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ProFatigue: A software program for probabilistic assessment of experimental fatigue data sets

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Abstract

In this work, the software program ProFatigue is presented as a practical tool for derivation of probabilistic S-N and ε-N fields from experimental fatigue data. The program provides an estimation of the parameters involved in the regression probabilistic Weibull fatigue model developed by the authors, allowing an advantageous application of both fields to the stress or strain based approaches in the fatigue design of structures and mechanical components. An extension to the analysis of more complex and varied lifetime problems as thermomechanical, multiaxial and fretting fatigue is possible by adopting suitable damage parameters proposed in the literature as driving force. Application to probabilistic assessment of cumulative damage and further program enhancement are now envisaged.

Keywords: Fatigue software; Probabilistic assessment; S-N field; ε-N field

1. Introduction

In most of the models used for the derivation of S-N and ε-N fields from assessment of experimental fatigue data the probability of failure is not explicitly included though the high scatter of the lifetime at the different stress ranges requires its consideration. This has consequences in the consideration of both fatigue fields by the stress and strain approaches for fatigue design of structures and mechanical components. As a suitable alternative, the probabilistic fatigue model developed by Castillo and Fernández-Canteli [1] fulfils both physical (weakest link principle and limited range) and statistical conditions related to extreme value analysis (stability, limit behaviour). In particular, the necessary compatibility between the cumulative distribution functions (cdfs) for fatigue lifetime at given stress range and stress range at given lifetime is fulfilled as an indispensable prerequisite allowing a functional equation to be established. Its solution provides only two possible Weibull models for minima:

$$\text{Model I: } F(N,\Delta\sigma) = 1 - \exp\left[-\left(\frac{(\log N - B)(\log\Delta\sigma - C) - \lambda}{\delta}\right)^{\beta}\right]; (\log N - B)(\log\Delta\sigma - C) \geq \lambda, \qquad (1)$$

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$$\text{Model II: } F(N, \Delta\sigma) = 1 - \exp\left[-\left(\frac{(\log N - B)(\log \Delta\sigma - C)^{\gamma}}{\delta}\right)^{\beta}\right]; \log N \geq B, \log \Delta\sigma \geq C, \tag{2}$$

where B is a threshold value of the lifetime, C is the endurance limit, or fatigue limit for $N \to \infty$, λ, δ, β are location, scale and shape Weibull parameters, respectively, and γ is a parameter that scales the normalization of the S-N field. The percentile curves are hyperbolas sharing the asymptotes $N = \exp(B)$ and $\Delta\sigma = \exp(C)$, whereas the zero percentile curve represents the minimum possible required number of cycles to achieve failure for different values of $\log \Delta\sigma$. If the lower bound of the results, represented by the location parameter in the Weibull model, can be relaxed, a Gumbel model may be assumed as a limiting case of the Weibull model, taking the forms:

$$\text{Model I: } F(N, \Delta\sigma) = 1 - \exp\left[-\exp\left(\frac{(\log N - B)(\log \Delta\sigma - C) - \lambda}{\delta}\right)\right]; (\log N - B)(\log \Delta\sigma - C) \geq \lambda, \tag{3}$$

$$\text{Model II: } F(N, \Delta\sigma) = 1 - \exp\left[-\exp\left(\frac{(\log N - B)(\log \Delta\sigma - C)^{\gamma}}{\delta}\right)\right]; \log N \geq B, \log \Delta\sigma \geq C. \tag{4}$$

These models offer the advantage of requiring one parameter less than the Weibull one, avoiding the always compromising task of assuming a lower bound associated with zero probability of failure and cover the whole lifetime region from the first cycle. The assumption of a Gumbel model is fully justified when β is greater than, say, 6, based on the practical coincidence of the data evaluation for both probability distributions, at least up to fairly low probabilities of failure, see [1].

A similar solution as model II, though based on micromechanical considerations, is proposed by Bolotin [2] in both Weibull and Gumbel alternatives:

$$\begin{aligned} \text{Weibull: } F(N|s) &= 1 - \exp\left[-\frac{M}{M_0}\left(\frac{N - N_0}{N_c}\right)^{\beta}\left(\frac{s - s_{th}}{s_c}\right)^{\alpha}\right], \\ \text{Gumbel: } F(N|s) &= 1 - \exp\left[-\frac{M}{M_0}\exp\left(\frac{N - N_0}{N_c}\right)^{1/m}\left(\frac{s - s_{th}}{s_c}\right)\right], \end{aligned} \tag{5}$$

where s can be understood as $\log \Delta\sigma$, N as $\log N$, and the quotient M/M_0 represents the scale effect.

The size effect is handled by assuming the weakest-link principle and statistical independence, so that simultaneous evaluation of experimental results obtained from testing specimens of different size is feasible. The consideration of the normalizing variable $V = (\log N - B)(\log \Delta\sigma - C)$ allows the whole S-N field to be reduced to a unique cumulative distribution function (cdf).

