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TECHNICAL NOTE

A relationship between tensile strength and mode I fracture toughness of rocks using the critical distance

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Abstract Correlations between material parameters are useful because they provide a first estimation of unknown parameters. Here, the correlation between the tensile strength and the mode I fracture toughness of rocks is studied. Some researchers have proposed empirical correlations based on a certain amount of empirical data and a fitting process. On the other hand, a few researchers have considered that the proportionality coefficient could be related to a rock property with units of length. Here, a linear relationship without an intercept at the origin between the tensile strength and the mode I fracture toughness of rocks is theoretically confirmed using the theory of critical distances (TCD). A comprehensive experimental database is presented and comparisons with this data from the literature (including tests at different temperatures) confirm the linear relationship and values of the critical distance of several millimeters (e.g., 3–15 mm). However, the scatter is large because there are different sources of uncertainty in the correlation, such as the testing method. Finally, the physical meaning of the critical distance is explored and its linear correlation with the grain size using the experimental database.

Article Highlights

- The linear relationship between the tensile strength and the mode I fracture toughness of rocks is theoretically confirmed using the TCD.
- The slope of the linear relationship depends on the critical distance, whose value is around 3–15 mm.
- Comparisons with experimental data from the literature confirm the linear relationship.
- There is some scatter in the comparison due to the different testing methods, sample geometries and rock types.

Keywords Fracture toughness · Tensile strength · Critical distance · The theory of critical distances · Grain size

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1 Introduction

Establishing correlations between material parameters is useful because they may help to provide a first rough estimation in case of a lack of available information. In most cases, these correlations are based only on a certain amount of empirical data and a fitting process. Therefore, the quality of the correlation depends on the suitability of the fitting equation and the breadth and heterogeneity of the data.

Some researchers have proposed empirical correlations between the tensile strength, σ_t, and the mode I fracture toughness, K_{IC} for rocks and soils. Most of them propose a linear relationship between them (Table 1). It is worth noting that the constant of proportionality has units of square root of length and the correlations are usually given assuming the following units: σ_t [MPa] and K_{IC} [MPa$\cdot\sqrt{\text{m}}$]. Zhang (2002) is probably the most well-known correlation because it is based on extensive experimental data. Xu et al. (2018) show that the coefficient of determination (i.e. the squared value of the Pearson coefficient of correlation) of Zhang (2002) is in fact $R^2=0.79$, instead of $R^2=0.94$ as stated in the original paper. Better coefficients of correlations are found when only one specific type of material is considered (e.g., Bhagat 1985; Harison et al. 1994). Xu et al. (2018) classify their data using the three main rock types, namely sedimentary, igneous and metamorphic, but they do not obtain good results and propose further study.

Other authors have proposed a linear relationship but with an intercept at the origin (e.g., Barry et al. 1992; Whittaker et al. 1992; Gunsallus and Kulhawy 1984; Bhagat 1985; Roy et al. 2017). Some authors have also considered power-law (e.g., Zhang et al. 1998) or exponential (e.g., Talukdar et al. 2018) relationships. Similar relationships have been presented for other geomaterials, such as for concrete (e.g., linear and parabolic relationships, Karimi et al. 2023).

More fundamental approaches show that the linear relationship is theoretically based and the proportionality coefficient could be related to a rock property with units of length, such as the crack propagation radius (Feng et al. 2019), the characteristic crack length (Wang and Hu 2017), the fracture process zone (FPZ) (Hu et al. 2022) or the grain size (Potyondy and Cundall 2004; Zhang et al. 2018). Despite these developments, there is still a wide variety of relationships used in the recent literature (e.g., Shi et al. 2022) and not a clear understanding of how this relationship depends on the rock type.

