New approach to study the dynamic performance of worm gear drive model

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Abstract: worm gear sets are used in many applications of the transmission field. These mechanisms have interesting advantages. Despite this, its modelization is still challenging. This is due to its complex geometry. The goal behind this is to have an accurate model that allows improvement of the design of worm gear sets. In this context, the paper presents a new mathematical model of a worm gear drive under rotational external excitations. Dynamic model is defined by fourteen degrees of freedom describing all rotations and translations of worm gear set, bearings, a motor and a receiver connected together. First, equations of motion are developed to describe the dynamic behavior. Lagrange formula is thus employed. Elastic deformations of meshed teeth are considered. The normal load associated with the meshing is established as function of the model degrees of freedom through the elastic deflection. Second, a numerical simulation is carried out. Newmark Beta method is used to solve equations. Numerical results are presented to discuss the model accuracy. The impact of the variation of friction coefficient and the stiffness of the worm gear is finally studied to discuss the consistency of the new formulation developed through this work.

Keywords: worm gear, model, dynamic, friction, normal load, meshing, stiffness, deflection

1. Introduction

In many applications of transmission power, worm gear drive is used for its interesting advantages. Its high transmission ratio and reduced volume are its most important assets. However, its complex geometry presents a challenging issue to deal with. Despite this, scientists were interested by this mechanical device and tried to enhance the advanced knowledge about it. Most researches on worm gear sets were focused on the design [1], the load/stress distribution [2], the wear behavior [3], the efficiency [4]-[5] or thermal analysis [6]. In the dynamic field, there is limited number of works done on worm gear. Most of research has been done on less complex sets such as spur gears. From simple to multiple stages of spur gears were studied. For simple category, one can mention the work of Tamminana et al. [7] who developed and validated both finite elements and discrete models to predict a spur gear dynamic. Wang et al. [8] proposed a three degrees of freedom (DOF) model for typical railway locomotive. Numerical study was then carried to detect the effect of some parameters on the dynamic behavior of the transmission set. Park et al. [9] investigated the dynamic characteristics of spur gears through a four DOF model and by introducing the time varying mesh stiffness and bearings stiffness. For more complex configurations considering multiple stages of spur gears, the work of Fakhfakh et al. [10] can be mentioned in which a twelve DOF model is used to study the dynamic behavior of healthy and defective system. Abboudi et al. [11] based their work on the previous one to study the dynamic of gear boxes with introduction of bearings and shafts inertia in the wind turbine application.

For worm gears sets, many assumptions were done concerning the modelization in order to investigate the dynamic behavior. Chung et al. [12] studied a model of two DOF. Only axial rotations of gears were considered, and no bearings were introduced. A comparison with experimental measurements was then done to illustrate a relatively correct model. Hammami et al. [13] investigated the dynamic behavior of a two DOF model with plastic material. Authors studied the evolution of modal frequencies by the variation of the plastic material characteristics. Stolyarchuk [14] presented a dynamic analysis of a worm gear drive through focusing on the sliding friction contact area. A mathematical formulation was developed considering the worm gear as a translational multibody system including two-dimensional logic. Benabid et al. [15] carried out their work on a eight DOF model to make diagnostics with vibration analysis. This number of DOF is chosen to combine modelling simplicity and dynamic accuracy. Fang et al. [16] developed a mathematical model of the ZN-type worm gear. Their study was based on cutting mechanism and parameters of the cutter tool. The most recent work done in this field is the work of Chakroun et al [17] who considered a fourteen DOF model and used modal analysis to study the dynamic behavior.

In this paper, a new approach is presented to model also a fourteen DOF mechanical system but by focusing in addition on local aspects. Considering the contact forces and teeth elastic deformations in meshing area will overcome the limitations of existing models. This developed model is thus more accurate and practically as fast as the others. It does not exclude any intervention coming from a DOF considered negligible in the other works. Added to this, more in-depth studies can be established in relation with the meshed area as contact forces and teeth deflections, that condition the global and the local performances, are taken into consideration. The general performance covers the dynamic behavior and the efficiency. The local one includes the stress distribution and the wear behavior...A new formulation relating contact forces and teeth deflections is developed here to include all DOF participation. The goal behind this approach is to make an accurate model of a worm gear drive to investigate its dynamic performance.

