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# Conceptual Model-Based Approach to Teaching Multiplication and Division Word-Problem Solving to A Student with Autism Spectrum Disorder

Irene Polo-Blanco and Steven Van Vaerenbergh
Universidad de Cantabria

Alicia Bruno
Universidad de La Laguna

María J. González
Universidad de Cantabria

*Abstract*: *Conceptual model-based problem solving (COMPS) was tested for its efficacy in teaching a student diagnosed with autism spectrum disorder to solve word problems involving multiplication and division. A single-case, multiple-baseline across behaviors design was conducted. The ability to solve each of three types of multiplication problems examined (equal groups, multiplicative comparison and Cartesian product) was addressed separately. The student's performance improved in all three, and it was maintained five weeks after the intervention. The student also generalized the effects of instruction to two-step (addition and multiplication) word problems. Knowledge transfer to an everyday situation was also assessed. The implications of these findings for teaching multiplicative word problems to students diagnosed with autism spectrum disorder are discussed.*

Problem solving is an essential feature of mathematics. The cognitive skills involved include working memory and executive functions such as inhibition and mental flexibility (Lee et al., 2009). Students with deficits in these skills may exhibit scant organizational skills, inability to concentrate, low motivation and difficulties in carrying a strategy through to completion or finishing a task (Swanson et al., 2001). Some of those deficits are characteristic of people diagnosed with autism spectrum disorder (ASD) and may interfere with development of the skills needed to solve mathematical problems (Happe et al., 2006). These students' executive dysfunction (Ozonoff & Schetter, 2007) or frequent shortcomings in understanding the vocabulary or the real situation to which a problem refers, for instance, are related to difficulties in identifying the arithmetic operations required to solve the problem or in performing the tasks needed to find the solution (Bae et al., 2015).

With a view to improving the problem-solving abilities of students with ASD, recent studies have assessed the efficacy of the methods used to teach such students (Gevarter et al., 2016). Most focus on simple arithmetic word problems involving addition and subtraction (Desmarais et al., 2019; Rockwell et al., 2011; Root et al., 2017; Xin, 2019). Very little research has been reported, however, on the effectiveness of those methods for teaching students with ASD to solve problems calling for multiplication and division. Whitby (2012), for instance, applied 'Solve it!' methodology to improve the problem-solving skills of secondary school students with ASD. Delisio, Bukaty and Taylor (2018), in turn, also reported good results when teaching ASD-diagnosed students to use a graphic organizer to solve word problems in which they had to multiply to draw comparisons or divide to calculate rates.

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Correspondence concerning this article should be addressed to Irene Polo-Blanco, Departamento Matemáticas Estadística y Computación, Facultad de Ciencias, Universidad de Cantabria, Avda de los Castros s/n, 39005, Santander (SPAIN). E-mail: irene.polo@unican.es

Research interest on teaching multiplication to students with mathematics learning disabilities has been growing in recent years. Alghamdi et al. (2020), for instance, successfully deployed schema-based strategy instruction (SBI) to teach three students with mathematics learning disabilities in the fifth-grade how to broach multiplication problems. Pivotal to the present study is the research conducted by Xin et al. (Xin, 2012, 2019; Xin et al., 2008; Zhang et al., 2014), who proposed using conceptual model-based problem solving (COMPS) to teach students with learning disabilities to solve arithmetic word problems.

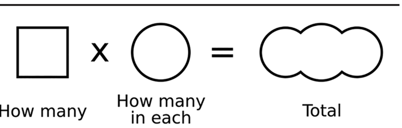
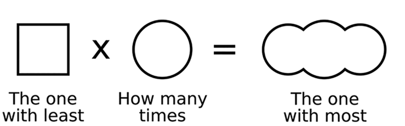
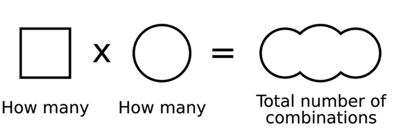
COMPS methodology deploys equation-like, conceptual model diagrams to improve students' problem-solving competence. Such diagrams help students recognize the multiplicative relationship in equations of the type: factor × factor = product. The diagrams illustrated in Table 1 refer to the three types of one-step multiplication problems (Vergnaud, 1983) used in this study: (1) equal groups (EG) problems establish proportionality between two measurements; (2) multiplicative compare (MC) problems define a relationship between quantities on a scale defined by the word 'times'; and (3) Cartesian product problems combine two measurements to generate a third (Table 1).

