

# Analysis of direct phase modulation with an injection-locked oscillator

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**Abstract**— The behaviour of low cost, direct phase modulators, based on injection-locked oscillators, is studied in detail, using a reduced-order semi-analytical formulation. The derived expressions allow the identification of critical design parameters, influencing aspects such as the sensitivity of the phase-shift interval versus the element tolerances or the amplitude modulation. An envelope-domain equation, derived from the same semi-analytical formulation, allows the understanding of qualitative changes in the modulator dynamics versus variations in the modulation frequency, in different operation conditions. The study is illustrated with a modulated push-push oscillator at 18 GHz. Very good agreement has been obtained between the theoretical results and the experimental measurements.

## I. INTRODUCTION

Recently, a number of works have proposed the use of injection-locked oscillators for compact and low cost direct phase modulators [1-4]. Among other applications, this concept is interesting for the implementation of phase-modulated active antennas and coupled-oscillators systems for beam steering. The operation principle is based on the fact that the phase of an injection-locked oscillator at fixed frequency  $\omega_s$  changes with the bias voltage of the devices used [5-6]. Thus, we can obtain a phase modulator by introducing the modulation signal in the bias line. One problem with this technique is the inherent limitation of the stable phase range to about  $180^\circ$ , which would prevent QPSK modulations. Another critical aspect is the nonlinearity of the phase characteristic versus the bias voltage, with a high sensitivity near the edges of the stable phase-shift interval, which often gives rise to a practical reduction of the theoretical phase shift range.

Several solutions have been proposed in the literature to overcome the above limitations [1-4]. One is the chain connection of two injection-locked oscillators, which can be difficult to implement in practice, as it requires the adjustment of the phase-locking range of the two oscillators, which should agree approximately. Another solution is the sub-synchronization of the oscillator at one half the oscillation frequency, which provides an inherent doubling of the phase modulation. The main drawback of this technique is the usually narrow bandwidth of sub-synchronized oscillators and the requirement for a relatively high injection power. Here we investigate the possibility of doubling the stable phase shift range by using an injection-locked push-push oscillator [7-8]. A push-push oscillator is composed of two sub-oscillator circuits at  $\omega_0$ , with  $180^\circ$  phase shift. Assuming perfect symmetry of the two sub-oscillators at  $\omega_0$ , the odd harmonic

terms are cancelled out at the circuit output, whereas the even harmonic terms are added in phase. The possible drawback is the lower output power of the push-push configuration. However, realizations with suitable power levels have been reported in the literature [9]. Because the circuit is injection-locked at the fundamental frequency, we can expect a relatively broad synchronization bandwidth, which should increase the robustness of the modulator.

The main objective of this work is the derivation of a general-application analytical formulation, providing insight into the behaviour of phase modulators based on injection-locked oscillators. The derived expressions should allow the identification of critical design parameters influencing aspects such as the sensitivity of the phase-shift interval versus the element tolerances or the amplitude modulation. With an envelope-domain extension of the formulation, we will also investigate qualitative changes in the modulator dynamics versus variations of the modulation frequency. The techniques will be applied to a push-push oscillator at 18 GHz.

## II. OPERATION OF THE PHASE MODULATOR BASED ON AN INJECTION-LOCKED OSCILLATOR

Initially we will consider an injection-locked oscillator, containing a varactor diode, in the absence of a modulation signal. In synchronized state, for a fixed frequency of the injection source  $\omega_s$ , there is constant phase shift between the oscillation and the injection source. This constant phase shift changes when varying the bias voltage of the varactor diode. The typical interval of stable phase values is  $\pi$  rad at the first harmonic and  $2\pi$  rad at the second harmonic component. For the circuit simulation we will select a sensitive observation node, where we will connect an auxiliary generator (AG), with voltage  $V_{AG}$ , frequency  $\omega_{AG}$  and phase  $\phi_{AG}=0$  [10-11]. Then, the outer tier of the system describing the injection-locked oscillator is given by:

$$Y(V_s, V_T, \omega_s, E_g e^{j\phi_s}) = 0 \quad (1)$$

with  $Y$  the ratio between the AG current and the AG voltage,  $V_s = V_{AG}$ , the voltage amplitude,  $\omega_s = \omega_{AG}$ , the synchronization frequency,  $V_T$  the tuning voltage and  $-\phi_s$  the phase shift with respect to the input generator. Note that the inner tier of (1) is constituted by the pure harmonic-balance (HB) system, accounting for the rest of circuit nodes and harmonic terms. At the time to design the phase modulator, two different aspects must be taken into account: the frequency bandwidth in synchronized state and the required range of variation of the

modulation signal, introduced at the varactor bias line. The synchronization bandwidth must be relatively broad in order to admit a high modulation frequency. We will initially assume a fixed value of the bias voltage,  $V_{T0}$ . For the HB simulation of the push-push oscillator, two AGs with the same amplitude values  $V_s$  and phase values  $0$  and  $\pi$ , are connected at identical nodes of two sub-oscillator circuits. Each one should fulfil equation (1) with the same  $V_s$  value and node phase  $0$  and  $\pi$ , respectively. The synchronized operation bandwidth is determined by sweeping  $\phi_s$  between  $0$  and  $2\pi$ , calculating  $\omega_s$  and  $V_s$  at each sweep step, in order to fulfil  $Y=0$ , as shown in the references [7-8]. The technique has been applied to the push-push oscillator in Fig. 1. The resulting synchronization bandwidth for the input power  $-16$  dBm is  $160$  MHz with this particular topology.

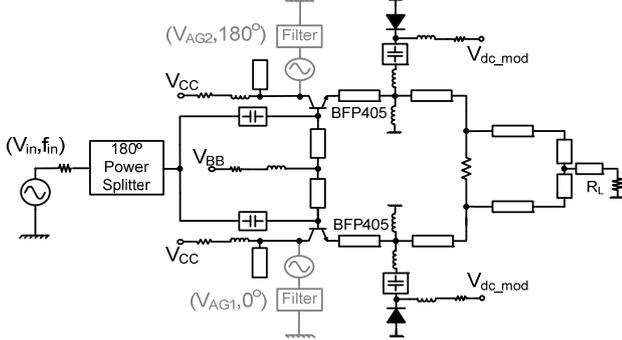


Fig. 1 Schematic of the push-push oscillator at 18 GHz. Two AGs with  $180^\circ$  phase shift are used for the HB analysis of this circuit.

Next we will determine the phase characteristic versus the dc bias voltage of the diode. To facilitate the analytical study, we will initially consider low amplitude of the injection source. Then, it becomes possible to linearize equation (1) about the free-running oscillation. At this small injection amplitude, we will choose an injection-locking frequency equal to the free-running frequency  $\omega_s = \omega_0$ , as the latter corresponds to the middle of the closed synchronized-solution curve. Thus, assuming  $\omega_s = \omega_0$ , we will obtain the following linearized system:

$$Y_V(V_s - V_0) + Y_{V_T}(V_T - V_{T0}) + Y_{E_g}E_g e^{j\phi_s} = 0 \quad (2)$$

where  $Y_V$ ,  $Y_{V_T}$  and  $Y_{E_g}$  are the complex admittance derivatives with respect to  $V_s$ ,  $V_T$  and  $E_g$ , evaluated in the free-running regime. By splitting (2) into real and imaginary parts and adding the squares of these parts, it is easily seen that the above expression corresponds to an ellipse in the plane defined by the bias voltage  $V_T$  and the oscillation amplitude  $V_s$ . This is in agreement with the results in Fig. 2a, obtained for different values of the input voltage  $E_g$ , with the harmonic balance method. Actually, for derivatives obtained through finite differences, as shown in [7], the ellipse overlaps with the one obtained with the HB simulations. For this HB simulations, we sweep  $\phi_s$  and calculate the oscillation amplitude and varactor bias value at each phase step in order to fulfil  $Y=0$  at the two AGs (Fig. 1). The stable operation range corresponds to the upper section of all the solution curves, between the two turning points [11-12]. Note that as