For this purpose, in this work the dynamic model is first under investigation. A mathematical approach is then developed to establish equations of motion. This is done by using Lagrange formula. Relation between elastic deflection of meshing teeth and normal load associated with the motion functionality is developed. For this, the geometries of worm and worm gear are taken into consideration. Second, a numerical simulation is carried out. The Newmark beta method is used to solve the fourteen equations of motion.

In real application, the dynamic of this model is affected by several factors. The motion between components in a worm gear drive is characterized by an important sliding. The study of the effect of friction coefficient seems to be judicious here. The impact of the gear stiffness is also studied by analyzing the dynamic behavior of different metallic materials employed in worm gear. The coherence of results allows to discuss the consistency of the new formulation developed in this manuscript.

2. Dynamic model

Worm gears can be found in many types of mechanical systems. In this study, the model under investigation is composed of worm gear drive, motor, receiver, transmission shafts and bearings. This type of configuration is widely found in the electro-mechanical applications. The gear meshing and bearings are modeled by linear springs. Two assumptions are taken here to make compromise between simplicity and reliability of the model. Error on tooth profile and clearance between teeth are not taken into consideration. And the shafts dynamic behavior is supposed negligible. Only friction damping is studied in this work.

The model is composed of two blocks (Fig. 1). The worm (w), the transmission shaft, the bearing (Bearing 1), and the motor make the first block. The second one consists of worm gear (g), transmission shaft, bearing (Bearing 2), and receiver.

The model presents fourteen DOF. Seven DOF for each block: three defining translations: $(x_i, y_i, z_i)_{i=1,2}$ respectively along axes $(\vec{X}, \vec{Y}, \vec{Z})$ and four for rotations: $(\theta_{jx}, \theta_{jy}, \theta_{jy})_{j=w,g}$ respectively around axes $(\vec{X}, \vec{Y}, \vec{Z})$ of worm, worm gear and bearings and $(\theta_k)_{k=R,M}$ of the motor/receiver.



Fig. 1: Dynamic model

When worm threads and worm gear teeth are engaged, loads are transmitted: the normal load F_n and the tangential one F_f representing frictional actions (Fig. 2). This frictional load is supposed to be proportional to normal one and is written as $F_f = \mu F_n$. These two loads vary by time along the action line of the meshing area. Normal force is composed of three orthogonal forces [18]:

$$F_x = F_n \cos\phi \sin\lambda + F_f \cos\lambda \tag{1}$$

$$F_{y} = F_{n} \cos\phi \cos\lambda - F_{f} \sin\lambda \tag{2}$$

$$F_z = F_n sin\phi \tag{3}$$

where, λ and ϕ are the lead and the pressure angles respectively.



Fig. 2 worm gear force components

Using the Lagrange formalism, the equations of motions of the model are written as follows:

$$m_1 \ddot{x}_1 + k_{x1} x_1 = F_x \tag{4}$$

$$m_1 \ddot{y}_1 + k_{y1} y_1 = F_y \tag{5}$$

(6)

(10)

(13)

(17)

 $m_1 \ddot{z}_1 + k_{z1} z_1 = F_z$

$$I_{wx}\ddot{\theta}_{wx} + k_{\theta wx}\theta_{wx} = F_z R_w \tag{7}$$

$$I_{wy}\ddot{\theta}_{wy} + k_{\theta wy}(\theta_{wy} - \theta_M) = F_x R_w$$
(8)

 $I_M \ddot{\theta}_M + k_{\theta wy} (\theta_M - \theta_{wy}) = \tau_M$

$$I_{wz}\ddot{\theta}_{wz} + k_{\theta wz}\theta_{wz} = -F_x R_w \tag{9}$$

Motor

$$m_2 \ddot{x}_2 + k_{x2} x_2 = -F_x \tag{11}$$

$$m_2 \ddot{y}_2 + k_{y2} y_2 = -F_y$$
(12)
$$m_2 \ddot{z}_2 + k_{z2} z_2 = -F_z$$
(13)

