Multimode fiber-based transmitter for free space optical communications

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ABSTRACT

The importance of free space optical links is increasing, for instance for the solution of the "last mile" network access. The influence of the atmosphere on the optical beam is one of the most relevant issues to be considered for the quality of the link. In this work we deal with optical wave distribution in a multimode fiber, which can be applied as a primary optical beam source in a transmitter for a free space optical link. The utilization of the fiber instead of a laser diode as the primary source of the optical beam presents certain advantages. First, the beam spot at the end of the fiber is circularly symmetrical and the optimal beam shape can be reached by a particular excitation of the fiber. Secondly, the head of the link can be based on a completely photonic concept, and as a consequence the electronic parts of the signal path of the head can be omitted. The optical intensity distribution and the complex degree of coherence at the end of the fiber are modeled. The results show that the optimal beam shape can be obtained in the transmitting fiber.

Keywords: Optical wireless links, optical beams, optical fibers, multimode fiber.

1. INTRODUCTION

Optical wireless links are thought to be relevant in the near future in the general field of communication systems. Satellite optical communications will also make great progress. Even mobile optical communication was already verified. On the contrary of the radiofrequency band, the optical band is free for atmospheric communications. The weather dependence of the links has been modeled and can be estimated to a certain extent. Optical wireless links are assumed to help in the solution of the "last mile" problem for network access.

The atmospheric influence on the optical beam provokes some disadvantages in optical wireless links. The availability of the terrestrial links depends strongly on the weather, and these links require line of sight between the transmitter and receiver. Optical beam scintillation also causes beam interruptions. Some methods can be applied for reliability improvement, such as multi-beam transmission, beam shaping, auto-tracking system, adaptive optics or polygonal topology.

This work is focused on the possibility of employing a transmitter based on a laser beam with a special distribution of optical intensity. One the beam distributions that could be of interest is called Top Hat beam. The Top Hat beam is an alternative to the Gaussian beam and presents certain advantages. The almost flat intensity distribution across the Top Hat beam profile ensures that mechanical vibrations of the transceiver consoles will cause only small power fluctuations in the received signal. In this work we propose the utilization of a multi-mode optical fiber as a primary source in a transmitting optical system. The influence of the optical fiber in the spatial distribution of light is analysed. Distribution of the optical intensity at the end of the fiber is studied, as a function of the fiber characteristics and the number of modes. The preliminary experimental results are in agreement with the simulation.

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2. WEAKLY GUIDED FIBER MODES

We assume that the optical fiber is a weakly guiding step-index fiber and that it is situated in a Cartesian coordinate system so that the axis of the fiber coincides with the z axis of the coordinate system. In this case the electric field of the individual modes that are propagated in the positive direction of the z-axis is described according to these equations:

- Modes with $l = 0$:

$$\begin{aligned}\mathbf{E}_{0,m}^{(1)}(r,\varphi,z) &= \mathbf{e}_{0,m}^{(1)} \exp\left(-i\beta_{0,m}^{(1)} z\right) = \mathbf{x}_0 \mathrm{F}_0(r) \exp\left(i\beta_{0,m}^{(1)} z\right),\\ \mathbf{E}_{0,m}^{(2)}(r,\varphi,z) &= \mathbf{e}_{0,m}^{(2)} \exp\left(-i\beta_{0,m}^{(2)} z\right) = \mathbf{y}_0 \mathrm{F}_0(r) \exp\left(i\beta_{0,m}^{(2)} z\right),\end{aligned} \tag{1a}$$

- Other modes with $l > 0$:

$$\begin{aligned}\mathbf{E}_{l,m}^{(1)}(r,\varphi,z) &= \mathbf{e}_{l,m}^{(1)}(r,\varphi) \exp\left(i\beta_{l,m}^{(1)} z\right) = \mathrm{F}_l(r)\left[\cos(l\varphi)\mathbf{x}_0 - \sin(l\varphi)\mathbf{y}_0\right] \exp\left(i\beta_{l,m}^{(1)} z\right),\\ \mathbf{E}_{l,m}^{(2)}(r,\varphi,z) &= \mathbf{e}_{l,m}^{(2)}(r,\varphi) \exp\left(i\beta_{l,m}^{(2)} z\right) = \mathrm{F}_l(r)\left[\cos(l\varphi)\mathbf{x}_0 + \sin(l\varphi)\mathbf{y}_0\right] \exp\left(i\beta_{l,m}^{(2)} z\right),\\ \mathbf{E}_{l,m}^{(3)}(r,\varphi,z) &= \mathbf{e}_{l,m}^{(3)}(r,\varphi) \exp\left(i\beta_{l,m}^{(3)} z\right) = \mathrm{F}_l(r)\left[\sin(l\varphi)\mathbf{x}_0 + \cos(l\varphi)\mathbf{y}_0\right] \exp\left(i\beta_{l,m}^{(3)} z\right),\\ \mathbf{E}_{l,m}^{(4)}(r,\varphi,z) &= \mathbf{e}_{l,m}^{(4)}(r,\varphi) \exp\left(i\beta_{l,m}^{(4)} z\right) = \mathrm{F}_l(r)\left[\sin(l\varphi)\mathbf{x}_0 - \cos(l\varphi)\mathbf{y}_0\right] \exp\left(i\beta_{l,m}^{(4)} z\right),\end{aligned} \tag{1b}$$

where

$$\mathrm{F}_l(r) = \begin{cases} \dfrac{\mathrm{J}_l\left(U_{l,m}\dfrac{r}{a}\right)}{\mathrm{J}_l\left(U_{l,m}\right)} & \text{for } r \le a,\\[2ex] \dfrac{\mathrm{K}_l\left(W_{l,m}\dfrac{r}{a}\right)}{\mathrm{K}_l\left(W_{l,m}\right)} & \text{for } r > a,\end{cases} \tag{2}$$

where J_l and K_l are Bessel functions of the first and second kind of the order l respectively, a is a core radius, $U_{l,m}$ and $W_{l,m}$ are modal parameters for the core and cladding, $\beta_{l,m}^{(q)}$ is the propagation constant, r, φ, z are coordinates in the cylindrical system and i is the imaginary unit. The symbol $\mathbf{E}_{l,m}^{(q)}$ stands for a complex representation of a real monochromatic electrical vector with angular frequency ω (the multiplicative factor $\exp(-i\omega t)$ in (1a) and (1b) was omitted for clarity). $\mathbf{e}_{l,m}^{(q)}(r,\varphi)$ is the complex amplitude of the q mode and $\mathbf{x}_0$ and $\mathbf{y}_0$ are the unit vectors of the Cartesian system.

According to equations (1b), there is a group of four modes for every modal numbers l and m for $l > 0$. We distinguished these modes by a superscript (q). If $l = 0$ then there are only two modes for modal numbers l and m. The propagation constant $\beta_{l,m}^{(q)}$ and the group velocity $g_{l,m}^{(q)}$ of individual modes can be expressed as[1]:

$$\beta_{l,m}^{(q)} = \beta_{l,m} + \delta\beta_{l,m}^{(q)}, \quad g_{l,m}^{(q)} = g_{l,m} + \delta g_{l,m}^{(q)}, \tag{3}$$

where $\beta_{l,m}$ is a scalar propagation constant (the solution of the scalar characteristic equation), $\delta\beta_{l,m}^{(q)}$ is the polarization correction, $g_{l,m}$ is the group velocity derived from a purely scalar treatment, and $\delta g_{l,m}^{(q)}$ is the polarization correction of the group velocity.

The scalar propagation constant is the same for modes that differ only by the index q, but polarization corrections may be different for different q. Furthermore the modal parameters $U_{l,m}$ and $W_{l,m}$ are also different for every mode and

they should be denoted also by the superscript q. However we didn't take into account the polarization corrections for modal parameters, because their influence is negligible.
The notation of modes by the subscripts l, m and the superscript (q) is often undesired in a mathematical formula due to its complexity. Therefore we introduce a single subscript j for every mode, so the j-th mode is marked as $\mathbf{E}_j$. We are also indexing by the subscript j parameters that belong to the individual mode, for example propagation constant β_j, group velocity g_j, etc.