Table 1 Correlations between the tensile strength, σ_t [MPa], and the mode I fracture toughness, K_{IC} [MPa$\sqrt{\text{m}}$]

Reference	Correlation	Comments
Backers (2004)	$K_{IC}=0.25\cdot\sigma_t$	Six rocks
Barry et al. (1992), Whittaker et al. (1992)	$K_{IC}=0.107\cdot\sigma_t+0.27$	Brittle rocks
Bhagat (1985)	$K_{IC}=0.28\cdot\sigma_t+4.87$	Coal
Feng et al. (2019)	$K_{IC}=\sqrt{2\pi r_{IC}}\cdot\sigma_t$	r_{IC}: critical crack propagation radius
Gunsallus and Kulhawy (1984)	$K_{IC}=0.0736\cdot\sigma_t+0.76$	Sedimentary rocks
Haberfield and Johnston (1989)	$K_{IC}=0.0761\cdot\sigma_t$	Eight rocks
Harison et al. (1994)	$K_{IC}=0.0706\cdot\sigma_t$	Compacted cohesive soil
Muñoz-Ibañez et al. (2020)	$K_{IC}=0.11\cdot\sigma_t$	Four rocks
Potyondy and Cundall (2004)	$K_{IC}=\sqrt{\pi\alpha R}\cdot\sigma_t$	R: particle radius; α: packing factor (≥ 1)
Roy et al. (2017)	$K_{IC}=0.11\cdot\sigma_t+0.23$	Crystalline and sedimentary rocks
Talukdar et al. (2018)	$K_{IC}/K_{IC0}=0.25\cdot e^{1.12\frac{\sigma_t}{\sigma_{t0}}}$	Tonalite
This research (TCD)	$K_{IC}=\sqrt{\pi L}\cdot\sigma_t$	L: critical distance
Wang and Hu (2017)	$K_{IC}=\sqrt{4a^*_\infty}\cdot\sigma_t$	a^*_∞: material constant (charact. crack length)
Wang et al. (2007)	$K_{IC}=0.3546\cdot\sigma_t$	Compacted gravelly clay
Zhang et al. (1998)	$K_{IC}=0.0295\cdot\sigma_t^{1.61}$	Several rock types
Zhang (2002)	$K_{IC}=0.1453\cdot\sigma_t$	Large database
Zhang et al. (2018)	$K_{IC}=2\sqrt{3G}\cdot\sigma_t$	G: Grain size. Large database

Here, the linear relationship is theoretically confirmed using the Theory of Critical Distances (TCD) (Sect. 2), which is a successful fracture mechanics theory based on a material characteristic parameter called the critical distance (L) (e.g., Taylor 2007), and comparison with experimental data is presented (Sect. 3). The results show the linear relationship between the tensile strength and the mode I fracture toughness with a constant of proportionality that depends on a material property called the critical distance, which is somehow related to the size of the FPZ and, in some cases, to the grain size. Experimental data for tests at different temperatures are also analysed. Difficulties in the validation of the theoretical relationship arise due to the scatter of the experimental data (Sect. 4). Next, the physical meaning of the critical distance is analysed, and in particular the empirical correlation between the critical distance and the average grain size (Sect. 5). Finally, some conclusions are drawn.

2 Theoretical relationship

The TCD is a group of fracture mechanics methodologies with some common features: the use of linear elastic analyses when performing fracture assessments and the use of the critical distance as a material characteristic parameter. Neuber (1958) and Peterson (1959) were the first to use these concepts, but the TCD has been recently scientifically analysed in more detail (e.g., Taylor 2007) and it has gained popularity with the development of finite element stress analyses. The expression for the critical distance L is as follows (e.g., Taylor 2007):

$$L = \frac{1}{\pi}\left(\frac{K_{IC}}{\sigma_0}\right)^2 \tag{1}$$

where σ_0 is a characteristic material strength parameter, usually called inherent strength.

Equation (1) is easily obtained considering the stress distribution along the crack propagation direction under mode I around the crack tip using linear elastic fracture mechanics:

$$\sigma = \frac{K_I}{\sqrt{2\pi r}} \tag{2}$$

where σ is the principal tensile stress (perpendicular to the crack propagation direction) and r is the radial distance from the crack tip.

