

# Application of the Fast Fourier Transform and parametric frequency estimation for the measurement of the Bragg wavelength of interferometrically interrogated Fiber Bragg Grating sensors

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## ABSTRACT

A theoretical investigation on two techniques for the interrogation of fiber Bragg gratings (FBG) is reported. We compare two methods for decoding the Bragg wavelengths of gratings: the Fast Fourier Transform (FFT) processing and a parametric frequency estimation method called MUSIC. Results demonstrate that the second technique has advantages over the first one.

**Keywords:** Multiplexing and sensor network techniques, FFT transform, MUSIC, optical fiber sensors.

## 1. INTRODUCTION

Various techniques for interrogating Bragg grating sensors have been proposed in the last years<sup>1</sup>. Among them, the interferometric method has always been an attractive choice. This approach is suitable for high resolution and high-dynamic strain sensing. The peak reflectance wavelengths from arrays of fiber Bragg grating (FBG) sensors can be demultiplexed and measured by passing the light through an optical fiber Mach-Zehnder interferometer and processing the resulting interferogram to determine the absolute wavelength. In 1995, Davis and Kersey<sup>2</sup> proposed an interferometric scheme to detect the wavelength shifts from a series of Bragg grating sensors utilizing a scanning Michelson interferometer. In this scheme the interferometer was used as a Fourier transform spectrometer (FTS). They reported 15 pm wavelength resolution (for gratings with reflectance near 1500 nm) using an electrical spectrum analyzer to process interferograms formed from 10 cm optical path difference (OPD) scans. Flavin<sup>3</sup> applied Hilbert transform processing achieving a wavelength resolution of 5 pm for an OPD scan of just 1,2 mm. This technique was applied to the interrogation of multiplexed gratings. Rochford and Dyer<sup>4</sup> demonstrated that the Hilbert transform method was capable of accurately measuring wavelengths separated by 2 nm or less.

In this paper, we report two processing algorithms for the interferometric techniques of interrogation of fiber Bragg grating (FBG) sensors which yield absolute measurement of the Bragg wavelength: the Fast Fourier Transform (FFT) method and a novel interrogation technique based on parametric frequency estimation called MUSIC. We have determined that the second method is capable of accurately measuring wavelengths separated by less than 2 nm. Also, we discuss processing advantages and tradeoffs of both techniques. They can be useful for measuring a very large number of densely multiplexed wavelengths reflected by gratings illuminated by a broadband source.

## 2. THEORY

Most of the frequency estimation methods can be grouped into two classes: parametric or high resolution methods and non-parametric or periodogram-based methods. The high-resolution methods are able to resolve spectral peaks separated in frequency less than  $1/T$  ( $T$  being the observation time of the signal), which is the resolution limit for the methods based on the periodogram. To this class belong methods such as MUSIC.

On the other hand, the main advantage of classical periodogram-based methods, which employ FFT algorithms, is that they have a low computational cost and, therefore, can be efficiently implemented. However, in general, periodogram or FFT-based methods cannot resolve closely spaced frequencies.

To obtain high-accuracy estimates with FFT-based approaches we have to deal with two different kinds of problems: first, it is the problem of crosstalk due to interference among the sinusoidal components. Second, since the FFT is



evaluated in a grid of discrete frequencies, it introduces a bias in the frequency estimates. The classical method of zero-padding to mitigate this effect may be computationally expensive if high accuracy is required.

Several techniques have been proposed to overcome this problem. The simplest one is based on interpolating the FFT samples surrounding the true frequency. Interpolated FFT methods have the advantage that they are easy to implement and very fast. However, when the sinusoids are not well separated in frequency, their results are not very accurate.

