## Super-resolution and apodization with discrete adaptive optics

M. P. CAGIGAL  $^{1,\ast}$  , A. Fuentes  $^1$  , V. F. Canales  $^1$  , P. J. Valle  $^1$  , M. A. CAGIGAS  $^2$  , and O. Castellanos  $^3$ 

<sup>1</sup>Departamento de Física Aplicada, Universidad de Cantabria, Avda. de los Castros 48, 39005 Santander, Spain

<sup>2</sup> Instituto de Astrofísica de Canarias, Vía Láctea s/n, 38200 La Laguna, Spain

<sup>3</sup> Universidad Estatal Península de Santa Elena, Vía Santa Elena – La Libertad, Km 1, La Libertad, Ecuador

\* Corresponding author: perezcm@unican.es

Compiled June 19, 2023

High-resolution imaging is of great importance in various fields. The use of pupil phase-only filters (PPF) exceeds the diffraction limit of the imaging system in a simple way. When dealing with distorted wavefronts, however, PPF require that aberrations be compensated for. In this paper, we introduce a novel technique consisting in the use of discrete adaptive optics with PPFs so that the compensating device implements the PPF at the same time. Analysis of the theory for point spread function reshaping using PPFs has enabled us to develop a new approach to characterizing apodizing filters. A validation experiment has been carried out, the first of its kind to our knowledge, in which a number of PPFs were combined with two levels of compensation. Our experimental results are discussed. © 2023 Optica Publishing Group

## http://dx.doi.org/

Reshaping an instrument's point spread function (PSF) is a technique applied in various fields such as microscopy [1, 2], medical ultrasound [3], optical trapping [4], lithography [5], data storage [6] and astronomy [7, 8]. An easy way to reshape the instrument PSF is through the use of pupil filters [9–12]. We focus specifically on pupil phase-only filters (PPFs) since they do not absorb energy. We pay particular attention to binary phase-only filters with phase values 0 and  $\pi$ , since they are simple to calculate and implement, do not produce focus shift along the system's optical axis and perform fairly well [13]. In two zone binary filters, it is sufficient to modify the border radius to change the shape of the PSF central peak; that is, to obtain a super-resolved or apodized PSF. We use figures of merit, such as Strehl ratio (S), and the axial and transverse gain, to estimate the binary PPF efficiency. So far, these figures have been obtained analytically by using a parabolic approximation of the PSF core [13]. This approach is effective for describing super-resolution but not for apodization, so our first goal is to introduce a new theoretical approach to design apodizing filters. The main drawback when dealing with distorted wavefronts is that PPFs are not effective unless the wavefront has been corrected to achieve

a Strehl ratio of 0.25 [14]. This condition requires the help of an adaptive optics (AO) system. The second goal of our paper is to show that performing the PPF and compensation simultaneously is as effective as using an AO system together with an additional separate system to create the PPF. It is also much simpler and cheaper. The final goal of this paper is to demonstrate that the use of discrete AO (DAO) [15, 16] is particularly advantageous for reproducing the PPF and the compensating phase screen (CPS) at the same time. The use of a deformable mirror (DM) in standard AO has certain drawbacks for CPS generation such as the limited dynamic range, the low spatial sampling and the influence function (particularly important in binary PPF). However, the use of a spatial light modulator (SLM), as in DAO, easily surmounts these limitations. Moreover, SLM behaves quite well in broadband.[17] DAO techniques have several additional advantages over AO: they are based on a point diffraction interferometer (PDI) wavefront sensor that is easy to manufacture and implement, no algorithm is needed to reconstruct the wavefront surface and the dynamic range is almost infinite. Finally, note that combining super-resolving or apodizing PPF with discrete CPS for aberration compensation is quite simple because both share the same set of discrete phase values.

1

We provide experimental validation for the binary and quaternary cases of DAO. In the laboratory set-up we used a deformable mirror to distort the incoming wavefront and a SLM to compensate for the distortion while at the same time introducing a PPF. This is the first time that such experiments have been carried out. The results confirm the feasibility of our proposal.