In the case of rocks or other quasi-brittle materials, the inherent strength (σ_0) can be assumed to roughly coincide with the tensile strength (σ_t) of the material (e.g., Taylor 2007; Justo et al. 2017). Consequently, replacing σ_0 by σ_t in Eq. (1) and rearranging terms, the linear relationship between σ_t and K_{IC} is found:

$$K_{IC} = \sqrt{\pi L} \cdot \sigma_t \tag{3}$$

An equivalent relationship was found by Feng et al. (2019) using the modified maximum tangential stress (MMTS) criterion (Smith et al. 2001), which is, in turn, equivalent to one of the methodologies of the TCD, namely the point method (e.g., Taylor 2007).

3 Comparison with empirical data from the literature

The experimental database used by Zhang (2002) has been extended to include the results gathered by Ameen et al. (2023), Backers (2004), Bearman (1999), Chandler et al. (2016), Iqbal and Mohanty (2007), Muñoz-Ibañez et al. (2020) and Pakdaman et al. (2019) (please, refer to Supplementary Information for the spreadsheet with this information). The relationship between σ_t and K_{IC} for those experimental data is compared in Fig. 1 with the linear relationship provided by Eq. (3). Despite the large

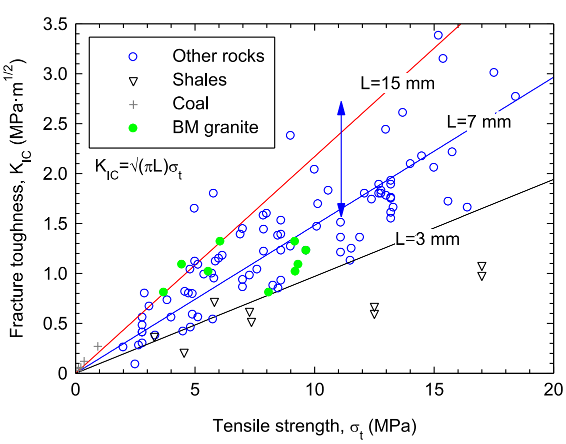

Fig. 1 Relationship between σ_t and K_{IC} (empirical data gathered from the literature)

scatter of the experimental results, which may be justified by the fact that they include many different types of rocks and different tests to obtain those two parameters (please, refer to next section on sources of uncertainty), most of the experimental values correspond to critical distances of several millimeters (e.g., 3–15 mm), in agreement with the existing information (e.g., Taylor 2007; Justo et al. 2017). Some types of rocks seem to be in the upper and lower bounds, for instance, the correlation for coals gives high critical distances (larger than 20 mm), while for shales, it usually gives low values (lower than 3 mm).

Since the correlation depends on the specific value of the critical distance, the linear relationship can only be confirmed for those cases with the same critical distance. On the other hand, both σ_t and K_{IC} vary with temperature, but L does not change (Justo 2020) until a certain limit temperature, at which microstructural changes start to develop. Therefore, tests on rock samples at different temperatures are a good benchmark for validation of the linear relationship proposed in Eq. (3). Table 2 gathers some experimental data of rocks tested at different temperatures from the literature. Those data show a good linear correlation between σ_t and K_{IC} (Fig. 2). For Fangshan gabbro only the values up to 400 °C have been plotted because that is considered to be the limit temperature beyond which L varies due to microstructural

Table 2 Experimental information of rocks at different temperatures for validation purposes