A novel parametric method to examine the spectral content of a measured signal is reported. This parametric algorithm is called MUSIC, which assumes a known number of tones in the measured signal. The name MUSIC is an acronym for Multiple Signal Classification. It was developed by Schmidt<sup>5</sup>. The MUSIC algorithm estimates the pseudospectrum from a signal or a correlation matrix using Schmidt's eigenspace analysis method. The algorithm performs eigenspace analysis of the signal's correlation matrix in order to estimate the signal's frequency content. This algorithm is particularly suitable for signals that are the sum of sinusoids with additive white Gaussian noise. The MUSIC pseudospectrum estimate is given by the following expression:

$$p_{music}(f) = \frac{1}{e^H(f) \left( \sum_{k=p+1}^N v_k v_k^H \right) e(f)} = \frac{1}{\sum_{k=p+1}^N |v_k^H e(f)|^2} \quad (1)$$

where  $N$  is the dimension of the eigenvectors and  $v_k$  is the  $k$ -th eigenvector of the correlation matrix. The integer  $p$  is the dimension of the signal subspace, so the eigenvectors  $v_k$  used in the sum correspond to the smallest eigenvalues and thus belong to the noise subspace. The vector  $e(f)$  consists of complex exponentials that form the eigenvector of a sinusoid of frequency  $f$ . Taking into account the fact that all the eigenvectors of a matrix are orthogonal, then if a frequency sweep is carried out, whenever the frequency value matches one of the signal sinusoids the product  $v_k^H e(f)$  will be zero, and the pseudospectrum will present a pole.

### 3. EXPERIMENT

We compare two techniques for obtaining the Bragg wavelength of the considered gratings: the Fast Fourier Transform (FFT) processing and a novel parametric frequency estimation method called MUSIC. Towards this goal we present some simulation results to compare the performance of the two methods. Some important parameters of the system, such as the resolution and the minimum allowed signal to noise ratio, are going to be studied. The results obtained by using both methods will be compared. Also, we will analyze whether crosstalk is present when there are two multiplexed FBGs.

#### 3.1 Signal to Noise Ratio (SNR)

In this analysis a FBG transducer ( $\lambda_B = 1524$  nm) was used. The simulated signal had 20000 points, but the number of points of the FFT used was  $2048 \times 256 = 524288$ . This means that zero-padding was applied to the signal.

As shown in Fig. 1 the Bragg wavelength,  $\lambda_B$ , was detected by the FFT method when the SNR was varied from -20 dB to 20 dB, whereas MUSIC only detected the correct  $\lambda_B$  when the SNR was higher than 2 dB. This limitation comes directly

