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HYBRID PARTICLE SWARM-BASED ALGORITHMS AND THEIR APPLICATION TO LINEAR ARRAY SYNTHESIS

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Abstract—A heuristic particle swarm optimization (PSO) based algorithm is presented in this work and the novel hybrid approach is applied to linear array synthesis considering complex weights and directive element patterns so as to analyze its usefulness and limitations. Basically, classical PSO schemes are modified by introducing a tournament selection strategy and the downhill simplex local search method, so that the hybrid algorithms proposed combine the strengths of the PSO to initially explore the search space, the pressure exerted by the genetic selection operator to manage and speed up the search, and finally, the ability of the local optimization technique to quickly descend to the optimum solution. Four classical real-valued PSO schemes are taken as reference and synthesis results for a 60-element linear array comparing those classical schemes and the hybridized ones are reported and discussed in order to show the improvements achieved by the hybrid approaches.

1. INTRODUCTION

The particle swarm optimization technique (PSO) is a modern heuristic search method that has aroused great attention among the electromagnetics community in many applications and research areas during the last decade, demonstrating throughout the literature its ability to manage high-dimensional and multimodal optimization problems in a near-optimal manner [1–8]. Unlike many other stochastic population based search algorithms, the PSO method, inspired by the

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behavior of organisms such as bird-flocking, has gained practitioners because it is easier to implement and tune, using only one operator to manage the swarm and carry out the optimization: the velocity of the particles. Furthermore, novel PSO based algorithms are continuously emerging to overcome the limitations of the well-known classical PSO schemes and thus, to improve overall performance [7–16].

Different heuristic PSO-based approaches can be found in the literature applied to array synthesis. For instance, in [7] real, binary and multiobjetive PSO implementations have been applied to non-uniform and thinned array design. Moreover, a modified PSO algorithm is successfully applied in [10] to amplitude-only synthesis in both linear and planar arrays, and in [17] PSO and genetic algorithm (GA) are compared when applied to phase-only, amplitude-only and complex synthesis of linear arrays. The authors have also applied classical [18] and hybrid versions [19] of the PSO optimizer to linear array synthesis.

In this paper, classical PSO schemes [20] have been modified by introducing one of the most effective selection strategies commonly used in GA, the tournament selection strategy [19, 21], and the downhill simplex method, a geometric approach that does not require derivatives [22]. Two hybrid approaches have been considered: 1) in the so-called HPSOS, the PSO based scheme is used to make the swarm explore adequately the hyperspace in combination with tournament selection to redirect and speed the search towards the apparently most profitable areas just until it reaches a certain residual error, and then the local search algorithm is launched in order to quickly lead to a solution; 2) a hybrid scheme, HPSO, identical to the previous one but removing the selection strategy to let the PSO scheme explore appropriately the multidimensional search space without any external pressure.

The behavior of this kind of hybrid approaches, which are suitable for solving high-dimensional problems like the one considered here (linear array feed synthesis), needs to be analyzed in statistical terms as shown in [23–25] due to the no free lunch theorem. In this work, this statistical analysis is carried out by taking 25 independent runs for each optimization scheme, averaging the results using several indicators to appropriately assess the accuracy achieved by the algorithms for the problem at hand.

The following sections include a brief mathematical description of the problem at hand, a general overview of the classical PSO schemes considered (global and local PSO with synchronous and asynchronous updates of the swarm), as well as the hybridized approaches proposed (HPSOS and HPSO); along with a summary of results comparing the performance of classical and hybrid optimization schemes, considering as the representative example the synthesis of the elements complex weights of a 60-elements z-directed linear array.