In our case we must consider a vector field rather than a scalar field. According to equations (1) we must use the theory that enables the analysis of vectorial fields. The pure scalar description of the coherence phenomena in the optical fiber is described in[2,3]. For a description of the electromagnetic beam the cross-spectral density matrix $\underline{\underline{\mathbf{W}}}$[4,5,6] was introduced. The cross-spectral density describes the coherence phenomena in the space-frequency domain. The description in the space-time domain is possible by means of the mutual coherence matrix $\underline{\underline{\Gamma}}(\mathbf{R}_1,\mathbf{R}_2,\tau)$[7,8,9,10]:

$$\underline{\underline{\Gamma}}(\mathbf{R}_1,\mathbf{R}_2,\tau)=\begin{bmatrix}\Gamma_{xx}(\mathbf{R}_1,\mathbf{R}_2,\tau) & \Gamma_{xy}(\mathbf{R}_1,\mathbf{R}_2,\tau)\\ \Gamma_{yx}(\mathbf{R}_1,\mathbf{R}_2,\tau) & \Gamma_{yy}(\mathbf{R}_1,\mathbf{R}_2,\tau)\end{bmatrix},$$
$$\Gamma_{\alpha\beta}(\mathbf{R}_1,\mathbf{R}_2,\tau)=\left\langle E^*_\alpha(\mathbf{R}_1,t)E_\beta(\mathbf{R}_2,t+\tau)\right\rangle,\quad (\alpha,\beta=x,y), \tag{4}$$

where $\mathbf{E}(\mathbf{R},t)=\mathbf{x}_0E_x(\mathbf{R},t)+\mathbf{y}_0E_y(\mathbf{R},t)$ stands for the fasorial representation of the real electric vector. $\mathbf{R}_1$, $\mathbf{R}_2$ are the position vectors and τ is the time difference t_2-t_1 of field observation at points $\mathbf{R}_1$ and $\mathbf{R}_2$. The brackets $\langle\ \rangle$ denote the ensemble average. As we are assuming that the electric field is stationary and ergodic, these brackets mean also time averaging. The elements of these two matrices $\underline{\underline{\mathbf{W}}}$ and $\underline{\underline{\Gamma}}$ are connected by the Fourier transform (Wiener-Khinchin theorem)[5,6,11].
We can express the resultant electric field at the end of the optical fiber of length z as:

$$\mathbf{E}(\mathbf{r},z,t)=\sum_j\int_0^\infty a_j(\omega)\mathbf{e}_j(\mathbf{r},\omega)\exp\left[i\beta_j(\omega)z-i\omega t\right]\mathrm{d}\omega\,, \tag{5}$$

where the summation is meant for all guided modes, $a_j(\omega)$ is the modal weight of the j-th mode, $\mathbf{r}$ is the position vector in the cross-section of the fiber (in a plane $z=const.$) and $\mathbf{e}_j(\mathbf{r},\omega)$ represents the electric field vector of the j-th mode (see Eq. (1)), where the angular frequency dependence was not shown. If we substitute the expression (5) into (4), then we obtain the mutual coherence matrix with elements:

$$\Gamma_{\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2,z,\tau)=$$
$$=\sum_j\sum_k\int_0^\infty\int_0^\infty\left\langle a^*_j(\omega)a_k(\omega')\right\rangle e^*_{j,\alpha}(\mathbf{r}_1,\omega)e_{k,\beta}(\mathbf{r}_2,\omega')\times \tag{6}$$
$$\times\exp\left[i\omega'(t+\tau)-i\omega t\right]\exp\left\{i\left[\beta_k(\omega')-\beta_j(\omega)\right]z\right\}\mathrm{d}\omega\,\mathrm{d}\omega',$$

where $e_{j,\alpha}$ means the α-component of the j-th mode. It is possible to determine the modal weight $a_j(\omega)$ from the knowledge of the distribution of the electrical field at the input of optical fibre[1]:

$$a_k(\omega)=\frac{1}{N_k}\iint_S\mathbf{E}_s(\mathbf{r},0,\omega)\cdot\mathbf{e}^*_k(\mathbf{r},\omega)\,\mathrm{d}^2\mathbf{r},$$
$$N_k=\iint_S\mathbf{e}_k(\mathbf{r},\omega)\cdot\mathbf{e}^*_k(\mathbf{r},\omega)\,\mathrm{d}^2\mathbf{r}. \tag{7}$$