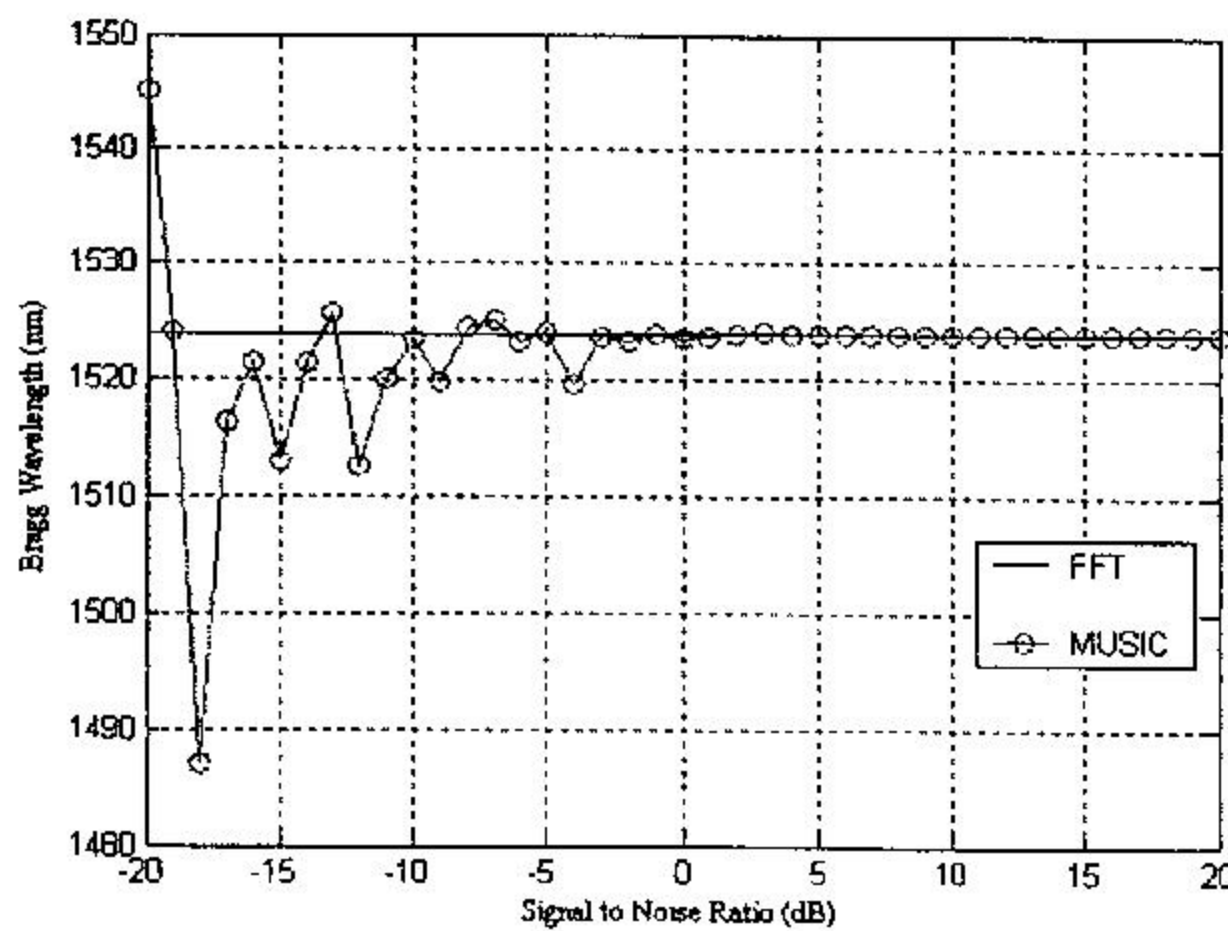


Figure 1: Bragg wavelength obtained with FFT and MUSIC

from the requirement, imposed by the algorithm, that the noise eigenvalues were the lower valued ones. However, this should be of no concern in a practical situation since the signals reflected by the FBGs will be higher than the noise.

The interpolation increases the number of points of the signal, so that the FFT is better defined. The Bragg wavelength detected with the FFT is 1524.005 nm, whereas the interpolated FFT gives 1523.925 nm. In this case, there is no great difference between the Bragg wavelengths obtained with and without interpolation. On the other hand, the interpolation does not affect MUSIC. However, the main drawback of interpolation is the higher computational cost. Although the results obtained in Fig. 1 with the FFT are better than those provided by MUSIC, it can be concluded that MUSIC is apt for its use in most practical cases because we usually work with signals that are not immersed in noise.



### 3.2 Resolution

We study the resolution of the system in two ways: the first one aimed to determine the minimum spectral separation between sensing FBGs, and the other one to obtain the actual measurement resolution. First, two FBG elements with reflectance near 1500 nm were multiplexed. We generated signals of length 20000 samples, also we applied zero-padding to perform a FFT of  $2048 \times 256 = 524288$  points, the sampling frequency was 20 KHz. The demultiplexing of the different FBGs was performed by selecting the largest peaks. The minimum wavelength separation between fiber Bragg gratings obtained with the FFT method is 2 nm. However, MUSIC, in the same conditions, provides a wavelength separation of 1 nm. We can conclude that if we use the high accuracy method called MUSIC it is possible to multiplex a large number of gratings using a single broadband source and a single receiving interferometer.