2. SYNTHESIS OF LINEAR ARRAYS

Linear array complex synthesis has been considered as the theoretical and reference problem to test the PSO-based approaches proposed. Basically, if mutual coupling effects are neglected and ϕ dependence omitted, the far-field radiation pattern of a z-directed linear array at a certain direction given by the angle θ, is given by

$$FF(\theta) = EP(\theta) \cdot AF(\theta) \tag{1}$$

in which $EP(\theta)$ represents the element pattern and $AF(\theta)$ is the array factor, which for a linear array consisting of N elements uniformly spaced a distance d on the z-axis is given by

$$AF(\theta) = \sum_{n=1}^{N} a_n \cdot e^{j(2\pi(n-1)(d/\lambda)\cos(\theta)+\alpha_n)} \tag{2}$$

with a_n and α_n representing the amplitude and phase of each element complex weight to be determined in case complex synthesis is considered. Thus, the goal is to optimize the couples (a_n, α_n) so that the $FF(\theta)$ satisfies certain far-field pattern specifications given in terms of upper and lower masks, UM and LM respectively, described by the limits imposed at P angular directions, θ_p. For both classical and hybrid PSO based algorithms, the vector C in (3) contains the whole set of parameters to be optimized and (4) presents the fitness or cost function to be minimized and used to weigh up the accuracy achieved by any vector C.

$$C = (a_1, \alpha_1, \dots, a_n, \alpha_n, \dots, a_N, \alpha_N) \tag{3}$$

$$F = \sum_{p=1}^{P} \min\left(|FF_p(\mathrm{dB})| - |UM_p(\mathrm{dB})|, 0\right)^2 + \sum_{p=1}^{P} \min\left(|LM_p(\mathrm{dB})| - |FF_p(\mathrm{dB})|, 0\right)^2 \tag{4}$$

3. PARTICLE SWARMS AND THE HYBRID APPROACH

An overview of classical real-valued PSO schemes along with a detailed description of the novel hybrid approach is presented in the following subsections.

3.1. Classical PSO Schemes

Let us consider a swarm consisting of K particles, i.e., a set of vectors C in (3), in which each particle is represented by its position in the D-dimensional search space, $\mathbf{X}_k = (\mathbf{x}_{k,1}, \ldots, \mathbf{x}_{k,D})$. Each particle moves iteratively (iteration i to $i+1$) to new positions $\mathbf{X}_k^{i+1}$ with a velocity $\mathbf{V}_k^{i+1} = \left(\mathbf{v}_{k,1}^{i+1}, \ldots, \mathbf{v}_{k,D}^{i+1}\right)$ as given by (5)–(6)

$$\mathbf{V}_k^{i+1} = w\mathbf{V}_k^i + c_1 r_1(pbest - \mathbf{X}_k^i) + c_2 r_2(gbest - \mathbf{X}_k^i) \quad (5)$$

$$\mathbf{X}_k^{i+1} = \mathbf{X}_k^i + \mathbf{V}_k^{i+1} \cdot \Delta\mathbf{t} \quad (6)$$

in which w is the inertial weight, c_1 and c_2 are acceleration constants that specify how much each particle is influenced by the best location ever found by itself, *pbest*, and by the best position ever found by the whole swarm, *gbest*; r_1 and r_2 represent two independent random numbers and $\Delta\mathbf{t}$ is the time step, usually chosen to be one [1, 20].

Depending on how and when particles cooperate and share information, four classical PSO schemes can be outlined: the PSO with either synchronous or asynchronous updates of the swarm and a global or a local topology (PSO-AG, PSO-SG, PSO-AL and PSO-SL, respectively), [20]. The PSO scheme with asynchronous updates and a global topology for the swarm is the most widely used in the literature as it proves to be the most efficient one in computational terms.

3.2. Hybrid Schemes

Classical PSO schemes have been modified by introducing one of the most effective selection strategies commonly used in genetic algorithms (GA), the tournament selection strategy (TS) [21], and a local optimizer, the downhill simplex method [22].

The selection mechanism reinforces the influence of the fitness function on the PSO process. As occurs in GA, the fitness-weighted selection process increases the search pressure over the swarm, propagating iteratively several copies of the best particles and thus, speeding up convergence. However, selection must be applied in such a way that some relatively unfit particles also propagate iteratively so as to preserve diversity within the swarm, avoiding a premature convergence to a local solution. That is why the hybrid scheme proposed uses TS with a subpopulation of two particles that compete on the basis of their fitness value. For larger subpopulations, the pressure exerted by the selection operator becomes too high because too many copies of the same particles propagate, driving the search towards deceptive solutions. Furthermore, if the effect of a local search algorithm is added, then the resulting hybrid algorithm (HPSOS) increases even more the pressure over the swarm, speeding up the optimization but, in contrast, the effect of both selection and local search together may be harmful as these techniques combined promote the premature convergence of the population.