$E_g$  increases, the solution curve deviates from a perfect ellipse, as the linearization is no longer valid, so it is possible to have a stable phase shift range slightly larger than  $180^\circ$ . The variation of the total stable range ( $\Delta\phi = \phi_{smax} - \phi_{smin}$ ) versus  $E_g$  is shown in Fig. 2b. On other hand, the tilt angle of the major axis of the ellipse increases the variation of the output power with the bias voltage, which will have an impact on the amplitude modulation. It is easily shown that this angle becomes zero for  $\pi/2$  phase shift between  $Y_V$  and  $Y_{V_T}$ . Because the derivatives are taken at the free-running oscillation, they will depend on the particular oscillator topology and operation point. The possible modifications of the existing push-push oscillator, in order to improve the performance, are beyond the scope of this paper.

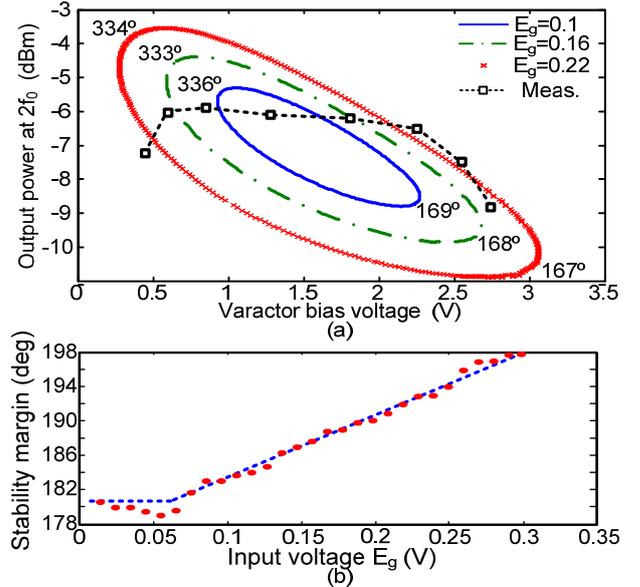


Fig. 2 Injection-locked oscillator. (a) Variation of the output power versus the bias voltage of the varactor diode. (b) Variation of the total stable phase-shift range versus the injection voltage  $E_g$ .

Starting from (2), and after some manipulations, we can obtain the equation that determines the phase shift variation with the dc bias voltage:

$$V_T - V_{T0} = \frac{E_g}{V_0} \frac{|Y_{E_g}| \sin(\phi_s - \alpha_G - \alpha_v)}{|Y_{V_T}| \sin(\alpha_T - \alpha_v)} \quad (3)$$

where  $\alpha_v$ ,  $\alpha_T$  and  $\alpha_G$  are, respectively, the angles of  $Y_V$ ,  $Y_{V_T}$  and  $Y_{E_g}$ . The phase variation versus the bias voltage is ruled by an arcsin function (Fig. 3), so the phase sensitivity increases when approaching the locking edges, which correspond to the turning points  $T_1$  and  $T_2$  in Fig. 3. The use of voltage values near the band edges may lead to desynchronization due to the tolerances of the circuit elements. For a QPSK modulation (and considering the later doubling action of the push-push configuration), we can center the modulation voltage about  $V_{T0}$  and limit the phase variation to the interval  $(-3\pi/8, 3\pi/8)$ , leaving a phase margin of  $\pi/8$  with respect to the each of the static stability edges. These edges correspond to  $\sin(\phi_s - \alpha_G - \alpha_v) = \pm 1$  in (3). The stability margin in terms of the tuning voltage  $\delta V_T$  is given by:

$$\delta V_T = \frac{E_g}{V_0} \frac{|Y_{Eg}| [1 - \sin(3\pi/8 - \alpha_G - \alpha_v)]}{|Y_{VT}| \sin(\alpha_T - \alpha_v)} \quad (4)$$