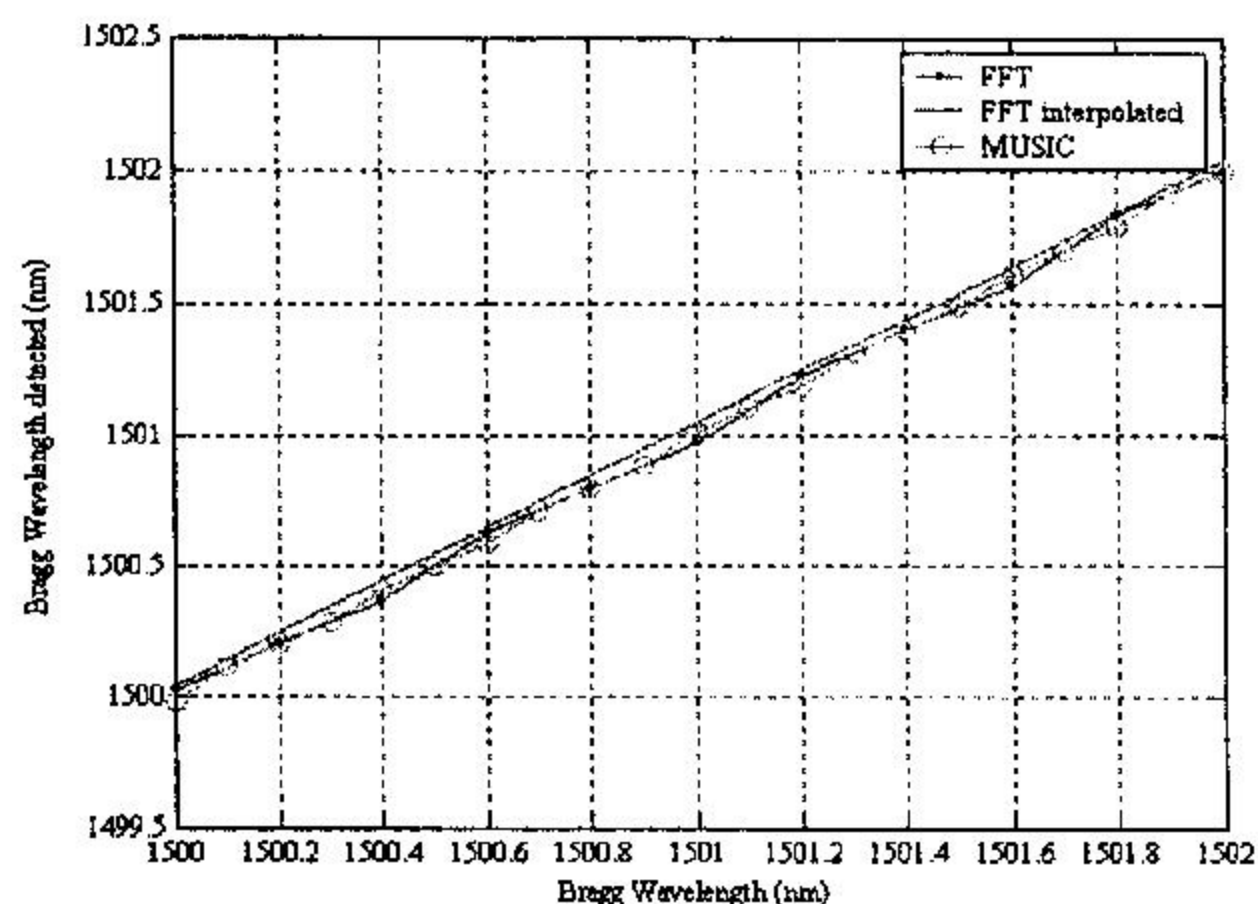


Figure 2: Bragg wavelength obtained with FFT and FFT interpolated when  $\Delta\lambda_B = 200$  pm and with MUSIC when  $\Delta\lambda_B = 100$  pm

The second approach consists in varying the Bragg wavelength of one FBG element as shown in Fig. 2. We used a FBG element with nominal peak reflectance of 1500 nm that was varied from 1500 to 1502 nm. The simulated signal had 20000 samples. We chose a sampling frequency of 20 KHz and we applied zero-padding as before. Wavelength changes of 200 pm in the case of the FFT were detected. It may seem that an increase in the number of points of the signal would result in an improvement of resolution. However, using an interpolation factor of 100, the maximum resolution obtained was still 200 pm. This is due to the fact that, in this case, the zero-padding defined sufficiently well the spectrum. Nevertheless, the interpolation is a good general method for improving the wavelength resolution of the FFT. On the other hand, the resolution provided by MUSIC, in these circumstances, is 100 pm, that is twice better than that of the previous case. These poor resolutions obtained in the reported results depend exclusively on the characteristics of the acquired

signal. Therefore, these are by no means absolute resolution limits, and are to be taken as relative values to allow for the performance comparison between the two algorithms. With a proper measured signal, practical resolutions of pm can be easily obtained.