As an alternative to HPSOS, another hybrid approach can be proposed, just by removing TS from the HPSOS (HPSO). For a single run, the HPSO allows the classical PSO scheme considered to explore appropriately the solutions space during the initial iterations and just when a certain fitness value has been achieved (F_{launch}), the downhill simplex method is launched to quickly descend the swarm to an optimum fitness value or solution (F_{goal}). The F_{launch} value is problem dependent and is selected by examining the fitness evolution, so that when the convergence slows down, i.e., the minimum fitness value achieved by the swarm stagnates or simply exhibits a very small improvement during several iterations, the local optimizer is launched. On the one hand, the HPSO will exhibit a slower convergence than the HPSOS but, on the other hand the success rate will be higher.

The following steps summarize the HPSOS algorithm considering the classical PSO-AG scheme as the basis:

i) Initialize the swarm. Generate K particles with random positions and velocities, $\mathbf{X}_k$ and $\mathbf{V}_k$. Evaluate their fitness, F_k, and assign $pbest_k = \mathbf{X}_k$ and make $gbest$ equal to that $\mathbf{X}_k$ with $(\min F_k)$.

ii) Until maximum number of iterations is reached
- ii.1) Repeat for all particles
  - ii.1.1) Update velocity $\mathbf{V}_k^{i+1}$
  - ii.1.2) Update position $\mathbf{X}_k^{i+1}$
  - ii.1.3) Evaluate fitness, $F_k = f(\mathbf{X}_k)$
  - ii.1.4) Update personal best?, $pbest_k = \mathbf{X}_k$
  - ii.1.5) Update global best, $gbest$?
- ii.2) Next particle
- ii.3) If residual error F_{launch} is met
  - ii.3.1) Use swarm to build vertexes of the Simplex
  - ii.3.2) Apply reflection, expansion and/or contraction opera-

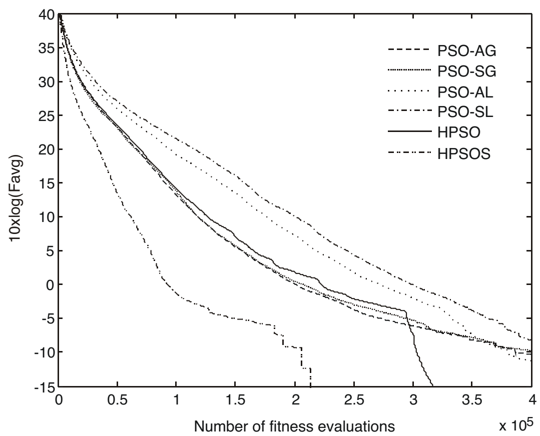


Figure 1. Comparison of classical and hybrid PSO schemes taking as reference the evolution of the normalized averaged fitness value computed considering only successful runs.

tions over the simplex to improve the solution during the remaining iterations or until F_{goal} is obtained

ii.3.3) Solution: current best vertex

ii.3.4) END

Otherwise

ii.3.5) Apply K tournaments to build the new swarm

iii) Next iteration

The HPSOS approach becomes the HPSO algorithm just by removing step ii.3.5.