Worm gear

$$I_{gx}\ddot{\theta}_{gx} + K_{\theta gx}(\theta_{gx} - \theta_R) = -F_y R_g$$
(14)

$$I_{gy}\ddot{\theta}_{gy} + k_{\theta gy}\theta_{gy} = -(F_x + F_z)R_g \tag{15}$$

$$I_{gz}\ddot{\theta}_{gz} + k_{\theta gz}\theta_{gz} = -F_x R_g \tag{16}$$

Receiver

Where, $(k_{xi}, k_{yi}, k_{zi})_{i=1,2}$ are the axial and radial stiffness, of worm gear set compounds, along global reference axes and $(k_{\theta jx}, k_{\theta jy}, k_{\theta jz})$ are the torsional stiffness around axes. m_1 and m_2 represent the weights of the first and second block respectively. $(I_{jx}, I_{jy}, I_{jz})_{j=w,g}$ are the inertial moments of worm and worm gear. (I_M, τ_M) and (I_R, τ_R) are respectively the inertia moment and the torque of motor and receiver. $(R_j)_{j=w,q}$ is the pitch radius. To highlight the elastic deformations of meshed teeth, the above equations of motion must be rewritten under a specific way detailed as follows below. First, the normal load must be expressed as function of the worm gear set DOFs. Normal load is related to deflections $(\delta_j)_{i=w,a}$, generated by teeth elastic deformations in the engaged worm and worm gear by:

 $I_R \ddot{\theta}_R + k_{\theta ax} (\theta_R - \theta_{ax}) = \tau_R$

$$\delta_w(t) = \frac{F_n(t)}{k_w(t)} \tag{18}$$

$$\delta_g(t) = \frac{F_n(t)}{k_g(t)} \tag{19}$$

 $k_w(t)$ and $k_g(t)$ are the meshed tooth spring constant respectively in worm and worm gear. To make next equations easier to read, the dependence on time of quantities will not be written. The assumption of equations (18) and (19) gives:

$$\delta_w + \delta_g = \frac{F_n}{k_{wg}} \tag{20}$$

Worm

Each meshed teeth pair is modelled by two springs $(k_j)_{j=w,g}$ joined in series. The total mesh stiffness k_{wg} , varying periodically in time and by the meshing gear pair number, is:

$$k_{wg} = \frac{k_g k_w}{k_g + k_w} \tag{21}$$

Gears can operate normally by maintaining contact of deformed teeth in both worm and worm gear. The total displacement must be the same for each teeth pair in contact simultaneously. This displacement is the sum of the rotation movement of the worm/worm gear, the bearing excitations, and deformations of the elastic teeth. To determine the relation between normal load and deflections in the model, displacements are projected on \vec{X} , \vec{Y} and \vec{Z} axes. Then we sum the three equations and extract the deflections sum $(\delta_w + \delta_g)$. For the displacement projected on the \vec{Y} axis: when the worm rotates by an angle of θ_{wy} the worm teeth move a distance of $l_w \theta_{wy}$. Bearing 1 translates worm a distance of $\delta_{bw|y}$. It also stimulates two rotational movements around \vec{X} and \vec{Z} axes. These rotations generate additional displacements $\delta_{bw|Rx}$ and $\delta_{bw|Rz}$ along the axial direction (Fig. 3). Deflection of tooth is normal to its direction and thus is equal to $\delta_w \sin(\frac{\pi}{2} - \lambda)$. On the other side, worm gear rotates from 0 to θ_{gx} . Its teeth move a distance of $R_g \theta_{gx}$ along \vec{Y} direction. Bearing 2 adds complementary movements. $\delta_{bg|y}$ is the deformation due to the translation. $\delta_{bg|Ry}$ and $\delta_{bg|Rz}$ are deformations generated by the bearing due to rotations around \vec{X} and \vec{Z} axes. The theory of the same displacement amplitude is thus written as:

$$l_{w}\theta_{wy} - \delta_{w}\sin\left(\frac{\pi}{2} - \lambda\right) - \delta_{bw|y} - \delta_{bw|Rx} - \delta_{bw|Rz}$$

$$= R_{g}\theta_{gx} + \delta_{g}\sin\left(\frac{\pi}{2} - \lambda\right) + \delta_{bg|y} + \delta_{bg|Ry} + \delta_{bg|Rz}$$
(22)

Rearranging equation (22), on can obtain:

$$\left(\delta_w + \delta_g\right)\sin\left(\frac{\pi}{2} - \lambda\right) = l_w\theta_{wy} - R_g\theta_{gx} - \delta_{bw|y} - \delta_{bw|Rx} - \delta_{bw|Rz} - \delta_{bg|y} - \delta_{bg|Rz} - \delta_{bg|Ry}$$
(23)



Fig. 3 Deformations generated by rotational excitations of bearings. (a) rotation of worm around \vec{X} axis, (b) rotation of worm around \vec{Z} axis, (c) rotation of worm gear around \vec{Z} axis, (d) (left) rotation of worm gear around \vec{Y} axis (right) top view of the rotation