The symbol $\mathbf{E}_s(\mathbf{r},0,\omega)$ in (7) means the electric vector at the input of the optical fiber, so this vector is given by the exciting source. We integrate in equation (7) over the infinite cross-section of the optical fiber. The equation (7) is valid only for a weakly guiding fiber. If we substitute the expression for a_j and a_k of (7) into equation (6) we obtain:

$$\Gamma_{\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2,z,\tau)=\sum_j\sum_k\int_0^\infty e^*_{j,\alpha}(\mathbf{r}_1,\omega)G_{jk}(\omega)\,e_{k,\beta}(\mathbf{r}_2,\omega)\times$$
$$\times\exp(i\omega\tau)\exp\{i[\beta_k(\omega)-\beta_j(\omega)]z\}\mathrm{d}\omega, \tag{8}$$

where

$$G_{jk}(\omega)=\frac{\iint\limits_S\iint\limits_S \underline{\underline{\mathbf{e}}}_j(\mathbf{r}_1,\omega)\,\underline{\underline{\mathbf{W}}}_s(\mathbf{r}_1,\mathbf{r}_2,\omega)\,\underline{\underline{\mathbf{e}}}_k^{\mathrm{H}}(\mathbf{r}_2,\omega)\mathrm{d}^2\mathbf{r}_1\mathrm{d}^2\mathbf{r}_2}{N_jN_k}, \tag{9}$$

where $\underline{\underline{\mathbf{W}}}_s(\mathbf{r}_1,\mathbf{r}_2,\omega)$ is the cross-spectral density matrix at the input of the fiber, superscript $^{\mathrm{H}}$ means the transpose and conjugate matrix and we introduce the following matrix for j-th mode of the electric vector:

$$\underline{\underline{\mathbf{e}}}_j(\mathbf{r}_1,\omega)=\begin{bmatrix} e_{j,x}(\mathbf{r}_1,\omega) & e_{j,y}(\mathbf{r}_1,\omega)\end{bmatrix}. \tag{10}$$

For the derivation of equation (8) we assumed that the fiber is excited by a stationary and ergodic source.

Equation (8) is the desired expression for the mutual coherence matrix at the end of the optical fiber of length z. We simplify this result provided the exciting source is cross-spectrally pure, quasi-monochromatic and spatially coherent. In this case, the cross spectral density matrix of the source is given by:

$$\underline{\underline{\mathbf{W}}}_s(\mathbf{r}_1,\mathbf{r}_2,\omega)=\underline{\underline{\mathbf{J}}}_s(\mathbf{r}_1,\mathbf{r}_2)\mathrm{S}(\omega),$$
$$J_{\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2)=E^*_\alpha(\mathbf{r}_1)E_\beta(\mathbf{r}_2),\quad(\alpha,\beta=x,y), \tag{11}$$

where $\mathrm{S}(\omega)$ is the spectral density of the source and $\underline{\underline{\mathbf{J}}}_s(\mathbf{r}_1,\mathbf{r}_2)$ is the mutual intensity (or equal-time mutual coherence matrix) of the source. This is the case when the excitation of the optical fiber is made by a laser diode. If the source is quasi-monochromatic the spectral density $\mathrm{S}(\omega)$ is negligible, provided the angular frequency is not around the center frequency ω_0. For that reason we can integrate equation (8) only in the vicinity of the center frequency ω_0. Furthermore, due to the same reason, we ignore the frequency dependence of the electric vector of the individual modes, hence we can write:

$$\mathbf{e}_j(\mathbf{r},\omega)=\mathbf{e}_j(\mathbf{r},\omega_0)\equiv\mathbf{e}_j(\mathbf{r})\ \text{ for all } j. \tag{12}$$