4. RESULTS

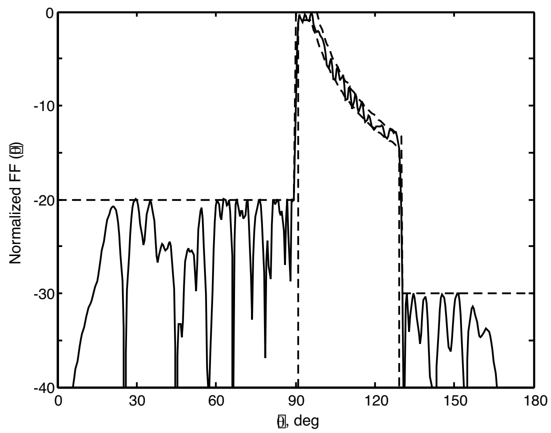
As a canonical problem to compare both classical PSO schemes and the hybridized approaches, let us consider the synthesis of the elements complex weights for a linear array consisting of 60 half-wavelength spaced radiators lying on the z axis and with $\sin\theta$ element patterns, in order to comply with the secant squared far-field pattern depicted in Fig. 2, which exhibits -20 dB max sidelobes from 0° to 90°, a tilt angle of 2°, a secant squared pattern with a 1 dB ripple level from 96° to 128°, and -30 dB max sidelobes from 130° to 180°. The goal is to minimize the cost function given in (4) by optimizing the 120-dimensions C vector of (3) that satisfies the far-field radiation pattern masks proposed. The dynamic range allowed for the elements complex weights are $a_n \in [0, 1]$ and $\alpha_n \in [0, 360°)$, respectively. Moreover, and based on previous work [20], the following parameters have been considered for the classical PSO schemes: inertial weight $w = 0.729$, acceleration constants $c_1 = c_2 = 1.49445$, maximum velocity for particles, V_{max}, equal to half the dynamic range on each dimension of the search space, reflecting walls, a swarm with $K = 121$ particles, and a neighborhood size for the local topologies of $N_n = 20$ particles [20]. Finally, twenty five independent runs have been considered to carry out the analysis so as to take into account the stochastic nature of the PSO algorithm, and the results have been properly averaged to compare classical and hybrid schemes, using parameters such as the success rate (SR), representing the percentage of runs that converge ($F_{goal} < 0.75$ in Equation (4) with a maximum of 400000 fitness function calls or 3306 iterations allowed), the average number of fitness function evaluations (NF_{avg}) necessary to reach the minimum residual error of $F_{goal} = 0.75$, and the averaged CPU time ($Tcpu_{avg}$), both, NF_{avg} and $Tcpu_{avg}$ computed considering only the successful runs.

Regarding the hybrid schemes HPSOS and HPSO, both are only combined with the classical PSO-AG scheme because it is the most efficient one [20] as shall be demonstrated forthwith. Furthermore, the downhill simplex method will be launched when the PSO-AG reaches a residual error of $F_{launch} = 9$.

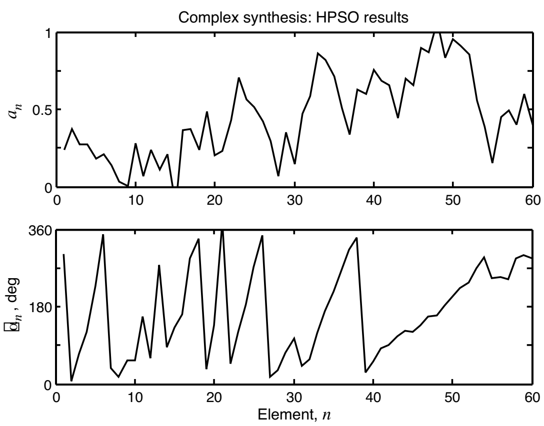
Table 1. Comparison of classical and hybrid PSO schemes (Intel Q6600 2.4 GHz).

Parameter	PSO-AG	PSO-SG	PSO-AL	PSO-SL	HPSOS	HPSO
SR (%)	88	80	72	60	56	96
NF_{avg}	197516.1	214381.7	271315.6	304492.4	118751.5	141010.8
$Tcpu_{avg}$(sec)	845.7	920.1	1165.4	1305.7	507.1	604.3

The results comparing the performance of the classical PSO schemes and the two hybridized ones are summarized in Table 1. Let us focus first on the four classical PSO schemes. According to the computational cost, which is directly related to the NF_{avg} factor, which at the same time is related to the $Tcpu_{avg}$, the PSO-AG outperforms the other three classical schemes, saving up to 8.1%, 27.4% and 35.2% of CPU time with regard to PSO-SG, PSO-AL and PSO-SL schemes, respectively. Moreover, the PSO-AG obtains the highest SR, which shows the robustness and ability of the scheme to avoid deceptive regions in such a high-dimensional and multimodal search space. Regarding the results obtained for the hybrid schemes,