The bearings deflections projected along y axis are written as:

$$\delta_{bw|y} = y_1 , \delta_{bw|Rx} = \theta_{wx} Y_{wx}, \delta_{bw|Rz} = \theta_{wz} Y_{wz}$$
(24)

$$\delta_{bg|y} = y_2, \, \delta_{bg|Rz} = \theta_{gz} Y_{gz}, \, \delta_{bg|Ry} = \theta_{gy} Y_{gy} \tag{25}$$

Considering two points M_w and M_g belonging to the active flank of the worm and the worm gear, respectively. The displacement of each point is denoted $U_i(M_i)$ (j = w, g) and is written referring to work done in [17]:

$$U_{w}(M_{w}) = \begin{pmatrix} x_{w} \\ y_{w} \\ z_{w} \end{pmatrix} + \begin{pmatrix} \theta_{wx} \\ \theta_{wy} \\ \theta_{wz} \end{pmatrix} \wedge \begin{pmatrix} l_{w}cos\lambda \\ -R_{w}tg(\phi-\gamma) + (p_{w}-l_{w}sin\lambda)cos(\gamma-\phi) \\ -R_{w} + (p_{w}-l_{w}sin\lambda)sin(\gamma-\phi) \end{pmatrix}$$
(26)
$$U_{g}(M_{g}) = \begin{pmatrix} x_{g} \\ y_{g} \\ z_{g} \end{pmatrix} + \begin{pmatrix} \theta_{gx} \\ \theta_{gy} \\ \theta_{gz} \end{pmatrix} \wedge \begin{pmatrix} l_{g}cos\lambda \\ R_{g}sin(\phi-\gamma) - (p_{g}+l_{g}sin\lambda)cos(\gamma-\phi) \\ R_{g}cos(\phi-\gamma) - (p_{g}+l_{g}sin\lambda)sin(\gamma-\phi) \end{pmatrix}$$
(27)

With, $l_{j=w,g}$ is the distance separating points $M_{j=1,2}$ from the middle of the line of action. One can thus obtain coefficients values as follows:

$$Y_{wx} = R_{bw} - (p_w - l_w sin\lambda) sin(\gamma - \phi)$$

$$Y_{wz} = l_w cos\lambda$$

$$Y_{gz} = -R_{bg} cos(\phi - \gamma) + (p_g + l_g sin\lambda) sin(\gamma - \phi)$$

$$Y_{gy} = 0$$
(28)

The same procedure described above is established for the projections of displacements on \vec{X} and \vec{Z} axes. On can thus obtain:

$$\left(\delta_w + \delta_g\right)\sin(\lambda)\cos(\phi) = -x_1 - x_2 - X_{wy}\theta_{wy} - X_{wz}\theta_{wz} - X_{gy}\theta_{gy} - X_{gz}\theta_{gz}$$
(29)

$$\left(\delta_w + \delta_g\right)\sin^2(\lambda)\cos(\phi) = -z_1 - z_2 - Z_{wx}\theta_{wx} - Z_{gx}\theta_{gx} - Z_{gy}\theta_{gy}$$
(30)

Where,

$$\begin{aligned} X_{wy} &= l_w cotg\lambda, \\ X_{wz} &= R_{bw} tg(\phi - \gamma) - (p_w - l_w sin\lambda) \cos(\gamma - \phi) \\ X_{gz} &= -R_{bg} sin(\phi - \gamma) + (p_g + l_g sin\lambda) \cos(\gamma - \phi) \\ Z_{wx} &= -R_{bw} tg(\phi - \gamma) + (p_w - l_w sin\lambda) \cos(\gamma - \phi) \\ Z_{gx} &= R_{bg} sin(\phi - \gamma) - (p_g + l_g sin\lambda) \cos(\gamma - \phi) \\ Z_{gy} &= l_g cos\lambda \end{aligned}$$
(31)

From the relation defined in equation (20) and the sum of equations (23), (29) and (30) written under the following form (32), one can obtain the final expression of F_N .

$$\delta_w + \delta_g = a \left[-x_1 - x_2 - y_1 - y_2 - z_1 - z_2 + a_1 \theta_{wx} + a_2 \theta_{wy} + a_3 \theta_{wz} + a_4 \theta_{gx} + a_5 \theta_{gy} + a_6 \theta_{gz} \right]$$
(32)