COMPS methodology is a two-stage process. In the first introductory stage, story grammars without unknowns are used to teach students to identify problem types and represent the quantities and relationships found in each using conceptual model diagrams. The problems (with unknowns) are introduced in the second stage. The teacher interacts with students by asking a series of questions about the situation described in the problem to help them focus on the key characteristics of each problem type. In a rate problem, for instance, the questions could be 'which part of the problem mentions the value of each unit? which the number of units? and which is the total or product?' Depending on problem structure, students position the numerical data on the conceptual model diagram, which helps them choose the operation needed to solve the problem. All the foregoing is supported by representation in the form of bar graphs or situational models illustrating the situation depicted in the problem for readier transition to the more symbolic conceptual model. Further support is provided by a cognitive heuristic DOTS (detect, organize, transform, solve) checklist to complete the problem-solving process (Xin, 2012).

COMPS methodology has yielded satisfactory results in helping students with learning disabilities (LD) solve multiplication word problems. Xin et al. (2008), for instance, examined the efficacy of word problem story

**TABLE 1**

**Conceptual Model Diagrams and Types of Multiplication Problems**

| *Problem Type* | *Sample Word Problem* | *Conceptual Model Diagram* |
|---|---|---|
| Equal groups (EG) | I have 6 pieces of candy in each bag. If I have 3 bags, how many pieces of candy do I have? |  |
| Multiplicative comparison (MC) | Luis has 6 pieces of candy. Pedro has 3 times more candy than Luis. How many pieces of candy does Pedro have? |  |
| Cartesian product (CP) | I have 4 T-shirts and 2 slacks. How many different combinations can I wear in all? |  |

grammar in helping five students with LD in the fourth and fifth grades represent problems in conceptual model equations. Their results showed that the model not only improved problem solving, but also fostered the learning of pre-algebraic concepts. Xin et al. (2011) compared COMPS-based to general heuristic problem-solving instruction to teach elementary school students with learning disabilities to solve multiplication word problems. They concluded that only the COMPS group's performance improved significantly between the pre- and post-tests in solving equal group and multiplicative compare problems and in the pre-algebra model expression test. Recently Xin et al. (2020) upgraded COMPS methodology to include online tutoring, which significantly improved the performance of the three participants in their study of equal group and multiplicative compare problems.

The present authors are unaware of the use of COMPS methodology to teach problem solving to students with ASD, in particular as regards multiplication problems, even though subjects with that disorder may benefit from such methodology, for the characteristics of ASD are deemed to be related to language comprehension difficulties and executive functional deficits. The assumptions adopted include: 1) teacher-student interaction via questions about story grammars helps the latter understand the situations described in problems; 2) the use of diagrams favors the use of mathematical vocabulary while affording visual support for understanding the mathematical operation that relates the quantities set out in the problem, and 3) the use of a checklist enumerating the problem-solving stages will lead to better planning and aid decision-making.

Further progress is required to determine the methodological adaptations that may enhance the aptitudes of students with ASD, such as their visual reasoning skills, and how to help them overcome word comprehension difficulties or executive functional deficits when solving problems involving multiplication. More specifically, despite their importance, multiplicative Cartesian product problems have not been addressed in any of the COMPS-based studies of which the present authors are aware. Lastly, the progress of students with ASD must be assessed not only in academic terms, but also in connection with the skills acquired to conduct their everyday lives independently (Bennett & Dukes, 2014; Kasap & Ergenekon, 2017). Learning to handle multiplication problems is particularly useful to enable ASD-diagnosed students understand frequently encountered mathematical concepts such as rate, proportion or slope (Bouck et al., 2018). Hence, the importance of determining whether their command of multiplication is transferred to real-life situations.

In light of the foregoing, this article describes a COMPS methodology-based intervention designed to help a 14-year-old ASD-diagnosed student improve his ability to solve one-step multiplicative rate, compare and Cartesian product problems. The specific research questions posed were: (1) Is the COMPS approach effective in teaching multiplication and division problems to a student with ASD? (2) Will improvement in problem-solving performance be generalized to two-step problems? (3) Will improvement in problem-solving transfer to real-life situations? and (4) Will improvement in problem-solving performance be maintained over time?