To estimate PPF efficiency, various figures of merit that characterize the PSF must be used. So far, these figures have been obtained analytically for binary PPFs from the filter zone radii by using a parabolic approximation to the PSF core [13]. However, we show that this approach is effective for describing superresolution but not for apodization, so we derive a new theoretical approach in order to obtain useful figures of merit for designing apodizing filters. We pay attention to the Strehl ratio *S*, which compares the height of the central core with that of the unobstructed pupil, and to the transverse gain factor *G*<sub>T</sub> (defined as in [13]), which gives a measure of the apodization or super-resolution performance in the transverse direction in the focal plane. Both are normalized so that they are equal to unity for the unobstructed pupil. A filter is considered to be super-resolving when  $G_T$  is greater than unity and apodizing in the opposite case. Our objective is to derive expressions for these figures of merit within the framework of scalar diffraction theory. Let us consider a general complex pupil function  $P(\rho)=A(\rho)e^{i\phi(\rho)}$ , where  $\rho$  is the normalized radial coordinate over the pupil plane,  $A(\rho)$  is the amplitude transmittance function and  $\phi(\rho)$  is the phase function. For a converging monochromatic spherical wave front passing through the center of the pupil, the normalized field amplitude *U* in the focal plane may be written as:

$$U(\nu) = 2 \int_0^1 P(\rho) J_0(\nu \rho) \rho d\rho$$
(1)

where  $J_n$  represents a Bessel function of the first kind (0th order in this case),  $\nu$  is the radial dimensionless optical coordinate in the focal plane given by  $\nu = k \text{ NA } r$  with  $k = 2\pi/\lambda$ , NA is the numerical aperture of the pupil, and r is the usual radial distance. In order to derive analytical expressions for the figures of merit, the field distribution in Eq. 1 is expanded in series near the geometrical focus based on moments of the pupil function,  $I_n$  [13]:

$$I_n = 2 \int_0^1 P(\rho) \rho^{2n+1} d\rho$$
<sup>(2)</sup>

Within a second-order approximation of the intensity distribution of the PSF core (parabolic approximation), the figures of merit obtained from these moments are [13]:

$$S = I_0^2 \qquad G_{\rm T} = 2 \frac{I_1}{I_0}$$
 (3)

For simplicity, we henceforth study two-zone binary phase-only filters, shown in Fig.1, though the analysis could be readily extended to n-zone filters. Two-zone  $0 - \pi$  filters are completely determined by the value of the radius,  $\rho_1$ , of the boundary between zones. In such a case,  $P(\rho) = -1$  if  $\rho < \rho_1$  and  $P(\rho) = 1$  if  $\rho_1 < \rho < 1$ , and Eq. 1 yields:

$$U(\nu) = 2\left(\frac{J_{1}(\nu)}{\nu} - 2\rho_{1}\frac{J_{1}(\rho_{1}\nu)}{\nu}\right)$$
(4)



**Fig. 1.** Left: Two-zone binary PPF with phase  $\pi$  in the internal circle and 0 in the external annulus. Right: PSF (squared modulus of Eq. 4) for different values of the radius of two-zone binary PPF.

while the pupil moments, Eq. 2, are real-valued functions:

$$I_n = \pm \frac{2 - 4\rho_1^{2n+2}}{2n+2}$$
(5)

where the plus sign corresponds to the filter with  $\pi$  phase in the inner circle and 0 in the outer annulus, and the minus sign to

the complementary filter. Finally, the figures of merit in Eq. 3 become:

$$S = (1 - 2\rho_1^2)^2 \qquad G_{\rm T} = \frac{1 - 2\rho_1^4}{1 - 2\rho_1^2} \tag{6}$$



**Fig. 2.** Transverse gain  $G_{\rm T}$  as a function of the radius  $\rho_1$ . Parabolic core approximation, Eq. 6, (black solid lines) and Gaussian sidelobe approximation, Eq. 10, (blue long-dashed line). Experimental data (dots and circles) of the transverse gain fit to Eq. 6 in the super-resolution regime (left branch) and to Eq. 10 in the apodization regime (right branch).