Several parameters influence the stability margin. For instance, it increases for smaller magnitude  $|Y_{VT}|$ , that is, for smaller sensitivity of the admittance function to the tuning voltage and for higher  $|Y_{Eg}|$ . For a fixed oscillator design, we can still enlarge the stability margin by increasing the input voltage  $E_g$ , in agreement with the observations in [4]. Fig. 3 illustrates the effect of increasing  $E_g$  on the stability margins in terms of  $V_T$ , for the phase value  $-3\pi/8$ . We will select  $E_g = 0.22$  V for the design of the phase modulator.

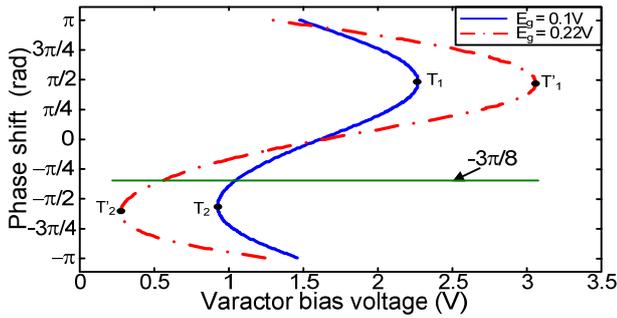


Fig. 3 Increase of the stability margins of the phase modulator through an increase of the amplitude of the injection source.

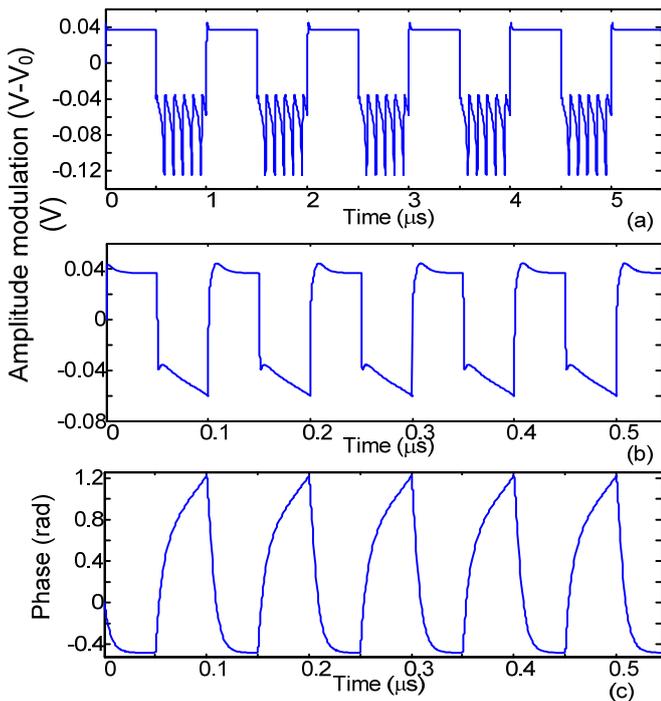


Fig. 4 Results of the analysis with (6) for a modulation signal going beyond the stable voltage range. Two different modulation frequencies are considered.