The potentiality of the model, initially restricted to the assessment of fatigue results obtained at constant stress level, becomes evident by successive interpretations, improvements, new applications and extensions to more advanced models since its initial proposal.

2. The ProFatigue program

In order to facilitate the applicability of the model to the practical assessment of the S-N field from fatigue data, the free use software program ProFatigue is developed [3]. The model parameters are estimated for any appropriate fatigue data thus allowing a probabilistic prediction of the lifetime as a function of the stress range for fixed stress level (R, σ_{mean}, etc.). The ProFatigue program can fit the parameters of model I of the Weibull (Eq. (1)) and the Gumbel (Eq. (3)) type to the fatigue data. Implementing model II as well as an extension to the case of varying stress level, this having been already developed by the authors [4], is envisaged as a future program enhancement. In the following only the Weibull model I is considered.

The product $V = (\log N - B)(\log \Delta\sigma - C)$ becomes a normalizing variable allowing the whole S-N field to be reduced to a unique cdf what suggests the model parameters being estimated in a two-step method: firstly, B and C, then the other three parameters of the normalized Weibull distribution using a standard procedure. It permits all

S-N data to be pooled as pertaining to a single distribution so that the model parameters are estimated with higher reliability. As soon as the five parameters are estimated, the analytical expression of the whole S-N field is known. The percentile curves can be interpreted as representing different initial crack sizes, for the moment unknown. Run-outs are considered in the parameter evaluation by means of the E-M algorithm [1]. Confidence intervals of the S-N field can be estimated by simulation using the boot-strap method in addition to the ProFatigue program. Since the model extends the Wöhler field up to an infinite number of cycles, an extrapolation of the lifetime outside the scope of number of cycles tested is possible allowing an evaluation of data results in the HCF or even VHCF region [1].

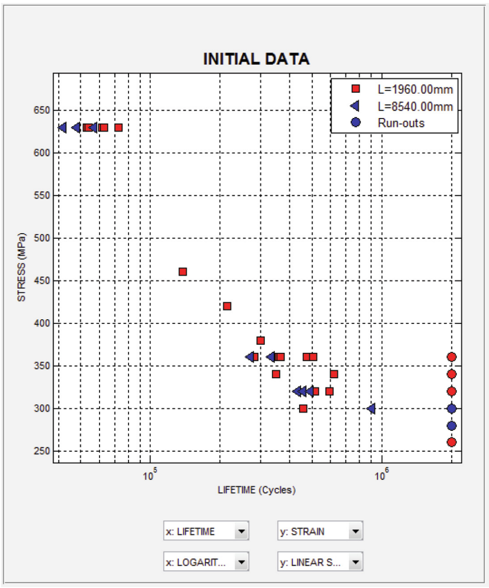
The weakest-link principle and statistical independence were assumed in order to take into account the size effect; in particular, simultaneous evaluation of experimental results obtained from testing specimens of different size can be considered by including the relation between the tested length L_i and a reference length L_0 leading to

$$F(N, \Delta\sigma) = 1 - \exp\left[-\frac{L_i}{L_0}\left(\frac{(\log N - B)(\log \Delta\sigma - C) - \lambda}{\delta}\right)^{\beta}\right] = 1 - \exp\left[-\left(\frac{(\log N - B)(\log \Delta\sigma - C) - \lambda}{\delta\left(\frac{L_0}{L_i}\right)^{1/\beta}}\right)^{\beta}\right]. \quad (6)$$

As an example of application, fatigue data obtained for prestressing wires of lengths 1960 mm and 8540 mm (see Fig. 1) are evaluated by the proposed probabilistic model following Eq. (6) and shown in Figs. 2a and 3a together with the S-N field represented by curves of constant failure probability. Run-outs are represented once as circles at the number of cycles where the test was suspended and once as stars at the expected lifetimes estimated by the program. Figures 2b and 3b represent the cumulative distribution functions of the normalizing variable V corresponding to each length.

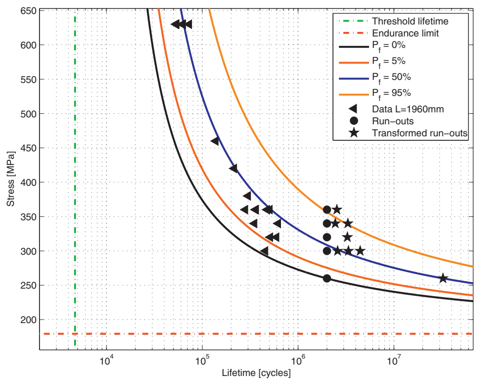
Since the fundamentals of the model rest on general internal functional properties of the Wöhler field, ProFatigue can be applied to a broad spectrum of lifetime problems of different nature, as it is confirmed by satisfactory application to a number of different materials as reinforcing and prestressing steels, various steel and aluminium alloys, concrete, polyethylene and composites, including size effect and extrapolation to the VHCF region, as reported in previous works of the authors furnishing reliable results. The results confirm the potentiality of the model for subsequent practical design, irrespective of the problem handled.