Rock	Temp (°C)	σ_t(MPa)	K_{IC}(MPa$\sqrt{m}$)	L(mm)*	References
Floresta sandstone	23	2.84 ± 0.42	0.37 ± 0.06	5.4	Justo (2020)
Floresta sandstone	70	3.25 ± 0.30	0.43 ± 0.07	5.6	Justo (2020)
Floresta sandstone	150	3.17 ± 0.59	0.46 ± 0.08	6.7	Justo (2020)
Floresta sandstone	250	3.55 ± 0.58	0.45 ± 0.08	5.1	Justo (2020)
Moleanos limestone	23	6.86 ± 1.08	0.73 ± 0.11	3.6	Justo (2020)
Moleanos limestone	70	8.18 ± 0.53	0.96 ± 0.06	4.4	Justo (2020)
Moleanos limestone	150	9.62 ± 0.87	0.95 ± 0.11	3.1	Justo (2020)
Moleanos limestone	250	8.88 ± 0.50	1.07 ± 0.17	4.6	Justo (2020)
Macael marble	23	9.97 ± 0.33	1.14 ± 0.13	4.2	Justo (2020)
Macael marble	70	7.52 ± 0.67	1.16 ± 0.24	7.6	Justo (2020)
Macael marble	150	4.92 ± 0.41	0.57 ± 0.07	4.3	Justo (2020)
Macael marble	250	4.50 ± 0.55	0.72 ± 0.12	8.1	Justo (2020)
Carrara marble	23	9.16 ± 0.71	0.74 ± 0.13	2.1	Justo (2020)
Carrara marble	70	7.26 ± 0.61	0.75 ± 0.14	3.4	Justo (2020)
Carrara marble	150	5.67 ± 0.77	0.62 ± 0.08	3.8	Justo (2020)
Carrara marble	250	5.00 ± 0.60	0.50 ± 0.17	3.2	Justo (2020)
Sichuan sandstone	20	8.20	1.11	5.8	Feng et al. (2019)
Sichuan sandstone	100	8.15	1.13	6.1	Feng et al. (2019)
Sichuan sandstone	200	7.18	0.99	6.1	Feng et al. (2019)
Sichuan sandstone	300	7.25	0.93	5.2	Feng et al. (2019)
Sichuan sandstone	400	6.93	0.84	4.7	Feng et al. (2019)
Sichuan sandstone	500	6.62	0.81	4.8	Feng et al. (2019)
Sichuan sandstone	600	5.10	0.64	5.0	Feng et al. (2019)
Fangshan gabbro 1	20	17.3	2.68	7.6	Zhang et al. (1998)
Fangshan gabbro 1	100	15.4	2.26	6.9	Zhang et al. (1998)
Fangshan gabbro 1	200	13.9	2.02	6.7	Zhang et al. (1998)
Fangshan gabbro 1	300	12.1	1.70	6.3	Zhang et al. (1998)
Fangshan gabbro 1	400	10.0	1.44	6.6	Zhang et al. (1998)
Fangshan gabbro 1	500	9.9	1.28	5.3	Zhang et al. (1998)
Fangshan gabbro 1	600	9.3	0.98	3.5	Zhang et al. (1998)

*Calculated using Eq. (3)

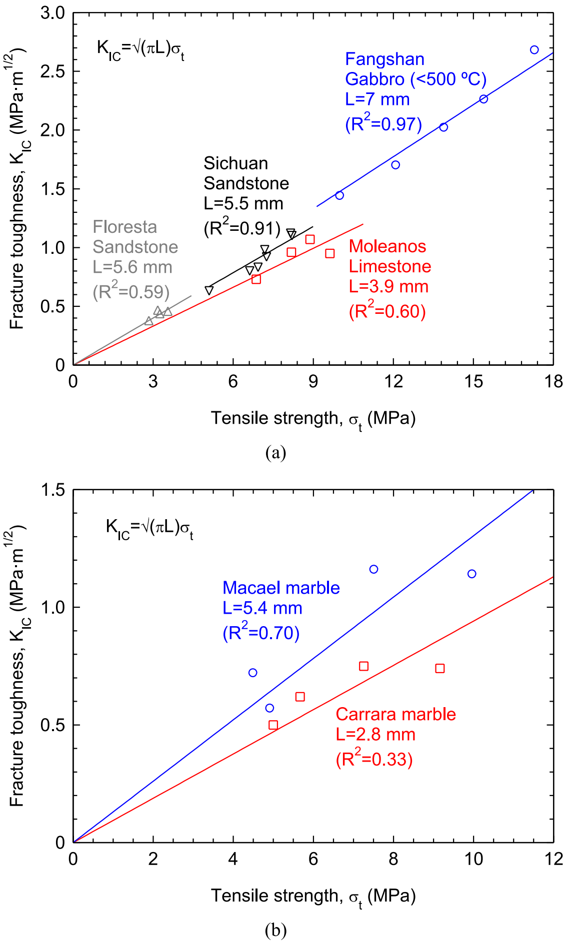

Fig. 2 Linear relationship between σ_t and K_{IC} for specific rocks at different temperatures (empirical data gathered from the literature): **a** Sandstones, limestone and gabbro; **b** Marbles