The transverse gain in Eq. 6 is represented in Fig. 2. Phase filters do not absorb energy, so when the core shrinks and decreases, the sidelobes increase, which distinguishes three regimes:

a) Super-resolution:  $0 < \rho_1 < 0.57$ . For such a radius range the PSF is narrower than that corresponding to the unobstructed pupil, at the cost of an increase of sidelobe intensity and a decrease in *S* (see Fig.1). To explain this behavior, we can use Eq. 4. The field amplitude is obtained from the subtraction of the Airy field corresponding to the clear pupil,  $J_1(\nu)/\nu$ , and twice that of the inner pupil of radius  $\rho_1$ ,  $\rho_1 J_1(\rho_1 \nu) / \nu$ . Consequently, the zeros of the PSF arise when the two terms are identical. It can be seen that this happens for a  $v_0$  value which is lower than the first zero of the Airy pattern, thus yielding super-resolution. This  $v_0$ value and the corresponding transverse gain can be estimated by using the second-order approximation of the PSF core, and we shall see that they fit quite well with experimental data. b) Sidelobe excess,  $0.57 < \rho_1 < 2^{-1/2}$ . In such a radius range, the sidelobe height surpasses that of the core (see Fig.1). This range is consequently useful only in some special applications. c) Apodization:  $2^{-1/2} < \rho_1 < 1$ . In this region, the width of the PSF is greater than that of the unobstructed pupil. For this regime, both terms in Eq. 4 reach the same value for a  $v_0$  value greater than the first zero of the Airy pattern, thus yielding apodization. This range could even be divided into two subregions, one with negative and the other with positive gain. It can be readily derived from Eq. 6 that the transition between the two subregions occurs when  $\rho_1 = 2^{-1/4}$ . In the first subregion,  $2^{-1/2} < \rho_1 < 2^{-1/4}$ , the PSF is composed of a central plateau with a higher border, which corresponds to the sidelobes of the fields in Eq. 4, while in the second subregion,  $2^{-1/4} < \rho_1$ , the PSF recovers the central peak shape. Eq. 6 predicts a high negative transverse gain for the first subregion (Fig. 2), which is unrealistic. In order to develop a correct expression of the transverse gain, we perform a Gaussian approximation of the first lobe of the clear pupil  $LG(\nu)$  and of the second term in Eq. 4,  $-2\rho_1 J_1(\rho_1 \nu) / \nu$  (which accounts for the effect of the pupil of

radius  $\rho_1$ ), LG<sub>1</sub>( $\nu$ ):

$$LG(\nu) = -\left[LG_{M} \exp\left(\frac{-(\nu - \nu_{M})^{2}}{2\sigma^{2}}\right)\right]^{1/2}$$

$$LG_{1}(\nu) = -2\rho_{1}^{2}\left[LG_{M} \exp\left(\frac{-(\rho_{1}\nu - \nu_{M})^{2}}{2\sigma^{2}}\right)\right]^{1/2}$$
(7)

where  $v_{\rm M} = 1.635\lambda/D$  is the position of the first lobe maximum of the Airy pattern,  $LG_{\rm M} = 0.0175$  is the normalized intensity at this position and  $\sigma = 0.18$  accounts for the lobe width. Introducing these approximations into Eq. 4, the position of the first zero of U(v),  $v_0$ , is given by:

$$LG(\nu_0) - LG_1(\nu_0) = 0$$
 (8)

which yields:

$$\nu_0 = \frac{\nu_{\rm M}(1-\rho_1) + \sqrt{\nu_{\rm M}^2(1-\rho_1)^2 - 2(1-\rho_1)^2 \sigma^2(\ln 4 + 4 \ln \rho_1)}}{1-\rho_1^2}$$
(9)

From Eq. 9 the transverse gain can be readily obtained:

$$G_{\rm T} = \left(\frac{\nu_{\rm Airy}}{\nu_0}\right)^2 \tag{10}$$

where  $v_{\text{Airy}}$  is  $1.22\lambda/D$ , the first zero of the Airy pattern. This gain is shown in Fig. 2. It yields more realistic results than those of Eq. 6 (long-dashed line), which overestimates the apodization capability of pupil filters. This new expression is now contrasted with experimental data.

