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ERRATUM TO "WELL-POSEDNESS OF EVOLUTIONARY NAVIER-STOKES EQUATIONS WITH FORCES OF LOW REGULARITY ON TWO-DIMENSIONAL DOMAINS"

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In this erratum, the authors correct an error in Theorem 2.9 of [1]. That paper focuses on the Navier-Stokes equations

$$\left\{\begin{array}{l}\dfrac{\partial \mathbf{y}}{\partial t} - \nu\Delta\mathbf{y} + (\mathbf{y}\cdot\nabla)\mathbf{y} + \nabla\mathfrak{p} = \mathbf{f} \ \text{ in } Q = \Omega\times I,\\ \operatorname{div}\mathbf{y} = 0 \ \text{ in } Q,\ \mathbf{y} = 0 \ \text{ on } \Sigma = \Gamma\times I,\ \mathbf{y}(0) = \mathbf{y}_0 \text{ in } \Omega,\end{array}\right. \tag{0.1}$$

under sufficiently low regularity assumptions in such a way that measure-valued forcing functions f in the spatial variable are admitted. Here $I = (0, T)$ with $0 < T < \infty$, and $\Omega \subset \mathbb{R}^d$ denotes a connected bounded domain with a C^3 boundary Γ, Concerning the notation we refer to [1]. Let us only recall that $\mathbf{B}_{s,r}(\Omega) = (\mathbf{W}_{s'}(\Omega)', \mathbf{W}_s(\Omega))_{1-1/r,r}$ denotes real interpolation spaces for $r, s \in (0, \infty)$.

In Theorem 2.9 of [1] we present the following result. Its proof contains a flaw which is corrected below.

Theorem 0.1. *Let us assume that* $q \geq 8$, $p \in (\frac{4}{3}, 2)$, $\mathbf{f} \in L^q(I; \mathbf{W}^{-1,p}(\Omega))$, *and* $\mathbf{y}_0 = \mathbf{y}_{N0} + \mathbf{y}_{S0} \in \mathbf{B}_{2,4}(\Omega) + \mathbf{B}_{p,q}(\Omega)$. *Then the variational solution* $\mathbf{y}$ *of* (0.1) *belongs to* $L^q(I; \mathbf{L}^4(\Omega))$ *and depends continuously in this topology on* $\mathbf{f}$ *and* $\mathbf{y}_0$. *Moreover, the estimate*

$$\|\mathbf{y}\|_{L^q(I;\mathbf{L}^4(\Omega))} \leq \eta_q\Big(\|\mathbf{f}\|_{L^q(I;\mathbf{W}^{-1,p}(\Omega))} + \|\mathbf{y}_{S0}\|_{\mathbf{B}_{p,q}(\Omega)} + \|\mathbf{y}_{N0}\|_{\mathbf{B}_{2,4}(\Omega)}\Big) \tag{0.2}$$

holds for an increasing monotone function $\eta_q : [0, \infty) \longrightarrow [0, \infty)$ *independent of* $\mathbf{f}$ *and* $\mathbf{y}_0$, *with* $\eta_q(0) = 0$.

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Proof. We can follow the proof in [1] up to $\mathbf{W}_{4,2}(0,T) \subset C([0,T]; (\mathbf{H}^{-1}(\Omega), \mathbf{H}_0^1(\Omega))_{\frac{3}{4},4})$. Then we observe that $(\mathbf{H}^{-1}(\Omega), \mathbf{H}_0^1(\Omega))_{\frac{3}{4},4} \subset \mathbf{L}^4(\Omega)$. Indeed, by ([2], p. 186, 317) we have $(\mathbf{H}^{-1}(\Omega), \mathbf{H}^1(\Omega))_{\frac{3}{4},4} = \mathbf{B}^{\frac{1}{2}}_{2,4}(\Omega)$, where $\mathbf{B}^{\frac{1}{2}}_{2,4}(\Omega)$ denotes a Besov space. Further the continuous embedding $\mathbf{B}^{\frac{1}{2}}_{2,4}(\Omega) \subset \mathbf{L}^4(\Omega)$ holds, see page 328 of [2]. Since $\mathbf{H}_0^1(\Omega) \subset \mathbf{H}^1(\Omega)$, is a closed subspace, the inclusion $(\mathbf{H}^{-1}(\Omega), \mathbf{H}_0^1(\Omega))_{3/4,4} \subset (\mathbf{H}^{-1}(\Omega), \mathbf{H}^1(\Omega))_{3/4,4}$ follows. Combining these facts we find $(\mathbf{H}^{-1}(\Omega), \mathbf{H}_0^1(\Omega))_{\frac{3}{4},4} \subset \mathbf{L}^4(\Omega)$, and $\mathbf{W}_{4,2}(0,T) \subset C([0,T]; \mathbf{L}^4(\Omega))$ follows. We can now return to the proof in [1] to obtain the desired result. □

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