changes. For Floresta sandstone, Moleanos limestone and Macael marble (Fig. 2), the Pearson coefficient of correlation (R^2) is not high because there are only 4 data for a limited range of temperatures (23–250 °C) and the scatter of the experimental results due to the inherent variability of the rock is high, see for example, the standard deviations in Table 2, which are around 15% of the mean value. These reasons also apply to Carrara marble (Fig. 2b), but additionally, the TCD does not provide good results for this rock (Justo et al. 2020a) because of its ductility and cohesive cracking, which justifies the poor correlation.

Other possible appropriate sets of experiments to benchmark the linear relationship in sedimentary rocks are those with different inclination angles between the loading axis and the bedding planes, such as those presented by Shi et al. (2022) for Shizhu shale (Fig. 3). The results show that the bedding planes influence both σ_t and K_{IC}, as expected, but not its relationship, which depends on the rock microstructure. Shi et al. (2022) obtained really high coefficients of correlation using an exponential function and a linear fit with a negative intercept at the origin, but in this case, the coefficient of correlation is still high, namely $R^2 = 0.96$, using only one free parameter instead of two and using a theoretically based expression.

Similar results are also found in other geomaterials, such as in concrete. Figure 4 presents results of polymer concretes (Karamzadeh et al. 2022) and

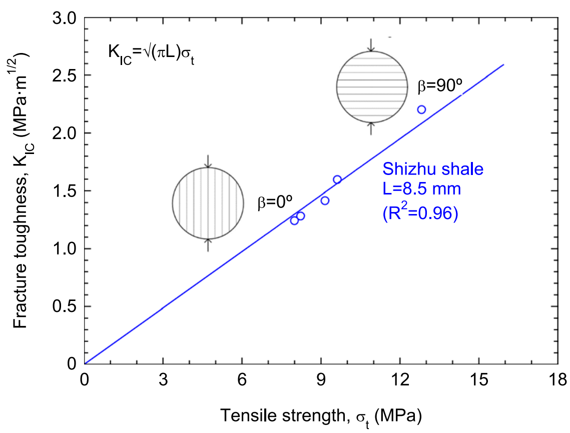

Fig. 3 Linear relationship between σ_t and K_{IC} for Shizu shale

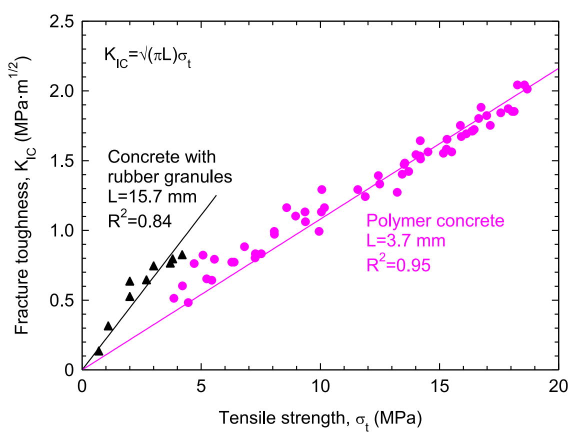

Fig. 4 Linear relationship between σ_t and K_{IC} for some concretes

concretes with rubber granules (Karimi et al. 2023). In the polymer concretes, the content of resin, fine and coarse aggregates and glass fibers is varied; then, the tensile strength and the fracture toughness vary, but its relationship keeps a linear trend. Analogously, the content of rubber granules and their size is varied for the other concretes. In these cases, the critical distance is also of the order of several mm.