We checked the previous theory in aberrated images. For this task, a DAO system simultaneously introduced the apodizing or super-resolving PPF and compensated for aberrations. We implemented the first two levels of DAO: Binary Adaptive Optics (BAO) [15], which achieves a half-wave correction by adding a binary CPS to the distorted wavefront, yielding S = 0.4 at most, and quaternary adaptive optics (QAO) [16], which uses a four-level CPS to increase the Strehl ratio to S = 0.81. The set-up we used is shown in Fig. 3 and is based on the Thorlabs AOK2 kit. The light source is a laser diode (635 nm) which illuminates a deformable mirror with 32 active actuators (Thorlabs DM32-35-Ux01). The light after leaving the mirror reaches an SLM (Hamamatsu X8267-16) and is then sent to a Shack-Hartmann (SH) wavefront sensor (Thorlabs WS150-SC) and to a PDI wavefront sensor. The PDI consists of a 4-f system with a mask placed on the intermediate common focus plane. We used a transmittance mask consisting of a chrome thin film, of thickness approximately  $\lambda/10$  (61 nm), coated on a transparent glass substratum with a central hole of diameter 7  $\mu$ m. The thin chrome film creates a semitransparent region around the central hole. This mask affects both the amplitude and the phase of the incoming field. The refractive index of chrome for the 635 nm red laser light is around 3.3, which introduces a phase retardation of about  $0.44\pi$  with respect to the field that crosses the central hole. A pellicle beam splitter sends part of the energy to the camera SC to form the compensated image. We placed DM, SLM and sensors on planes conjugate to that of the pupil plane with the help of a series of 4-f systems built with pairs of achromatic lenses. The distortion introduced by the deformable mirror is simultaneously measured by the SH and the PDI wavefront sensors. The DAO compensation is based on the successive binarization of PDI interferograms. The first interferogram is binarized using as a threshold the local average intensity estimated over an area of 20 pixel radius. To perform

the second binarization, we repeated the same procedure but the average is now estimated over an area of 5 pixel radius. The experimental procedure and the technique's performance were previously analysed in [16].



**Fig. 3.** Experimental set-up. LASER: laser diode 635 nm. DM: deformable mirror, creates wavefront distortions. SLM: spatial light modulator for pupil modulation PPF+CPS. SC: scientific camera. S-H. Shack-Hartman wavefront sensor for wavefront checking. PDI: point diffraction interferometer, wavefront sensor to generate interferograms for CPS.

We combined a super-resolving or apodizing PPF with binary and quaternary CPSs and we analyzed the effect of the different combinations on the PSF. We applied these combinations to different aberrated wavefronts and we obtained similar results in all cases. As an example, we show the effect of the PPF on a wavefront affected by defocus. The peak-to-valley distance in this distorted wavefront was  $4\pi$ . Figs. 4(a) and (b) show some transversal profiles of the PSF obtained from the aberrated wavefront when the BAO and QAO CPS are applied, and when they are combined with a super-resolving PPF with different values of the radius  $\rho_1$ . We see in Fig. 4(a) that a clear PSF peak appears when we apply the BAO CPS (solid line) corresponding to S = 0.53. The peak height and width decrease, at the same time as the intensity of the sidelobes increases, as the radius of the PPF,  $\rho_1$ , increases (long, medium and short dashed lines). When we apply the QAO CPS, Fig. 4(b), we observe a similar behavior except for a higher *S* value (S = 0.84). We note that the QAO-compensated peak (solid line) reduces its height and width as the PPF radius increases, as shown by long, medium and short dashed lines respectively.

The same analysis was carried out for the combination of BAO and QAO CPS with apodizing PPFs. Various apodizing PPFs were added to the BAO CPS, Fig. 4(c), so that the peak intensity decreased its height and increased its width as  $\rho_1$  decreased (dashed lines). We observe that an apodizing PPF also causes a small deformation of the PSF. Similar behavior was found when a QAO CPS corrected the distorted wavefront, as shown in Fig. 4(d). In this case, the apodized curves were not as deformed as in the case of BAO compensation. We also observe that the effect of apodizing PPFs is really low, in particular for QAO compensation. We have to evaluate the effect of the PPF on the  $G_T$  value. Experimental G<sub>T</sub> values are plotted in Fig. 2 (circles and dots for BAO an QAO, respectively) along with the theoretical curves. Experimental values provide a good fit to the super-resolution branch given by Eq. 6. However, apodization values are much closer to the theoretical values corresponding to Eq. 10.