### III. DYNAMIC EFFECTS IN THE PHASE-MODULATED OSCILLATOR WITH INJECTION-LOCKED SIGNAL

When a modulation signal is introduced through the varactor bias line, the solution phase becomes time varying. Thus, the

phase corresponding to the observation node voltage can be written  $0 + \phi(t)$ , so the phase shift with respect to the synchronizing source is given by  $\phi(t) - \phi_s$ . Due to the presence of the modulation signal, the circuit should be analysed with the envelope-transient method [11]. For a better understanding of the dynamic effects, we can extend the formulation (2) to the envelope domain. This requires the assumption of a small range of variation of the tuning voltage  $\Delta V_T(t)$  (with respect to the free-running value) and linear behaviour with respect to the input source, in similar way to (2). Then, the envelope-domain system can be written as:

$$Y[V_0 + \Delta V_0 + \Delta V, V_{T0} + \Delta V_T, j\omega_s + s](V_s + \Delta V)e^{j\phi(t)} = -Y_{Eg} E_g e^{j\phi_s} \quad (5)$$

where all the increments are time varying and  $s$  is a complex frequency increment, acting as a time derivator. Performing a Taylor series expansion of (5), we obtain:

$$Y_V V_s (\Delta V_0 + \Delta V) + Y_{VT} V_s \Delta V_T + Y_\omega (V_s \dot{\phi} - j \Delta \dot{V}) = -Y_{Eg} E_g e^{j(\phi_s - \phi)} \quad (6)$$

Note that unlike the case of a phase-noise analysis [7], no linearization is carried out with respect to the phase shift. As the modulation frequency decreases, the system tends to the linearized system consider in (2), since the time derivatives  $\dot{\phi}$ ,  $\Delta \dot{V}$  vanish from (6).

The dynamic effects observed when varying the modulation frequency are captured by the simplified equation (6). For low modulation frequency and  $\Delta V_T$  variations leading the system beyond the stability margin, the time-varying phase  $\phi(t)$  will contain a frequency component at the beat frequency  $\omega_u = \sqrt{(\omega_0 - \omega_s)^2 - F_0^2}$ , with  $\omega_0$  the free-running oscillation and  $F_0$  a constant term, depending on the coefficients in (6). However, for modulation frequency fulfilling  $\omega_m > \omega_u$ , the system dynamics will not be fast enough for the beat frequency  $\omega_u$  to be observable. This is illustrated by introducing a two-level modulation signal, so that one of the levels is beyond the stability margin. As shown in the simulation of Fig. 4a, obtained from the integration of (6), for a relatively low frequency  $f_m = 1$  MHz, the system is in unlocked stage during the time interval for which the voltage level is beyond the stable range. The observed oscillation takes place at the beat frequency  $\omega_u$ . For a higher modulation frequency (Fig. 4b), the oscillation at  $\omega_u$  is not observed. However, the upper section of the phase pulse (corresponding to varactor levels outside the stable range) is not flat, but grows in time as  $\omega_u t$ . With the rapid change to a varactor voltage within the stable section, the phase tends to a constant value, after a short transient, as expected in injection-locked operation. Thus, for a high modulation frequency, there is an apparent increase of the stable operation range. The same qualitative behaviour has been obtained with the envelope transient technique, based on harmonic balance. For this analysis, the two AGs are connected at the initial time value only, which enables the initialization of the two oscillations. Unlike (6), the modulation-voltage range is not small, but comprised between 0.55 and 3.2 V. The results are shown in Fig. 5a and Fig. 5b.

In a different test, we have adjusted the voltage levels of the modulation signal in order to obtain the phase values  $-3\pi/4$ ,  $-\pi/4, \pi/4, 3\pi/4$ . The four levels belong to the stable section of Fig. 3. Fig. 5c and Fig. 5d show the variation of the output phase at the double frequency  $2f_s$  for two different values of the modulation frequency. We have tested the robustness of the phase modulator with Monte Carlo analyses in terms of the element tolerances and we never observed unstable behaviour. The circuit is also stable in temperature. The circuit was built and experimentally characterized. The measured output power is superimposed in Fig. 2. The phase characteristic, measured with analyser HP70000, is shown in Fig. 6. The measured phase noise spectrum is shown in Fig. 7.