4 Sources of uncertainty

As well documented in the literature, the sources of uncertainty of the proposed correlation may be the rock variability, the testing method and the sample geometry. For example, Aliha et al. (2012, 2018), Iqbal and Mohanty (2007), Ouchterlony (1990) and Pakdaman et al. (2019) have studied the influence of different tests to obtain K_{IC}, Coviello et al. (2005), Perras and Diederichs (2014) and Shams et al. (2023) have considered different tests to measure σ_t and Akbardoost et al. (2014), Muñoz-Ibáñez et al. (2020), Pérez-Rey et al. (2023) and Zhang et al. (2021) have shown the influence of the sample size in these tests. Tutluoglu et al. (2022) have shown the influence of the loading span when measuring K_{IC} using disc and semi-disc bend geometries. Besides, when measuring K_{IC}, the notch length, width and tip shape may influence the results (e.g., Justo 2020; Muñoz-Ibáñez et al. 2020). The TCD or analogous concepts, such as the modified maximum tangential stress criterion, may help to improve the test interpretation, account for the notch tip shape or correlate results between different tests (e.g., Aliha et al. 2012, 2018; Tutluoglu et al. 2022).

As a result, correlations in Figs. 2 and 3 are much narrower than in Fig. 1 because they consider the same rock, the same tests and the same sample geometry, i.e. only rock variability affects the results. For example, a vertical arrow in Fig. 1 represents the results for a gabbro obtained using different tests to get K_{IC} (Pakdaman et al. 2019). It may be observed that the results spread along half of the common range. Similarly, Pérez-Rey et al. (2023) have used different tests and sample sizes to measure σ_t and K_{IC} of Blanco Mera granite (BM granite). The results are differentiated in Fig. 1 to show their spread. A more detailed analysis reveals that the sample size changes the values but not notably their ratio and that the main differences are between the direct tensile strength and that obtained using the Brazilian test.

In the next section the physical meaning of the critical distance is explored and its correlation with the crystal or grain size. It is also worth noting that measuring the grain size distribution is a complex process (e.g., Safari et al. 2021) and there may also exist some variability in the reported values in the literature. Besides, the mean grain size will be here considered, but obviously, the grain size distribution (Justo et al. 2022) or its maximum value also influence the rock microstructure.

5 Physical meaning of the critical distance

The critical distance somehow represents the size of the dominant source of microstructural heterogeneity in the material (e.g., Askes and Susmel 2015). For example, Taylor (2017) conceptually considered different sources of heterogeneity, namely periodic barriers to crack growth, weak points and reinforcing fibers, and correlated their spacing with the critical distance using thought experiments. An important source of microstructural heterogeneity is the crystal or grain size and broad correlations have been found between the critical distance and the grain size for different materials based on experimental data (Taylor 2017). In rocks, there is also an approximate empirical correlation of 10–1, the critical distance being of the order of several millimeters and the grain size of the order of several tenths of millimeters (e.g., Justo et al. 2017). The correlation between the critical distance and the grain size have also been shown numerically using discrete numerical analyses (e.g., Potyondy and Cundall 2004; Justo et al. 2020b, 2022), which seems logic because in these numerical models, the grain or crystal size is usually the only source of microstructural heterogeneity.

On the other hand, the relationship proposed by Zhang et al. (2018) (Table 1) may be combined with the relationship proposed here (Eq. 3) to obtain a correlation between the critical distance (L) and the mean grain size (G):

$$L = 12/\pi \cdot G \tag{4}$$

However, this is a general correlation for average values (e.g., mean grain size) and encompassing different types of rocks and laboratory tests.