## Method

A single-case, multiple-baseline across behaviors design (Horner & Baer, 1978) was applied to assess the effectiveness of COMPS instruction in improving the problem-solving performance of a student diagnosed with ASD. As in similar studies (Rockwell et al., 2011), the three target behaviors correspond to the student¨s solving of each of the three types of multiplicative problems (Table 1). The aim was to identify a functional relationship between the intervention and the improvement in the student's performance.

### *Participant*

At the time of the experience, the subject, Peter (a pseudonym), was a 14-year and 4-month-old male who had been diagnosed with ASD at the age of 6. His DSM-IV criteria-based clinical assessment revealed no co-morbidity. Rated as severely autistic on the Childhood Autism Rating Scale (Schopler et al., 1988), Peter exhibited a wide repertoire of stereotyped behaviors, a propensity toward repetitive conduct and special interest in certain issues. He was enrolled in a mainstream school until he was 10, with significant individual curricular adaptation in his mother tongue and mathematics. In the interim, he has been enrolled in special education. According to the school counselor's latest report, his social and emotional behavior was very closely aligned with that of his peers, although he showed scant interest in play or establishing relationships. He reacted well to established routine and felt at ease when he able foresee what was to happen next. Stagnation of his cognitive and emotional development had affected his ability to process symbols. He was also diagnosed with intellectual disability, with an IQ of 54 on the Wechsler Intelligence Scale for Children (WISC-V; Wechsler, 2015).

He devoted four hours weekly to mathematics under an adapted curriculum. Thanks to his good reading comprehension he could read and understand problems, although he stumbles over certain words. Prior to the experience, Peter had learned to add, to distinguish between addition and subtraction and to perform those operations using memorized numerical facts. In his introductory multiplication training, the operation was defined as reiterated addition. He was also able to solve some multiplication problems with drawings and counting. He had not memorized the multiplication table or received formal instruction on multiplication or division algorithms. He was able to understand some equal group (EG) multiplication word problems, which on occasion he solved successfully using informal strategies. The timing was therefore deemed right for Peter to learn to solve all three types of multiplication and division problems.

### *Dependent and Independent Variables*

The dependent variable was the student's ability to successfully solve three types of multiplication and division word problems: equal group (EG), multiplicative compare (MC) and Cartesian product (CP). Two indicators were used to measure performance: explicit identification of the arithmetic operation needed to solve the problem (distinguishing whether the values should be multiplied or divided) and correct performance of the operation to find the right numerical answer. The independent variable was the COMPS-based problem-solving intervention conducted for each type of problem, described in detail in the section on procedure.

### *Design and Data Collection*

The study was conducted in the first semester as a weekly extracurricular activity. It consisted of a total of 31 sessions. The instructor who conducted the study had engaged in teaching for over 20 years and had experience teaching mathematics to students with ASD. The five experimental stages defined were: baseline, training (for the three types of problem), follow-up, maintenance (5 weeks after training) and generalization (application of the knowledge acquired to two-step problems). Two final (and additional) sessions were held in a domestic context to determine whether the knowledge acquired would be transferred to a real-life situation.

In the baseline stage, Peter's performance in the three types of one-step problems (EG, MC and CP) was assessed, along with his ability to generalize to two-step problems, such as 'Ana bought 4 bags of jelly beans for her birthday party. Each bag had 3 strawberry-flavored beans and 2 lemon-flavored beans. How many jelly beans did she buy altogether?'

Training sessions were not begun until the baseline steadied, i.e., when the score for probes was the same in at least two consecutive baseline sessions. EG problems were the first to be addressed. Instruction with story grammars was followed by the introduction of multiplication and then division problems, and the subsequent interchange of the two types of operation. Once Peter had mastered EG problems (100% success in at least two consecutive sessions), EG follow-up was undertaken before proceeding to MC or CP problem training. When improvement in EG problems performance was observed, and the baseline steadied in MC problems, MC instruction was begun, following the same sequence as in EG. After that, the same procedure was applied for CP problems. When all three types of problem reached the follow-up stage, one final follow-up session was held in an untrained setting. In that session, Peter solved a probe independently in the classroom led by the school's usual tutor.

Five weeks after training was concluded, maintenance in the three types of problems was assessed, followed by a session to evaluate generalization via application to two-step problems.

