be a greater presence of this competency.

The teachers' progress seems to be influenced by the advisor's academic background, pedagogical expertise, ability to transfer research outcomes into teaching, and experience supporting others. The academic background was crucial for integrating disciplines in contextualised problems. The pedagogical expertise facilitated the promotion of mathematical strategies and the exploitation of technology for teaching. Their ability to transfer research outcomes into mathematics lessons helped to modify tasks, activating competencies at a higher level. The experience of supporting others was essential for maintaining a balance between teacher self-confidence and project enhancements, which required commitment and constancy. Commitment was necessary to begin the projects and constancy to complete them successfully. The fact that the teachers originally designed the projects also helped maintain this balance.

Based on this study, we conclude that in order to exploit mathematics through STE(A)M projects, teachers' professional development programmes should provide personalised training and support during the project planning stage. We recommend a combination of two STE(A)M advisors: one with research expertise and the other with school teaching experience. However, the present outcomes should be interpreted with caution and not generalised as they are the result of two experiments alone. To provide further consistency, it would be necessary to evaluate how the two teachers apply the acquired expertise to different projects and to conduct large-scale studies.

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