Compared to conventional models such as that of Basquin or the up-and-down method, the ProFatigue program proves to be advantageous providing higher reliability and capability to reproduce the whole Wöhler field.

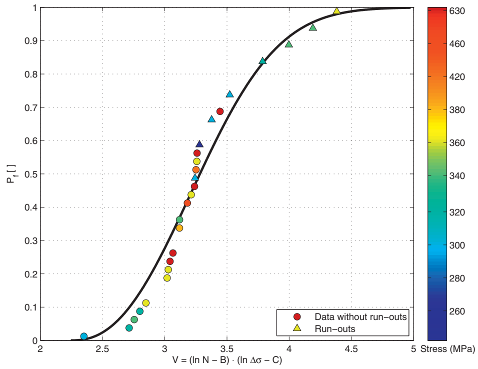


	LIFETIME	STRESS (MPa)	LENGTH (mm)	RUN-OUTS	INCLUDE
1	53000	630	1960	☐	☑
2	54000	630	1960	☐	☑
3	62000	630	1960	☐	☑
4	63000	630	1960	☐	☑
5	73000	630	1960	☐	☑
6	138000	460	1960	☐	☑
7	215000	420	1960	☐	☑
8	300000	380	1960	☐	☑
9	281000	360	1960	☐	☑
10	359000	360	1960	☐	☑
11	364000	360	1960	☐	☑
12	474000	360	1960	☐	☑
13	506000	360	1960	☐	☑
14	350000	340	1960	☐	☑
15	619000	340	1960	☐	☑
16	512000	320	1960	☐	☑
17	594000	320	1960	☐	☑
18	455000	300	1960	☐	☑
19	42000	630	8540	☐	☑
20	48000	630	8540	☐	☑
21	57000	630	8540	☐	☑
22	272000	360	8540	☐	☑
23	332000	360	8540	☐	☑
24	334000	360	8540	☐	☑
25	431000	320	8540	☐	☑
26	455000	320	8540	☐	☑
27	492000	320	8540	☐	☑
28	909000	300	8540	☐	☑
29	2000000	360	1960	☑	☑
30	2000000	340	1960	☑	☑

Fig. 1: Input data for S-N fatigue analysis for prestressing wires with two different lengths

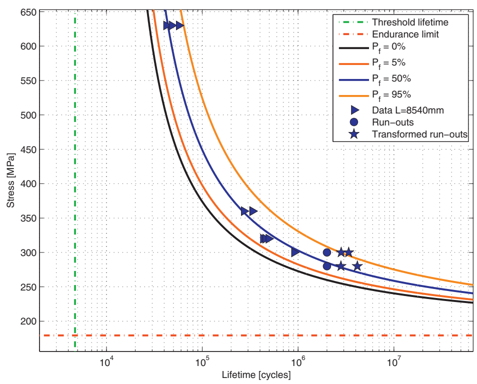


(a) S-N field

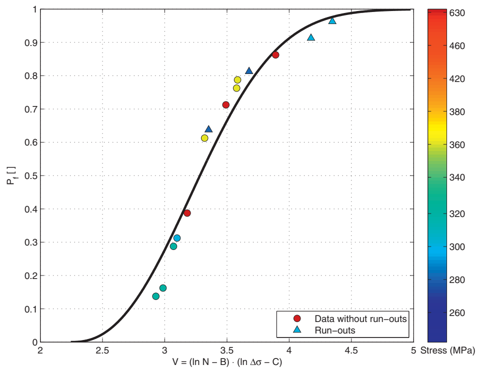


(b) Cdf of the normalizing variable V

Fig. 2: Model application on fatigue data for prestressing wires - length = 1960 mm



(a) S-N field

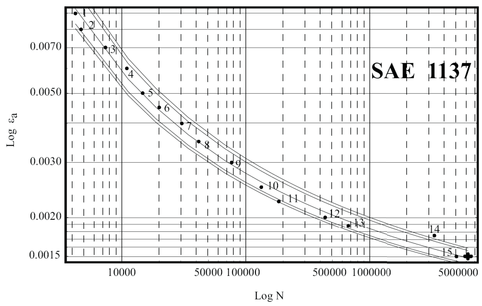


(b) Cdf of the normalizing variable V

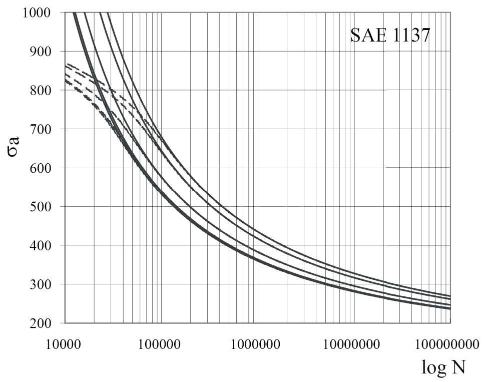
Fig. 3: Model application on fatigue data for prestressing wires - length = 8540 mm