The frequency dependence of the propagation constant cannot be ignored, so we expand the propagation constant in a Taylor series and then we consider only the first two terms

$$\beta_j(\omega)=\beta_j(\omega_0)+(\omega-\omega_0)\,g_j^{-1}(\omega_0)=\beta_j+(\omega-\omega_0)\,\tau_j, \tag{13}$$

where $\beta_j(\omega_0)\equiv\beta_j$, $\tau_j(\omega_0)\equiv\tau_j$ is the group delay of the j-th mode for a fiber of 1 meter length. Afterwards we take into account the natural shape of the spectral line of the laser diode. The spectral density of the source can be expressed as:

$$S(\omega) = \frac{\Delta\omega}{\pi\left[\Delta\omega^2 + (\omega - \omega_0)^2\right]}, \tag{14}$$

where $\Delta\omega$ is the width of the spectral line in the angular frequency domain. If we substitute equations (11), (13) and (14) into (8), we obtain:

$$\Gamma_{\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2,z,\tau) = \sum_j \sum_k a_j^* \, a_k \, e_{j,\alpha}^*(\mathbf{r}_1) \, e_{k,\beta}(\mathbf{r}_2) \times$$
$$\times \exp\left[i(\beta_k - \beta_j)z\right] \exp(-i\omega_0\tau) \left|\gamma_s(\tau + \Delta\tau_{jk} z)\right|, \tag{15}$$

where $a_j = a_j(\omega_0)$ for all j, and γ_s stands for the complex degree of temporal coherence of the source:

$$\gamma_s(\tau) = \int_0^\infty S(\omega)\exp(-i\omega\tau)\,d\omega = \exp\left(-|\tau|\,\tau_s^{-1} - i\omega_0\tau\right), \tag{16}$$

where τ_s is the coherence time of the exciting source, that is approximately equal to the reciprocal value of the spectral line bandwidth in the angular frequency domain $\Delta\omega$ [5,11].

3. OPTICAL INTENSITY AT THE END OF THE OPTICAL FIBER

If we know the mutual coherence matrix we can calculate the optical intensity according to:

$$I(\mathbf{r},z) = \mathrm{Tr}\left\{ \underline{\underline{\Gamma}}(\mathbf{r},\mathbf{r},z,0) \right\}, \tag{17}$$

where Tr means the trace of the matrix.

The circular symmetry of the optical intensity, i.e., the independence of the optical intensity on the angular variable, is one of the natural requirements. It is evident that the optical intensity has circular symmetry at the output aperture of the lens provided the optical intensity has also circular symmetry at the end of the optical fiber. The optical intensity at the end of the optical fiber can be expressed on the basis of (15) and (17), after some algebraic modifications, as:

$$I(\mathbf{r}) = \sum_j I_j(\mathbf{r}) + \sum_{\substack{j,k \\ j\neq k}} A_j A_k \, \mathbf{e}_j(\mathbf{r})\cdot\mathbf{e}_k(\mathbf{r}) \left| \gamma_s(d\,\Delta\tau_{jk}) \right| \cos\left[\alpha_j - \alpha_k + (\beta_j - \beta_k)d\right], \tag{18}$$

where d is the length of the fiber, A_j and α_j are the modulus and the phase of the modal weight a_j, I_j is the optical intensity of the individual modes and $\Delta\tau_{jk} = \tau_j - \tau_k$. The summations include all guided modes.

Equation (18) expresses the optical intensity as a sum of the optical intensity of the individual modes and some interference part (double summation). The optical intensity of each mode described by (1) is circularly symmetrical, so the potential circular non-symmetry of the resultant beam is caused by the interference part.

The distribution of the optical intensity is simulated for a step index optical fiber with core radius 4,7um, the refractive index of the core is 1,467 and the difference between core and cladding refractive index is 0,01. We considered a single mode laser source with center wavelength 1550nm. The non-coherent superposition of the guided modes on the last picture is shown to demonstrate the idea of the beam shaping. The optical intensity distribution in the case of three modes is shown in figures 1 and 2. We considered this specific distribution of the power between guided modes: 50% of the total power is launched into dominant mode LP_{01}, the mode LP_{11} brings 25% of the total guided power and LP_{21} brings 25% of the guided power again.