The study ended with assessment of transfer of the knowledge acquired to a real-life situation. Specifically, the student applied an MC strategy in a domestic environment, doubling the ingredients for one dessert recipe and dividing the quantities by three in another.

*Probes and Scoring*

Assessment during the baseline stage, upon conclusion of training in each type of problem and during maintenance was based on problem-solving with probes consisting of six multiplication and division problems adapted from Mulligan and Mitchelmore (1997): two involved EG, two MC and two CP, with one multiplication and one division problem per type. During the training stage, each session ended with a probe containing four problems of the type addressed in the session. The highest score for each problem was two: one for correctly identifying the operation and one for correctly performing the operation.

The two-step baseline and generalization stage problems were designed to cover all three types of problems, EG, MC and CP. The highest score for these problems was three: one for correctly identifying each operation and one for finding the right answer.

The real-life sessions were videorecorded and assessed qualitatively in terms of the student's reasoning. The recipes as rewritten by Peter were also used as a tangible record of his calculations.

*Procedure*

*Baseline.* The student received a probe with six problems covering all three types of problems in each baseline session. He was told he had to solve them alone and encouraged to do his best. Help for problems was confined to orally providing the meaning of any words he was unfamiliar with.

*Training.* The COMPS approach entailed direct instruction, consisting of teacher modeling, guided practice and continuous teacher feedback (Engelmann, 1980). The two stages of COMPS intervention (story grammar without and problems with unknowns) were implemented.

*Stage 1. Story Grammar.* The first training session for each type of problem was devoted to working with the respective story grammar. These sessions included representation (diagrams and drawings) for readier transitioning to the COMPS conceptual model diagram (Xin, 2012). The types of representation used in each problem type by the instructor to guide the student when transitioning to such schematic diagrams are listed in Table 2. For instance, for the EG problem in which the diagram contained the phrase 'how many x how many in each = total', the word-problem story grammar questions and instructions posed by the instructor were: 'How many cases do I have? Write that number in the box'; 'How many pencils can I put in each case? Write that number in the circle'. 'How many pencils do I have in all? Write that number in the cloud'.

*Stage 2. Problems.* The worksheets furnished during the training sessions were divided into sections as defined in the DOTS (detect, organize, transform, solve) checklist (Xin, 2012), as follows:

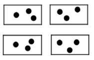
1. Problem story grammar and conceptual model diagrams (detect and organize). The student read the problem alone but received guidance in the form of the word-problem story grammar questions as he positioned the numbers on the diagram. If in the initial sessions the student encountered comprehension difficulties, specific diagrams representing the problem of the type used in the story grammar sessions (Table 2) were provided.
2. Operation. Using the conceptual model diagram, the student was asked to write in the operation needed to solve the problem (transform).
3. Solution. The student explicitly wrote down the equation for the operation to be

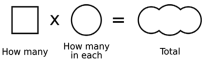
**TABLE 2**

**Story Grammar Components in COMPS Instruction by Type of Problem**

*Equal Group*

I have four 4 cases. I can put 3 pencils in each case. I have 12 pencils in all.

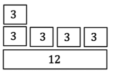


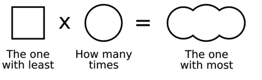



Word-problem story grammar questions: 'How many cases do I have? Write that number in the box.' '¿How many pencils can I put in each case? Write that number in the circle.' 'How many pencils do I have in all? Write that number in the cloud.'

*Comparison*

Pedro has 3 pieces of candy and Steven 4 times more than Pedro. Steven has 12 pieces of candy.

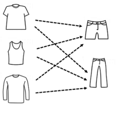


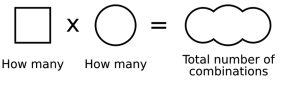




Word-problem story grammar questions: 'Who has the least? Pedro: Write the number Pedro has in the box.' 'Who has the most? Steven. Write the number Steven has in the cloud.' 'How many times more pieces does Steven have than Pedro? Write that number in the circle.'

*Cartesian Product*

I have 3 T-shirts and 2 slacks. If I wear one T-shirt and one pair of slacks every time I dress, I can dress 6 different ways.






Word-problem story grammar questions: 'How many T-shirts are there? Write that number in the box.' 'How many slacks? Write that number in the circle.' 'How many different combinations are there? Count the arrows and write the total number of arrows in the cloud. That's the total number of possible combinations.'

performed and solved for the unknown quantity (solve).