Replacing the expression of the normal load in equations of motions (4)-(17) written above, the total equation of motion can thus be written under the matrix form:

$$[M]\{\ddot{q}(t)\} + [K(t)]\{q(t)\} = \{F(t)\}$$
(33)

The degrees of freedom vector $\{q(t)\}$ is written as:

$$\{q(t)\} = \left\{x_1, y_1, z_1, x_2, y_2, z_2, \theta_{wx}, \theta_{wz}, \theta_{gy}, \theta_{gz}, \theta_{wy}, \theta_M, \theta_{gx}, \theta_R\right\}^t$$
(34)

External loads vector is given by:

$$\{F(t)\} = \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \tau_M, 0, \tau_R\}^t$$
(35)

The mass matrix is defined as a diagonal matrix:

$$[M] = \begin{bmatrix} M_l & 0\\ 0 & M_\theta \end{bmatrix}$$
(36)

composed of:

$$[M_l] = diag(m_1, m_1, m_1, m_2, m_2, m_2)$$
(37)

$$[M_{\theta}] = diag(I_{wx}, I_{wz}, I_{gy}, I_{gz}, I_{wy}, I_M, I_{gx}, I_R)$$
(38)

The stiffness matrix is the sum of two matrix:

$$[K(t)] = [K_c] + [K_t(t)]$$
(39)

The matrix of constant stiffness of worm and worm gears is:

$$\begin{bmatrix} K_c \end{bmatrix} = \begin{bmatrix} K_l & 0\\ 0 & K_\theta \end{bmatrix}$$
(40)

where,

$$[K_l] = diag(k_{x1}, k_{y1}, k_{z1}, k_{x2}, k_{y2}, k_{z2})$$
(41)

$$[K_{\theta}] = \begin{bmatrix} k_{wx} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & k_{wz} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_{gy} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_{gz} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & k_{wy} & -k_{wy} & 0 & 0 \\ 0 & 0 & 0 & 0 & -k_{wy} & k_{wy} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & k_{gx} & -k_{gx} \\ 0 & 0 & 0 & 0 & 0 & 0 & -k_{gx} & k_{gx} \end{bmatrix}$$
(42)

The matrix of variant stiffness has this shape:

$$[K_t(t)] = f(t).K_t \tag{43}$$

$$K_t = \begin{bmatrix} P & P & P & P & P & P & A_{wx} & P & A_{yz} & P & A_{gy} & P & A_{gz} & P & A_{yy} & 0 & P & A_{gx} & 0 \end{bmatrix}$$
(44)

$$P = \begin{bmatrix} P_t & P_R \end{bmatrix}' \tag{45}$$

$$P_t = \begin{bmatrix} -t_1 & -t_2 & -t_3 & t_1 & t_2 & t_3 \end{bmatrix}$$
(46)

$$P_{R} = \begin{bmatrix} -t_{3} * R_{w} & -t_{1} * R_{w} & (t_{1} + t_{3}) * R_{g} & t_{1} * R_{g} & -t_{1} * R_{w} & 0 & t_{2} * R_{g} & 0 \end{bmatrix}$$
(47)

Parameters in (45) are defined in the table below:

Table 1 Parameters of the variant stiffness matrix

 $\begin{array}{cccc} t_1 & K_{E1}R_w \\ t_2 & K_{E2}R_w \\ t_3 & K_{E3}R_w \\ K_{E1} & t_{11}\sin\left(\frac{\pi}{2}-\lambda\right)K_{wg} \\ K_{E2} & t_{12}\sin\left(\frac{\pi}{2}-\lambda\right)K_{wg} \\ K_{E3} & t_{13}\sin\left(\frac{\pi}{2}-\lambda\right)K_{wg} \\ t_{11} & \cos\phi\sin\lambda + \mu\cos\lambda \\ t_{12} & \cos\phi\cos\lambda - \mu\sin\lambda \\ t_{13} & \sin\phi \end{array}$

Fluctuation of the stiffness by time is defined by f(t) function. It describes the evolution of the length of the line of action for every meshing tooth pair period. It depends on geometrical parameters. In this work, it is taken as in the study of [17]. The goal is to compare results later.