As a final comparison, we show in Fig. 5 the PSF corresponding to the aberrated wavefront (a), to the BAO compensation



**Fig. 4.** PSF transverse profiles for different super-resolving filters combined with BAO (a) and QAO (b) compensating filters. PSF transverse profiles for different apodizing filters combined with BAO (c) and QAO (d) compensating filters.

(b), to the BAO compensation combined with a super-resolving filter (c) and to BAO compensation combined with an apodizing filter (d). Because of the BAO compensation, the energy is concentrated and a clear PSF peak appears (Fig. 5(b)). When the BAO filter is combined with the corresponding super-resolving (Fig. 5(c)) or apodizing filter (Fig. 5(d)), the PSF peak intensity decreases and the width decreases or increases respectively. In Fig. 5(c), we also see that a sidelobe clearly appears because of the super-resolving filter. Fig. 6 shows the PSF corresponding to the aberrated wavefront (a), to the QAO compensation (b), to the QAO compensation combined with a super-resolving filter (c), and to QAO compensation combined with an apodizing filter (d). We see that the behavior is similar to that of BAO compensation although the PSF height is greater in this case.



**Fig. 5.** PSF corresponding to the aberrated wavefront (a), the BAO compensation (b), the BAO compensation combined with a superresolving filter (c) and the BAO compensation combined with an apodizing filter (d).

We have demonstrated the attainment of super-resolution or apodization simultaneously with aberration correction by DAO with no extra elements. In our experimental set up, the binary PPF that allows the PSF reshaping was implemented by using the SLM of the DAO system. The advantage of this combination



**Fig. 6.** PSF corresponding to the aberrated wavefront (a), the QAO compensation (b), the QAO compensation combined with a super-resolving filter (c), and the QAO compensation combined with an apodizing filter (d).

is its simplicity. We found that PSF Strehl ratio increases with compensation, and that reshaping filters are effective when they are combined with BAO or QAO compensating phase screens. Moreover, we have introduced a new theoretical approach to describe the transverse gain of apodizing filters. This approach provides theoretical values that fit much better with experimental results than those of the general theory. Finally, the features and simplicity of this technique make it ideal for obtaining improved images with adaptive optics.

Disclosures. The authors declare no conflicts of interest.

## REFERENCES

- R. Gordon-Soffer, L. E. Weiss, R. Eshel, B. Ferdman, E. Nehme, M. Bercovici, and Y. Shechtman, Sci. Adv. 6, eabc0332 (2020).
- Y. Fang, C. Kuang, Y. Ma, Y. Wang, and X. Liu, Front. Optoelectron. 8, 152–162 (2015).
- D. A. Guenther and W. F. Walker, IEEE Trans. on Ultrason. Ferroelectr. Freq. Control. 54, 343 (2007).
- 4. M. Zheng, Y. Ogura, and J. Tanida, Opt. Rev. 15, 105 (2008).
- 5. Y. Zha, J. Wei, and F. Gan, Opt. Commun. **304**, 49 (2013).
- 6. W. Yang and G. Fuxi, *Data storage at the nanoscale: Advances and applications* (Pan Stanford Publishing, 2015).
- 7. B. Xu, Z. Wang, and J. He, Sci. Reports 8, 15216 (2018).
- Z. Li, Q. Peng, B. Bhanu, Q. Zhang, and H. He, Astrophys. Space Sci. 363, 92 (2018).
- 9. G. T. Di Francia, Il Nuovo Cimento 9, 426 (1952).
- 10. Y. Zabar, E. Ribak, and S. Lipson, Micron 38, 176 (2007).
- C. J. R. Sheppard, G. Calvert, and M. Wheatland, J. Opt. Soc. Am. A 15, 849 (1998).
- D. M. de Juana, J. E. Oti, V. F. Canales, and M. P. Cagigal, Opt. Lett. 28, 607 (2003).
- M. Cagigal, J. Oti, V. Canales, and P. Valle, Opt. Commun. 241, 249 (2004).
- 14. V. F. Canales, D. M. de Juana, and M. P. Cagigal, Opt. Lett. **29**, 935 (2004).
- 15. G. D. Love, N. Andrews, P. Birch et al., Appl. Opt. 34, 6058 (1995).
- M. P. Cagigal, A. Fuentes, M. A. Cagigas, P. J. Valle, X. Prieto-Blanco, and V. F. Canales, Opt. Express 27, 24524 (2019).
- D. Spangenberg, A. Dudley, P. Neethling, E. Rohwer, and A. Forbes, Opt. Express 22, 13870 (2014).