Given the student's abstract language comprehension difficulties, the unknown quantity was not represented by a letter as proposed in the COMPS model (Xin, 2012). Rather, the respective item on the diagram was left empty.

*Maintenance.* Five weeks after the last training session, the student was asked to answer a probe unaided to assess whether the knowledge acquired was retained over time.

*Generalization to Two-Step Problems.* The maintenance session was followed by a session in which the student received a probe that he was to solve unaided. The two-step problem comprising the probe was equivalent to the one used to establish his baseline skills.

*Transfer to Real-Life Situations.* Inspired by an earlier study by other authors (Patton et al., 1997), the functional problem posed entailed adapting recipes for desserts, to engage the student in an activity he enjoyed. Two extracurricular sessions were held in the student's own home. A teacher specializing in students with disabilities who knew Peter supervised the operation. In the first session, he was given a recipe for chocolate cookies with a list of ingredients for one person and asked to make cookies for two. In the second, he was shown a recipe to make three sponge cakes, which he was to adapt to make just one. In both sessions, the student was given the original recipe in writing and told to write down the new recipe before starting to make the dessert. The teacher guided him, step by step and with each ingredient, using MC problem language (such as 'if for 3 cakes we need 6 eggs, how many eggs do we need for one?') to ask him to tell her the new quantity. In other words, the student had to answer a multiplicative compare question for each ingredient in each recipe, multiplying in the first session and dividing in the second.

### *Reliability*

All the sessions were videorecorded. Interobserver reliability data were collected during the baseline, instruction, generalization and maintenance stages. A pre-service education graduate who had no knowledge of the research assumptions re-coded 30% of the probes in each stage. Interobserver agreement was calculated by dividing the number of agreements by the number of agreements plus disagreements and multiplying times 100. Interobserver reliability was 100% during the baseline, 97% during the instruction, 97% during the follow-up, 100% during the maintenance and 94% during the generalization stages. Mean interobserver reliability across all five stages was 97% for all three types of problems (EG, MC and CP).

Procedural reliability measured instructor conduct against the behavior specified, in other words, whether he: (1) posed the number of problems and used the quantities stipulated; (2) furnished the session material as stipulated; (3) allowed the student to solve the problems independently; (4) followed the problem-solving protocol; (5) highlighted the key features of each problem type; and (6) congratulated the student and/or rewarded him with words of encouragement after he solved the problem. The aforementioned pre-service education graduate assessed procedural reliability based on the videos of the instruction sessions. Calculated by dividing the number of teacher behaviors observed in 33% of the instruction sessions by the number of behaviors stipulated and multiplying times 100, procedural reliability was found to be 100% across the three problem types.

### *Social Validity*

One of the researchers interviewed the instructor weekly during the training stage to better coordinate and monitor training. The meetings addressed COMPS intervention program items such as student acceptance of the methodology and possible adaptations to overcome the difficulties observed. Minutes of the weekly interviews recorded the items discussed, which were then used to design subsequent training sessions.

Upon conclusion of the study, the instructor filled in an open-response questionnaire to evaluate: (1) COMPS program utility for teaching the student to solve problems; (2) the student's attitude and motivation throughout; (3) the applicability of the methodology for teaching other students with disabilities; and (4) the instructor's satisfaction with the results and areas in need of improvement.

The transfer to real-life situation instructor's opinions were also recorded. She was asked to evaluate the experience and the utility of engaging in similar activities with students with limitations comparable to those of the subject of the present case study.

## Results

The results for the baseline, training, follow-up, maintenance and generalization stages are summarized in Figure 1. Intervention efficacy was assessed on the grounds of visual analysis of inter-stage changes and intra-stage variability in the dependent variable.