The generated database (please, refer to the Supplementary Information) has been used here to compare the critical distance and the mean grain size of different rocks (Fig. 5). In this case, there are important uncertainties both in the grain size and in the testing methods used to obtain the critical distance (as mentioned in the previous section). The scatter of the results does not allow for refine analyses, but just as general and logical comments:

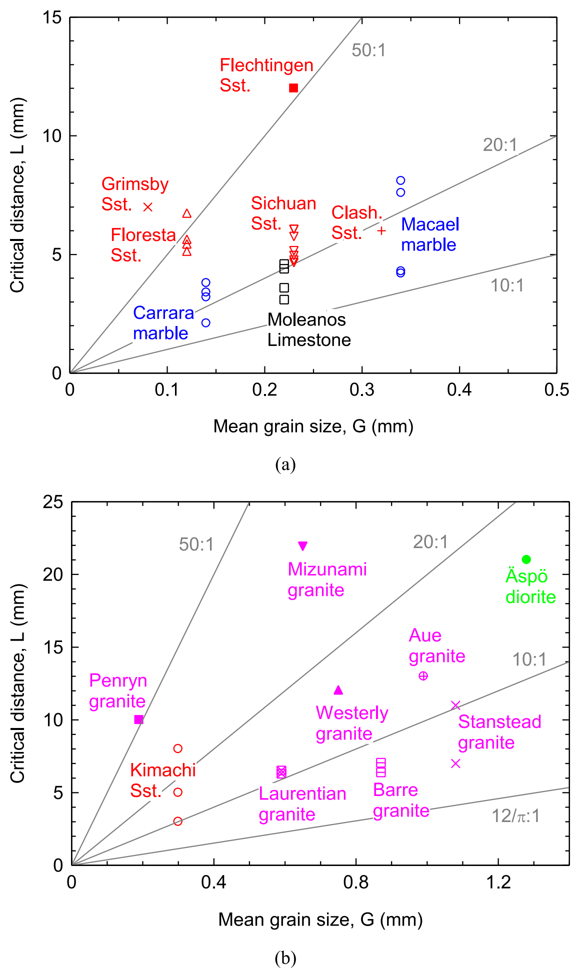

Fig. 5 Relationship between mean grain size and critical distance

- The ratio between the critical distance and the mean grain size in rocks is usually between 50 and 5.
- The correlation in Eq. (4) is clearly in the low range (Fig. 5b).
- In low porosity monocrystalline rocks, the correlation should exist.
- In polycrystalline rocks, a non-uniform distribution of the minerals influences the rock microstructure, and consequently the critical distance.
- In porous rocks, the distance between pores may be the dominant source of heterogeneity (e.g., Justo 2020) and the correlation of the critical distance should be with the average distance between pores.
- In rocks with very small mean grain sizes (e.g., $G < 100$ µm), G does not seem to be the dominant source of heterogeneity, and the critical distance is likely correlated with the distance between other defects or larger grains of less abundant minerals.

6 Conclusions

A linear relationship between the tensile strength and the mode I fracture toughness of rocks is presented using the TCD. Comparisons with experimental data from the literature confirm the linear relationship (particularly if only one specific type of rock, the same testing methods and the same sample geometries are considered). For example, values for the same rock at different temperatures, or with different inclination angles between the loading axis and the bedding planes, show variations in both σ_t and K_{IC}, but not in their ratio. The ratio between σ_t and K_{IC} depends on the rock microstructure. Using the TCD, the ratio may be correlated with the critical distance, which is of the order of several millimeters (e.g., 3–15 mm). The linear relationship may not hold for some types of rocks that do not show a quasi-brittle behaviour and for which the TCD does not provide satisfactory results (e.g., Carrara marble).

Finally, the critical distance depends on microstructural features, and for some rocks, it is correlated with the grain size. Here, the generated database is used to compare the critical distance and the mean grain size of different rocks. Although there is an important scatter because of the uncertainties in both the grain size and the testing methods, the critical distance is between 5 and 50 times the mean grain size in rocks. However, this correlation may be more complex (e.g., it may have to consider the grain size distribution) and for some types of rocks, the grain size may not be the most relevant microstructural feature (e.g., it may be the distance between large voids in porous rocks) and then, the critical distance is correlated with those other microstructural features.

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Data availability All data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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