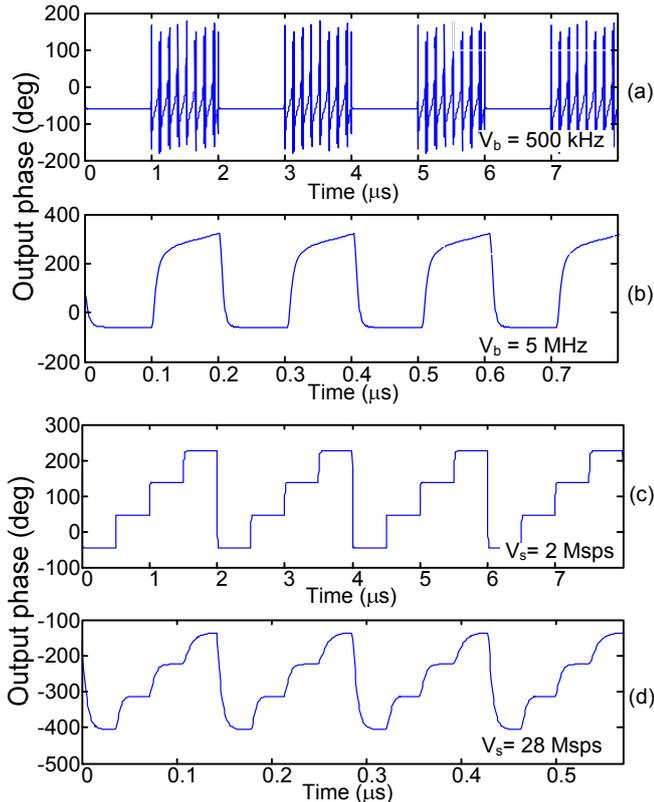


Fig. 5 Envelope-transient simulations. (a) and (b) Modulation signal going beyond the stable voltage range. (c) and (d) Operation within the stable margins.

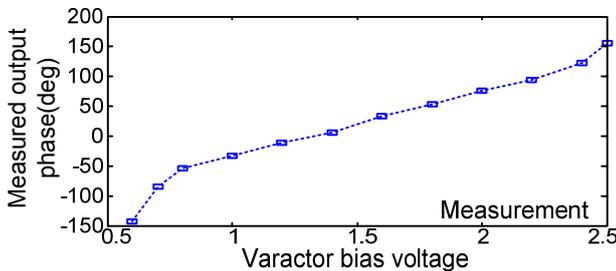


Fig. 6 Experimental phase characteristic versus the varactor bias voltage.

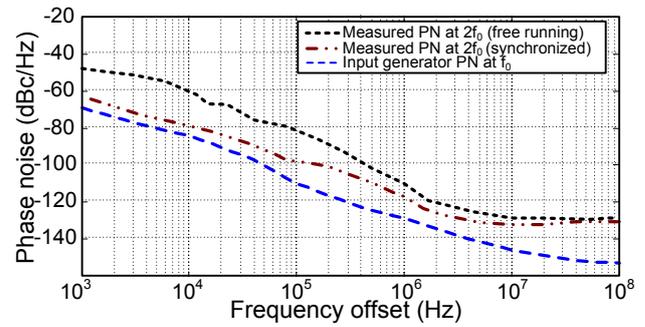


Fig. 7 Phase-noise (PN) spectrum of the injection-locked push-push oscillator.

#### IV. CONCLUSIONS

The behavior of phase modulators based on injection-locked oscillators has been analyzed in detail. The main limitation is the high sensitivity of the phase characteristic versus the bias voltage near the stability edges. A simplified formulation has enabled the derivation of design criteria to increase the stability margins. The behavior in the presence of a modulation signal has been analyzed with the envelope-transient method. A reduced order envelope-domain formulation has been used to explain the apparent stabilization of the circuit beyond the static stable phase-shift range. The techniques have been applied to an injection-locked push-push oscillator.

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