### *Baseline*

Peter's performance during the baseline stage was low and steady with no visible trend. His success rate in all the EG sessions was consistently 25%. More specifically, he found the right solution to the multiplication problems but was unable to correctly identify the operation. For the division problems, he neither identified the operation nor solved the problem correctly. Peter's initial scores for the MC

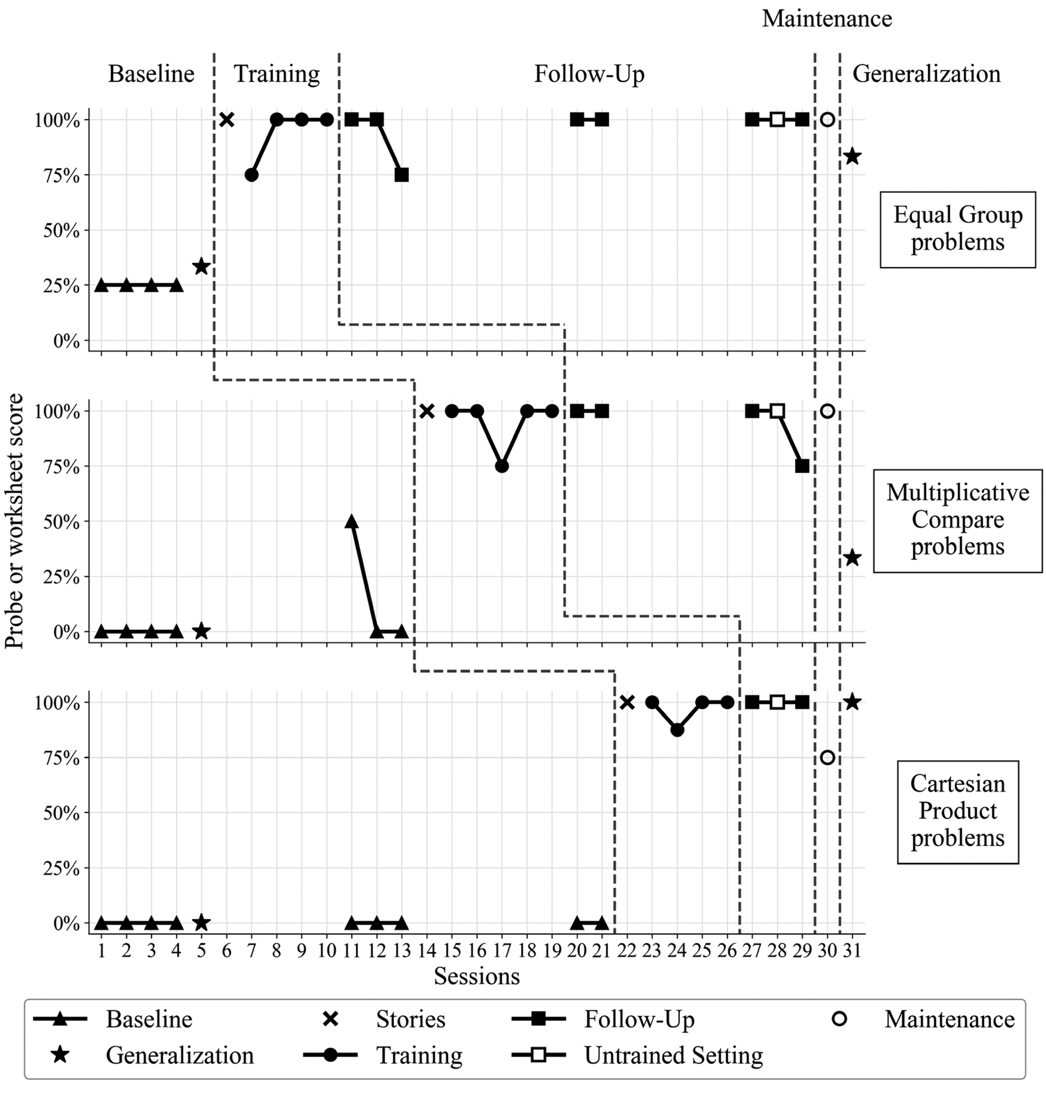



**Figure 1. Results for each problem type.**

problems were very low. In the first four sessions, he answered all the MC problems incorrectly (0% success rate in every session). In the fifth session, his score was 50%, after correctly solving one of the problems. In the next two sessions, however, the score dropped back to 0%. Peter steadily scored 0% in the PC problems during the baseline stage.

His baseline generalization skills in the two-step problems were assessed in session 5. He earned a score of 33% in EG generalization problems, correctly solving the additive step but not the multiplicative step. His score for the other two-step problems (MC and CP) was a consistent 0%.