3. Applications to the ε-N field

Since the basic physical and statistical considerations used to derive the Weibull S-N model also apply in the case of the ε-N field, due to the parallelism in the functional problem being dealt with, the same Weibull regression model



(a)



(b)

Fig. 4: (a) ε-N field for SAE1137 steel using the proposed model [5]; (b) Resulting S-N field from the ε-N field [6]

has been implemented in the ProFatigue program for the strain-life case simply by considering strain range as driving force. Nevertheless, the analytical probabilistic definition of the whole strain-life field as percentile curves applies both for the low cycle and high cycle fatigue regions, because the upper limit in the ε-N, opposite to the S-N field, is practically unrestricted, see [1].

Similarly to the former case handled for the S-N field, Fig. 4 (a) represents the ε-N field using ProFatigue as an advantageous alternative to the Coffin-Manson approach [5]. Moreover, using the σ-ε cyclic curves considering a Ramberg-Osgood model, it is possible to derive the corresponding stress life curves, i.e. the probabilistic S-N field, which in this case presents the typical change in curvature in the low-cycle fatigue region, see Fig. 4 (b), thus reproducing the LCF region.

A possible alternative at disposal to overcome the apparent inability of the model to reproduce the LCF region consists in setting energy parameters as the driving force [7].

4. Further achievements of the ProFatigue program

The following achievements are also provided by the ProFatigue program:

4.1. Support for fatigue program planning

The ProFatigue program facilitates optimizing planning of testing programs by means of an adequate testing strategy (cluster, cascade or mixed methodology have been already investigated). A preliminary estimation of the median S-N curve, i.e. that related to a failure probability $P_f = 50\,\%$, can be also provided by the program when the number of experimental results is yet scarce (say between 5 and 10). Without requesting the Weibull parameters this gives advice for the choice of a suitable stress range for subsequent fatigue tests.

4.2. Applicability of the ProFatigue program to assessment of complex fatigue problems

The compatibility condition is a prerequisite for guaranteeing validity in the present model the same as for the derivation of any models proposed for the assessment of the different lifetime problems. As a consequence, an application of the ProFatigue program to complex fatigue problems is possible if the adequate damage parameter, for instance Fatemi-Socie, SWT, ΔJ, etc., is adopted as generalized driving force according to the problem handled, for assessment of data in multiaxial fatigue, thermomechanical fatigue, fretting fatigue, UniGrow model, etc. and subsequent use in stress or strain based approaches. This allows the results of all these phenomena to be handled in a probabilistic way. In particular, an application to the definition of the LCF region can be contemplated [7].

The necessity of using energetic parameters seems to confirm the unsuitability of the stress range, as conventional primary parameter, when local plasticity effects are present in the fatigue process and the presumable energetic character of the damage induced by fatigue [8].

4.3. Support for probabilistic cumulative damage calculation

The definition of the S-N field, identifying percentile curves with curves representing different initial crack sizes, i.e. of damage, and the normalization of the S-N field as a unique cumulative distribution function by means of the normalizing variable $V = (\log N - B)(\log \Delta\sigma - C)$ given by the ProFatigue program, enables us to consider damage as probability of failure, in accordance to the conventional concept of ultimate limit state, and predict lifetime under varying loading using a probabilistic concept in the analysis of the Miner number. Simultaneous calculation of the Miner number and the normalized variable, which is related to probability of failure, allows to relate the cumulative damage provided by the Miner number to probability of failure.

5. Conclusions

- The probabilistic fatigue model of Castillo et al., based primarily on fulfilling the compatibility condition between both lifetime and stress range probability distributions, proves to be adequate to avoid primary errors in

establishing fatigue models according to the stress and strain based conventional approaches promoting successive advances in their complexity and facilitating its application for the development of the fatigue software program ProFatigue.
- The ProFatigue program allows estimation of the model parameters and, accordingly, to define analytically an expression of the S-N and ε-N field to be applied subsequently in the application of the stress based and strain based approaches in fatigue design of structures and mechanical components.
- Further extensions or improvements of the model can be envisaged allowing successive advances in its complexity, as for instance, for more complex fatigue cases as in UniGrow model, multiaxial, thermomechanical, fretting, etc. for any appropriate experimental fatigue data set to be estimated, as well as for determination of confidence levels and optimization of fatigue program planning or probabilistic damage assessment in practical fatigue design.

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