

# ERRATUM TO “WELL-POSEDNESS OF EVOLUTIONARY NAVIER-STOKES EQUATIONS WITH FORCES OF LOW REGULARITY ON TWO-DIMENSIONAL DOMAINS”

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In this erratum, the authors correct an error in Theorem 2.9 of [1]. That paper focuses on the Navier-Stokes equations

$$\begin{cases} \frac{\partial \mathbf{y}}{\partial t} - \nu \Delta \mathbf{y} + (\mathbf{y} \cdot \nabla) \mathbf{y} + \nabla \mathbf{p} = \mathbf{f} & \text{in } Q = \Omega \times I, \\ \operatorname{div} \mathbf{y} = 0 & \text{in } Q, \mathbf{y} = 0 \text{ on } \Sigma = \Gamma \times I, \mathbf{y}(0) = \mathbf{y}_0 \text{ in } \Omega, \end{cases} \quad (0.1)$$

under sufficiently low regularity assumptions in such a way that measure-valued forcing functions  $f$  in the spatial variable are admitted. Here  $I = (0, T)$  with  $0 < T < \infty$ , and  $\Omega \subset \mathbb{R}^d$  denotes a connected bounded domain with a  $C^3$  boundary  $\Gamma$ . Concerning the notation we refer to [1]. Let us only recall that  $\mathbf{B}_{s,r}(\Omega) = (\mathbf{W}_{s'}(\Omega)', \mathbf{W}_s(\Omega))_{1-1/r,r}$  denotes real interpolation spaces for  $r, s \in (0, \infty)$ .

In Theorem 2.9 of [1] we present the following result. Its proof contains a flaw which is corrected below.

**Theorem 0.1.** *Let us assume that  $q \geq 8$ ,  $p \in (\frac{4}{3}, 2)$ ,  $\mathbf{f} \in L^q(I; \mathbf{W}^{-1,p}(\Omega))$ , and  $\mathbf{y}_0 = \mathbf{y}_{N0} + \mathbf{y}_{S0} \in \mathbf{B}_{2,4}(\Omega) + \mathbf{B}_{p,q}(\Omega)$ . Then the variational solution  $\mathbf{y}$  of (0.1) belongs to  $L^q(I; \mathbf{L}^4(\Omega))$  and depends continuously in this topology on  $\mathbf{f}$  and  $\mathbf{y}_0$ . Moreover, the estimate*

$$\|\mathbf{y}\|_{L^q(I; \mathbf{L}^4(\Omega))} \leq \eta_q \left( \|\mathbf{f}\|_{L^q(I; \mathbf{W}^{-1,p}(\Omega))} + \|\mathbf{y}_{S0}\|_{\mathbf{B}_{p,q}(\Omega)} + \|\mathbf{y}_{N0}\|_{\mathbf{B}_{2,4}(\Omega)} \right) \quad (0.2)$$

holds for an increasing monotone function  $\eta_q : [0, \infty) \rightarrow [0, \infty)$  independent of  $\mathbf{f}$  and  $\mathbf{y}_0$ , with  $\eta_q(0) = 0$ .

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*Proof.* We can follow the proof in [1] up to  $\mathbf{W}_{4,2}(0, T) \subset C([0, T]; (\mathbf{H}^{-1}(\Omega), \mathbf{H}_0^1(\Omega))_{\frac{3}{4},4})$ . Then we observe that  $(\mathbf{H}^{-1}(\Omega), \mathbf{H}_0^1(\Omega))_{\frac{3}{4},4} \subset \mathbf{L}^4(\Omega)$ . Indeed, by ([2], p. 186, 317) we have  $(\mathbf{H}^{-1}(\Omega), \mathbf{H}^1(\Omega))_{\frac{3}{4},4} = \mathbf{B}_{2,4}^{\frac{1}{2}}(\Omega)$ , where  $\mathbf{B}_{2,4}^{\frac{1}{2}}(\Omega)$  denotes a Besov space. Further the continuous embedding  $\mathbf{B}_{2,4}^{\frac{1}{2}}(\Omega) \subset \mathbf{L}^4(\Omega)$  holds, see page 328 of [2]. Since  $\mathbf{H}_0^1(\Omega) \subset \mathbf{H}^1(\Omega)$ , is a closed subspace, the inclusion  $(\mathbf{H}^{-1}(\Omega), \mathbf{H}_0^1(\Omega))_{3/4,4} \subset (\mathbf{H}^{-1}(\Omega), \mathbf{H}^1(\Omega))_{3/4,4}$  follows. Combining these facts we find  $(\mathbf{H}^{-1}(\Omega), \mathbf{H}_0^1(\Omega))_{\frac{3}{4},4} \subset \mathbf{L}^4(\Omega)$ , and  $\mathbf{W}_{4,2}(0, T) \subset C([0, T]; \mathbf{L}^4(\Omega))$  follows. We can now return to the proof in [1] to obtain the desired result.  $\square$

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## REFERENCES

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