*Training*

Peter's score rose immediately and fairly steadily to 100% in all types of problems during the training stage. The only EG problem he answered incorrectly was in session 7, where he mistook multiplication for addition. He committed only one (finger counting) error in the MC problems during the training stage, in session 17. His training stage score for the CP problems was 100% in all but session 24, where he calculated the result correctly but identified the operation incorrectly (division instead of multiplication).

*Follow-Up*

Peter's follow-up stage performance was fairly stable. His EG score was 100% in all but session 13, where he divided incorrectly, although his score went back up to 100% in the following sessions. He started out with a steady score of 100% for the MC problems in the early follow-up stage, which nonetheless tended slightly downward in the last session. Specifically, in session 29 he correctly identified the operation involved in an MC problem, but then committed an error when dividing. He replied correctly to all the CP problems in the follow-up stage, consistently scoring 100%.

*Maintenance*

During maintenance (session 30), Peter's performance in EG problems was consistent with his 100% training and follow-up scores. He also scored 100% for the MC problems, correcting the downward trend identified in the last follow-up session. The CP problem results in the maintenance stage were a high 75%, although slightly lower than the follow-up score: the reason was that he mistook division for multiplication in one of the problems.

*Generalization to Two-Step Problems*

The generalization probe results (session 31) were higher than recorded during the baseline stage in all three types of problems and high in all the EG (75%) and CP (100%) types. The MC problem results were better than during baseline, but the success rate was just 30%.

*Transfer to Real-Life Situations*

Peter successfully rewrote the quantities of the ingredients for both recipes during the session involving knowledge transfer to a real-life situation, correctly specifying the amounts both when multiplying and dividing. He occasionally needed the instructor to repeat the question using words similar to those used in the problems, such as 'double' and 'triple'. In addition, his overall attitude and concentration were good, for he was highly motivated by the activity, which he enjoyed.

*Social Validity*

The instructor expressed his satisfaction with the method, previously unknown to him, when answering the post-experience questionnaire. In his opinion, the COMPS intervention had helped the student improve his multiplication problem-solving skills and raised his self-confidence when broaching such problems. He deemed the visual representation used as support in the CP problems to be particularly useful. He felt that emphasis based on visual representation of the problem (with trees or graphs, for instance) had helped Peter understand the situation and that thanks to this method he had been able to combine visual representation with an algebraic solution to the problem. He also pointed out that the student had satisfactorily generalized his knowledge by applying it to two-step problems after being taught to deal with single operation problems only. In his opinion, if Peter had been briefly trained to solve two-step situations with a method analogous to the one used for one operation situation instruction, his performance in the former could well have been highly satisfactory.

He observed that although the student showed scant interest in the experience at first, as he developed the skills needed to solve different types of problems his motivation and self-confidence rose substantially. One of the strengths of the method identified by the instructor was the utility for the student of synopsizing and diagramming information drawn from the text, which helped him understand the problem and the meaning of each data item. He added that even in MC problems where the student did not spontaneously use visual representation, the COMPS approach had helped him understand them. He felt that including this type of methodology in the instruction delivered to students with similar limitations would be very beneficial.

The teacher who conducted the transfer to real-life situation sessions also expressed satisfaction with the experience. She found using mathematics in activities that appealed to the student, such as cooking in Peter's case, to be a promising strategy. Furthermore, she believed, on the grounds of her own experience with people with functional diversity, that applying mathematics in activities that favor independence, such as cooking, shopping and handling money, was particularly beneficial. She deemed the results satisfactory, for the activity proceeded very smoothly and Peter spontaneously showed that he understood the changes made in the recipes and how to apply them to make the respective desserts. She added that a wider array of similar activities entailing academic content could be used to favor the independence of people with disabilities.

### Discussion and Conclusions

A functional relationship was observed between COMPS instruction and improvement in a student's ability to solve EG-, MC- and CP-type multiplication and division problems. His low initial baseline performance in all problem types rose rapidly to 100% from the time COMPS training was introduced and remained over 75% during the follow-up and maintenance stages. The student's rare errors in the latter sessions were due primarily to performing the operation incorrectly, rather than to choosing the wrong one, suggesting the need for more procedural drill. The improvement observed can therefore be attributed to COMPS methodology.