3. Numerical simulation

3.1 Dynamic performance

To investigate the dynamic performance of the model described above, a numerical simulation is carried out under MATLAB software. The worm gear set design parameters used are detailed in Table 2 [17], [19]. The Newmark beta solver is used to determine the amplitude of vibrations related to the DOF of the model. Fluctuation of the mesh stiffness by time is presented in Fig.4. Dynamic study is done on a period of 2s. The friction coefficient depends normally on relative velocity in the meshing during sliding. Many empirical expressions were developed to estimate it. However, it is considered here constant to linearize the problem. The influence of this coefficient will be presented later.

	worm	Worm gear
Module (mm)	3	
teeth number	1	50
Pitch radius (mm)	8	25
Material	Steel(S45C)	Bronze (CAC702)
Rotation speed (rpm)	1500	30
Stiffness of bearings (N/m)	$k_{x1} = k_{z1} = 10^8$	$k_{y2} = k_{z2} = 10^8$
Stiffness of shafts (N/m)	$k_{y1} = 2,5.10^8$	$k_{y2} = 2.5.10^8$

Table 2 model design parameters

Torsional Stiffness of bearings (Nm/rd)	$k_{wx} = k_{wz} = 4.10^7$	$k_{gx} = k_{gz} = 4.10^7$
Torsional Stiffness of shafts (Nm/rd)	$k_{wy} = 8, 4. 10^5$	$k_{gx} = 8.4.10^5$
Teeth stiffness (N/m)	$k_{w} = 10^{8}$	$k_g = 0.5 * 10^8$
Angle of pressure	20°	
Worm's lead angle	3°58'	
Normal pressure angle	14°33'	
Worm's lead (mm)	9.4	











The displacements amplitudes of worm are displayed in Fig. 5 and Fig. 6. Results are very close to those found in the literature [15]-[17]. Fig. 7 presents a comparison between two configurations. In both, the worm is with steel material. However, the worm gear is either cast iron (FC200 with $k_g = 0.35 * 10^8$) or bronze (CAC702 with $k_g = 0.5 * 10^8$) for this numerical example. The goal here is to check the consistency of the developed approach when the characteristics of materials are changed. Curves show a decrease in the displacement amplitudes of bearing when cast iron is used compared to the bronze one. This material has a softening effect of vibrations amplitudes which is the case here.



Fig. 7 Evolution of the angular displacement of the bearing around Y axis for steel-bronze and steel-cast iron configurations

3.2 The impact of the variation of the friction coefficient

To study the influence of the friction coefficient on the dynamic behavior of the model, a range of coefficient values is scanned [0.05,0.08], and displacements and rotations amplitudes are calculated. In Fig. 8, the effect of the friction coefficient on the rotation amplitudes of the worm (steel) and the worm gear (bronze) is presented. The evolution of theses amplitudes is presented through deviation defined as the ratio of the maximum amplitude on the end value: $\max(\theta(\mu)) / \theta(\mu_{=0.08})$. It is deduced that the higher is the coefficient, the smallest is the amplitude for both rotations. Faster decreasing of amplitudes is also observed for the worm gear rotation compared to the worm amplitudes. This can be explained by the proportionality of motion between components.



Fig. 8 Evolution of the amplitude of the maximum angular displacement by the variation of the friction coefficient



Fig. 9 Evolution of the deviation of the maximum amplitude of the angular displacement θ_{gx} by friction coefficient variation for both steel-bronze and steel-cast iron configurations

In Fig.9, the effect of the friction coefficient on the deviation of the angular displacement θ_{gx} of the worm gear is presented. Both materials of bronze and cast iron are the object of comparison here. Curves show here again the softening effect of the cast iron that exhibits lower values of amplitudes for the range of friction values scanned.

The purpose of this investigation is to highlight the consistency of the results of the new formulation as in the previous section for the change of the material of one component. The results seem coherent. The amplitude of the displacements evolves inversely to the evolution of the coefficient of friction.

4. Conclusion

Through this paper, a new mathematical model of a worm gear drive is presented. The dynamic behavior of fourteen degrees of freedom model is under investigation focusing on contact force and teeth elastic deformations in the meshing area. The objective is to develop a more accurate model taking into consideration the geometry of worm gear sets, contact friction, inertia moments and shafts rigidities. A numerical simulation is carried out showing reasonably correct results. Softening the mesh with a cast iron material shows a drop in the amplitudes of vibrations as expected. The influence of the friction coefficient on the amplitudes is also studied showing a decreasing in amplitudes with higher values of friction coefficient while maintaining motion proportionality.

Declarations

Conflict of interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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