Vietnam Journal of Mathematics (2023) 51:565–566
https://doi.org/10.1007/s10013-022-00575-2

CORRECTION

Correction to: State Error Estimates for the Numerical Approximation of Sparse Distributed Control Problems in the Absence of Tikhonov Regularization

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Published online: 3 October 2022

Correction to: Vietnam Journal of Mathematics (2021) 49: 713–738
https://doi.org/10.1007/s10013-021-00491-x

Unless otherwise stated, all the references to equation numbers, lemmas and sections are those of the paper being corrected. The numbers of the equations in this note follow the order of the original paper.

The proof of (3.11) given in Appendix A is based on the inequality (A.2). To prove this inequality, we distinguish three cases. There is an error in the proof of Case 3: after (A.4), we say that

> ...for $\varepsilon > 0$ small enough, $\|y_u - \bar{y}\|_{L^\infty(\Omega)} < \varepsilon$ implies $\|z_w\|_{L^\infty(\Omega)} < 2\varepsilon$ (which follows from [9, Lemma 2.4], the definition of w and the maximum principle).

This is not correct because [9, Lemma 2.4] can be applied to $u - \bar{u}$, but we do not know if it can be applied to w. Let us see how to complete the proof of (A.2) in this case.

After (A.4) the proof goes in the following way. Let us define $C_{\Omega,1}$ and $C_{\Omega,\infty}$ as the continuity constants of the mapping $v \mapsto z_v$ in $L^1(\Omega)$ and from $L^2(\Omega)$ to $L^\infty(\Omega)$, respectively, where z_v is the solution of (2.4), i.e.,

$$\|z_v\|_{L^1(\Omega)} \leq C_{\Omega,1}\|v\|_{L^1(\Omega)} \quad \forall v \in L^1(\Omega), \tag{A.5}$$

$$\|z_v\|_{L^\infty(\Omega)} \leq C_{\Omega,\infty}\|v\|_{L^2(\Omega)} \quad \forall v \in L^2(\Omega). \tag{A.6}$$

The original article can be found online at https://doi.org/10.1007/s10013-021-00491-x

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In adddition, we know that there exists $\delta > 0$ such that,

$$\|y_u - \bar{y}\|_{L^\infty(\Omega)} \le \delta \quad \Rightarrow \quad \|z_{u-\bar{u}}\|_{L^\infty(\Omega)} \le 2\|y_u - \bar{y}\|_{L^\infty(\Omega)}. \tag{A.7}$$

The proof of this result is done [9, Lemma 2.4] for the parabolic case, the adaptation for the elliptic case being immediate.

Let us define the constants

$$K = 2C^2_{\Omega,\infty}(\beta - \alpha)|\Omega|, \quad \varepsilon_0 = \frac{\rho\tau}{4cC_{\Omega,1}}, \quad \text{and } \varepsilon = \min\left\{\delta, 4\frac{\varepsilon_0^2}{K}\right\},$$

where ρ is the one given on the statement of Lemma 2, τ is the one given in (3.8), α and β are the control bounds, and c is given in (A.4). From now on, we suppose

$$\|y_u - \bar{y}\|_{L^\infty(\Omega)} \le \varepsilon.$$

Using (A.3), the fact that $u - \bar{u} \in G^\tau_{\bar{u}}$, that $\varepsilon \le \delta$, and (A.7), we have

$$\tau\|w\|_{L^1(\Omega)} \le J'(\bar{u}; u - \bar{u}) \le \tau\|z_{u-\bar{u}}\|_{L^1(\Omega)} \le \tau\|z_{u-\bar{u}}\|_{L^\infty(\Omega)}|\Omega| \le 2\tau|\Omega|\varepsilon.$$

With this inequality, and taking into account that $\|w\|_{L^\infty(\Omega)} \le \beta - \alpha$, and the definitions of ε and K, we obtain

$$\|w\|_{L^2(\Omega)} \le (\beta - \alpha)^{1/2}\|w\|^{1/2}_{L^1(\Omega)} \le (\beta - \alpha)^{1/2}(2|\Omega|)^{1/2}\varepsilon^{1/2} \le \frac{2\varepsilon_0}{C_{\Omega,\infty}}.$$

Taking into account the previous estimate and (A.6), we deduce that $\|z_w\|_{L^\infty(\Omega)} \le 2\varepsilon_0$, and hence, using (A.5), we obtain

$$\|z_w\|^2_{L^2(\Omega)} \le 2\varepsilon_0\|z_w\|_{L^1(\Omega)} \le 2\varepsilon_0 C_{\Omega,1}\|w\|_{L^1(\Omega)}.$$

Finally, (A.2) is deduced using inequalities (A.4) and (A.3), the previous inequality, and the definition of ε_0 as follows:

$$\begin{aligned}
\rho J'(\bar{u}; u - \bar{u}) + F''(u_\theta)(u - \bar{u})^2 &\ge \frac{\delta}{8}\|z_{u-\bar{u}}\|^2_{L^2(\Omega)} - c\|z_w\|^2_{L^2(\Omega)} + \rho\tau\|w\|_{L^1(\Omega)} \\
&\ge \frac{\delta}{8}\|z_{u-\bar{u}}\|^2_{L^2(\Omega)} + (\rho\tau - 2c\varepsilon_0 C_{\Omega,1})\|w\|_{L^1(\Omega)} \\
&= \frac{\delta}{8}\|z_{u-\bar{u}}\|^2_{L^2(\Omega)} + \frac{1}{2}\rho\tau\|w\|_{L^1(\Omega)} \\
&\ge \frac{\delta}{8}\|z_{u-\bar{u}}\|^2_{L^2(\Omega)}.
\end{aligned}$$

References

9. Casas, E., Mateos, M.: Critical cones for sufficient second order conditions in PDE constrained optimization. SIAM J. Optim. **30**, 585–603 (2020)