The student embraced that methodology naturally, with higher levels of concentration and interest when problem-solving training began than during the baseline sessions. He was initially able to understand only EG multiplication problems using drawings and counting. Once COMPS training began, however, he could distinguish when to multiply and when to divide in such problems. His high performance with CP problems was particularly striking, given the lack of understanding he exhibited in the baseline sessions. Once training in such problems was introduced, the situation drawings and conceptual model diagrams furnished by the instructor helped him understand the situation and identify the operation needed to solve the problem. Although studies with typically developing students have concluded that CP problems are more difficult (Mulligan & Mitchelmore, 1997; Nesher, 1992), the student in the present case study exhibited no such difficulty after training.

Peter had considerable trouble understanding MC problems in the baseline sessions. Although his performance improved substantially after COMPS instruction, he needed one full session more than in the other types of problems to master MC (mastery =100% success in mixed problems). Contrary to the way he proceeded with the other types of problems, in MC he used only the conceptual model diagrams, not the drawings furnished during instruction. That enabled him to identify the operation required to successfully solve those problems, although his failure to use drawings to represent the situation, as he did with the EG and CP problems, might be an indication of difficulties in understanding MC problems. The origin of such obstacles might lie in understanding keywords in such problems ('double', 'times more than'), a finding reported by other authors (Stern, 1993) and associated with language comprehension difficulties characteristic of ASD (Alderson-Day, 2014; Jones el al., 2009; Polo-Blanco et al., 2019, 2021).

The student was able to generalize what he learned, applying the knowledge to two-step problems, although he exhibited less improvement in the MC than in the other two types. Such behavior is consistent with the observations during the one-step training sessions, where he exhibited greater comprehension difficulties around multiplicative compare problems that he might have carried over to the two-step situations. The results suggest in any event that the student could readily improve his command of more complex problems if he were exposed to specific COMPS instruction in that regard.

The findings of the present study are in line with those of other authors' assessments of the effectiveness of the COMPS approach for students with mathematics learning disabilities (Xin, 2012; Xin et al., 2011; Xin et al., 2020). The present contribution consists of the provision of first-time evidence of the efficacy of COMPS methodology to help a student with ASD solve problems involving multiplication. Most prior research on students with ASD focus on addition (Kasap & Ergenekon, 2017; Rockwell et al., 2011). This study advances research on how students learn to deal with multiplication. The joint study of the three types of problems, including CP, often addressed in research on students characterized by typical development (Nesher, 1992) but absent where the subjects are students with learning disabilities, is another original feature of the present research. A further contribution is the finding that the skills acquired were also successfully transferred to real-life situations.

The COMPS approach relies heavily on representation in the form of conceptual model diagrams and specific drawings, guiding problem solving with story grammar questions and a detailed checklist. Those three features are particularly suitable in the context of ASD, in light of certain traits characteristic of subjects with the disorder, such as their visual processing skills, language comprehension difficulties and executive functioning deficits. The method also proved adaptable to Peter's specific needs for it envisages, for instance, explaining the meaning of a term unknown to the subject or suggesting the use of visual representations or manipulatives.

Given the single case design of the study, further research is consequently needed to replicate and assess the efficacy of COMPS methodology in helping other students with similar characteristics learn to solve multiplication problems.

A number of studies have revealed the need to discover effective mathematics teaching practice for students with ASD (see, for example, Gevarter et al., 2016). That need exists not only for addition in the early years of schooling, but also to enable students with ASD to understand more complex operations. In this regard, the present study shows that suitable intervention in the form of the COMPS approach, for instance, may help students with similar characteristics learn to solve fairly complex problems involving multiplication. The findings are therefore promising in terms of the breadth of the learning pathways open to students with severe limitations such as the subject of this study, diagnosed both for ASD and intellectual disability.

While acquiring academic skills is important, where students with intellectual disabilities are concerned the application of such skills to real-life functional situations is also essential (Bouck et al., 2018). Hence the need for reflection on how to significantly transfer skills acquired in an academic context to extracurricular situations and conduct studies to assess the efficacy of the respective approaches. Special education teachers should also devote time to teaching functional mathematics skills applicable to domestic, work and community situations. This study contributes to that aim, for it addressed the transfer of the skills acquired to a real-life context (cooking, in this case). The student understood the need to multiply or divide the quantities of the ingredients in recipes for two desserts, successfully performing the necessary operations in two separate sessions. Future research might pursue the present findings in deeper detail by studying other functional applications of mathematics.

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