From the simulation results, we can conclude that if we use the high accuracy method called MUSIC it is possible to measure grating wavelengths separated by 1 nm, or even less. The resolution provided by MUSIC is better than those of the FFT and the interpolated FFT methods. For instance, unlike the FFT methods, MUSIC is able to detect wavelength changes of 100 pm, in the experiments reported herein. In order to increase the maximum resolution, interpolation is applied in the case of the FFT. The linearity obtained interpolating the FFT is better than that obtained by the FFT method, however the computational cost of interpolation is higher.

### 3.3 Crosstalk

Crosstalk effects between two fiber Bragg gratings will now be considered. One FBG has a fixed Bragg wavelength of 1500 nm and the Bragg wavelength of the second is swept in steps of 1 nm in order to analyze the variation of the Bragg wavelength of the first FBG. We applied zero-padding to perform a FFT of  $2048 \times 256 = 524288$  points, the sampling frequency was 20 KHz for the FFT methods and also for MUSIC. At first, the wavelength separation between the fiber Bragg gratings was set to a value of 7 nm. For a wavelength change from 1507 nm to 1550 nm, the measured wavelength of the 1500 nm grating remained steady to within 0.15 nm using the FFT method. This value is close to the 200 pm wavelength resolution reported for a single grating using the FFT method. The same process has been carried out for wavelength separations of 4 nm and 2 nm giving maximum wavelength changes of 0.195 nm and 1.0536 nm, respectively. This means that in the worst case, using the FFT, a 0.07% variation of the first Bragg wavelength due to crosstalk effects occurs. Applying interpolation to the FFT, the detected maximum wavelength changes for wavelength separations of 7 nm, 4 nm and 2 nm, were 0.1946 nm, 0.3751 nm and 1.9997 nm, respectively. This method increases the



amount of crosstalk. Thus, in the worst case a 0.133% variation of the 1500 nm Bragg wavelength is obtained. Applying MUSIC a maximum wavelength change of 0.0229 nm was observed for wavelength separations of 7 nm and 4 nm. This means a 0.00152% variation of the first Bragg wavelength, which is practically negligible. Therefore, MUSIC behaves better than the FFT. For a wavelength separation of 2 nm the change obtained with MUSIC was 1.0507 nm. This corresponds to a 0.07% variation of the first Bragg wavelength due to crosstalk effects. This result is similar to that obtained with the FFT. In general, the performance of the MUSIC algorithm in this issue is better than the FFT methods. Table I summarizes the wavelengths obtained applying the FFT, the interpolated FFT and MUSIC methods when the initial wavelength separation between gratings is varied.

TABLE I  
MEASURED VALUES OF WAVELENGTHS

$\Delta\lambda_B$ (nm)	$\lambda_{FFT}$ (nm)	$\lambda_{INTERPOLATED FFT}$ (nm)	$\lambda_{MUSIC}$ (nm)
7	1500.15	1500.1946	1500.0229
4	1500.195	1500.3751	1500.0229
2	1501.0536	1501.9997	1501.0507

#### 4. CONCLUSIONS

We can conclude that closely spaced wavelengths reflected from fiber Bragg gratings can be accurately demultiplexed using two methods, FFT and MUSIC. With the second one we demonstrated the demultiplexing of grating wavelengths separated by half the distance required by the FFT. Besides, the resolution provided by MUSIC is better than those of the FFT and the interpolated FFT methods.

Finally, crosstalk analysis has showed wavelength variations of 0.07% in the worst case for the FFT method and MUSIC and of 0.133% for the interpolated FFT method. However, MUSIC can yield wavelength variations of 0.00152%, which are practically negligible. In general, the performance of the MUSIC algorithm in this issue is better than the FFT methods.

Among the proposed methods the best compromise between performance and computational cost is provided by MUSIC. We can conclude that if we use the high accuracy method called MUSIC it is possible to multiplex a large number of gratings using a single broadband source and a single receiving interferometer.

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