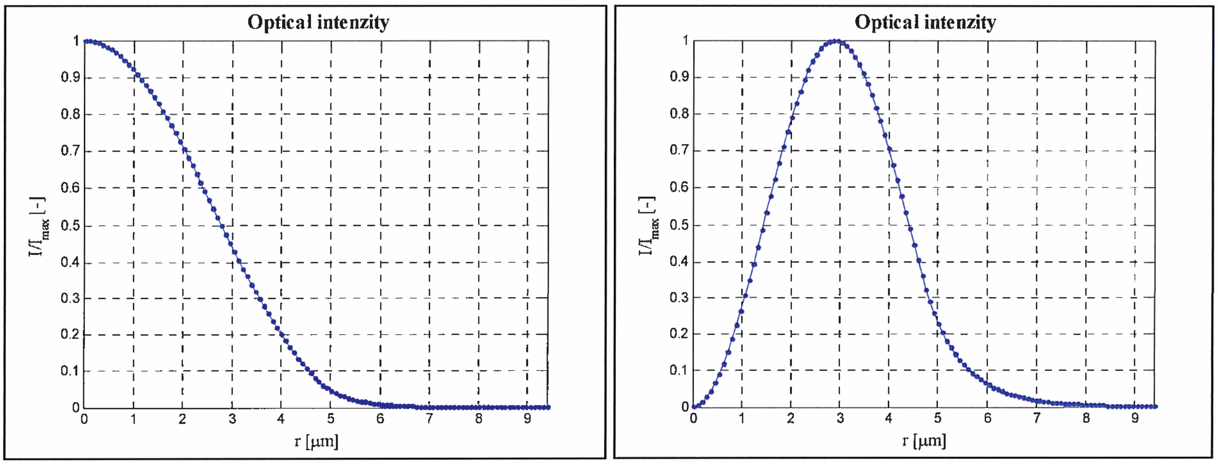

Fig. 1. Distribution of the normalized optical intensity at the end of the fiber for LP_{01} mode on the left side and for the mode LP_{11} on the right side, as a function of the radial coordinate.

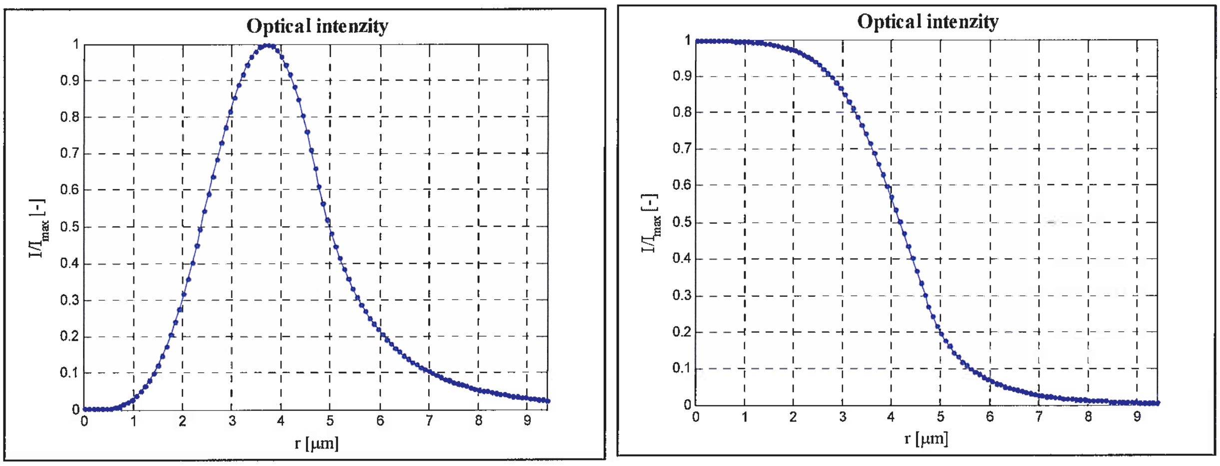

Fig. 2. Distribution of the normalized optical intensity for the mode LP_{21} (left picture), and of the optical intensity for non-coherent superposition of the modes LP_{01}, LP_{11} and LP_{21} (right picture), as a function of the radial coordinate. The power distribution between modes is: LP_{01} (50% of the total power); LP_{11} (25% of the total power); LP_{21} (25% of the total power).

4. CONCLUSIONS

We investigated the distribution of optical intensity and coherence properties of the light at the end of a weakly-guiding optical fibre. The transmitting fiber can be used for irradiation of a transmitting lens in the frame of a free space optical link transceiver. By this model the optimal length of the transmitting fiber can be calculated. The optical intensity distribution in the case of three modes was presented. The resulting expression for optical intensity distribution takes into account the mutual coherence between modes. The research presented shows how the optimal beam shape can be obtained already in the transmitting fiber.

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