## **FULL REFERENCES**

- R. Gordon-Soffer, L. E. Weiss, R. Eshel, B. Ferdman, E. Nehme, M. Bercovici, and Y. Shechtman, "Microscopic scan-free surface profiling over extended axial ranges by point-spread-function engineering," Sci. Adv. 6, eabc0332 (2020).
- Y. Fang, C. Kuang, Y. Ma, Y. Wang, and X. Liu, "Resolution and contrast enhancements of optical microscope based on point spread function engineering," Front. Optoelectron. 8, 152–162 (2015).
- D. A. Guenther and W. F. Walker, "Optimal apodization design for medical ultrasound using constrained least squares. part ii: Simulation results," IEEE Trans. on Ultrason. Ferroelectr. Freq. Control. 54, 343– 358 (2007).
- M. Zheng, Y. Ogura, and J. Tanida, "Three-dimensional dynamic optical manipulation by combining a diffractive optical element and a spatial light modulator," Opt. Rev. 15, 105–109 (2008).
- Y. Zha, J. Wei, and F. Gan, "A novel design for maskless direct laser writing nanolithography: Combination of diffractive optical element and nonlinear absorption inorganic resists," Opt. Commun. **304**, 49–53 (2013).
- 6. W. Yang and G. Fuxi, *Data storage at the nanoscale: Advances and applications* (Pan Stanford Publishing, 2015).
- B. Xu, Z. Wang, and J. He, "Super-resolution imaging via aperture modulation and intensity extrapolation," Sci. Reports 8, 15216 (2018).
- Z. Li, Q. Peng, B. Bhanu, Q. Zhang, and H. He, "Super resolution for astronomical observations," Astrophys. Space Sci. 363, 92 (2018).
- G. T. Di Francia, "Super-gain antennas and optical resolving power," Il Nuovo Cimento 9, 426–438 (1952).
- Y. Zabar, E. Ribak, and S. Lipson, "Measurement of the spatial response of a detector pixel," Micron 38, 176–179 (2007).
- C. J. R. Sheppard, G. Calvert, and M. Wheatland, "Focal distribution for superresolving toraldo filters," J. Opt. Soc. Am. A 15, 849–856 (1998).
- D. M. de Juana, J. E. Oti, V. F. Canales, and M. P. Cagigal, "Design of superresolving continuous phase filters," Opt. Lett. 28, 607–609 (2003).
- M. Cagigal, J. Oti, V. Canales, and P. Valle, "Analytical design of superresolving phase filters," Opt. Commun. 241, 249–253 (2004).
- V. F. Canales, D. M. de Juana, and M. P. Cagigal, "Superresolution in compensated telescopes," Opt. Lett. 29, 935–937 (2004).
- G. D. Love, N. Andrews, P. Birch *et al.*, "Binary adaptive optics: atmospheric wave-front correction with a half-wave phase shifter," Appl. Opt. 34, 6058–6066 (1995).
- M. P. Cagigal, A. Fuentes, M. A. Cagigas, P. J. Valle, X. Prieto-Blanco, and V. F. Canales, "Quaternary adaptive optics," Opt. Express 27, 24524–24537 (2019).
- D. Spangenberg, A. Dudley, P. Neethling, E. Rohwer, and A. Forbes, "White light wavefront control with a spatial light modulator," Opt. Express 22, 13870–13879 (2014).