(a)



(b)

Figure 2. Representative results for a secant squared far-field radiation pattern considering a linear array with 60 elements and a single run. (a) Normalized far-field pattern synthesized with the HPSO scheme. (b) Associated amplitude and phase for the optimized complex weights.

different conclusions can be summarized. First of all and using the $Tcpu_{avg}$ factor as the metric to perform the comparison, it can be concluded that any of the hybrid approaches proposed in this work, HPSOS or HPSO, clearly outperforms classical schemes. For instance, the HPSOS scheme is 40.1%, 44.9%, 56.5% and 61.1% faster than PSO-AG, PSO-SG, PSO-AL and PSO-SL schemes, respectively. For the HPSO algorithm, the improvements in terms of computational cost reduce slightly and the HPSO scheme saves 28.5%, 34.3%, 48.1% and 53.4% of CPU time with regard to PSO-AG, PSO-SG, PSO-AL and PSO-SL schemes, respectively. However, on comparing the SR values, the HPSO turns out to be more robust than the HPSOS scheme. Unlike HPSO, for which only one run did not converge to a valid solution, almost half of the HPSOS runs reached a misleading solution. The reason why the HPSOS offers such an unstable behavior is associated with the selection operator. Tournament selection makes the swarm move from the beginning towards those specific regions indicated by the best particles in an attempt to speed up the search, but at the same time it avoids visiting other regions that may contain the global solution. In short, the TS applied in the HPSOS scheme makes the swarm concentrate around such an enclosed region when the local search algorithm is launched that the simplex is made up of vertexes very close to each other, restricting the search of the downhill simplex method and making it more difficult to reach the F_{goal} limit. Finally, the reason why the HPSO even increases the SR with regard to the classical PSO-AG is justified by the right selection of the F_{launch} value for the HPSO scheme. If the downhill simplex method is launched too early, a premature convergence to a deceptive solution can appear as the global search technique has not yet suitably explored the search space. If, however, the local search algorithm is not launched until the PSO scheme has reached a relevant minimum, then the benefits of the HPSO techniques are wasted.

Complementary results that agree with those summarized in Table 1 are shown in Fig. 1, in which the convergence of the whole set of algorithms is compared when representing the evolution of the averaged fitness value, F_{avg}, against the number of cost function evaluations. The results show again that the HPSOS and even the HPSO exhibit on average a far faster convergence than any of the classical PSO schemes considered, demonstrating the improvements achieved and the usefulness of the hybridized approaches.

Finally, Fig. 2(a) shows as an illustrative example the far-field radiation pattern obtained for a single run with the HPSO algorithm, including in Fig. 2(b) the associated complex weights (a_n, α_n) optimized by the algorithm.

5. CONCLUSION

Two hybrid particle swarm based optimization techniques that combine the capacity of PSO to explore the search space in the early stages, the ability of a selection operator to drive the swarm faster in case it is considered, and the skill of the local downhill simplex method to quickly descend to a solution, have been presented in this work as useful approaches and alternatives to well-known classical PSO schemes, taking as reference to carry out the analysis its application to linear array synthesis using complex weights.

The results obtained and presented in this paper demonstrate that any of the hybridized approaches proposed, that is, the one combining the PSO with asynchronous updates and a global topology (PSO-AG) with tournament selection and the downhill simplex method (HPSOS) or the one that combines the PSO-AG with the local search method (HPSO), obtain accurate results, outperform classical PSO based schemes, are far less CPU time consuming and prove to be an efficient and powerful alternative to the classical PSO algorithms.

Regarding the hybrid approaches, the improvements achieved in terms of computational cost are more significant for the HPSOS but, on the contrary, this shows a more unstable behavior than the HPSO algorithm, influenced by the application of tournament selection. In fact, the main drawback of the hybrid selection-based approach (HPSOS) is related to the reduction experimented by the *SR*, due to the effects of the search pressure exerted by the selection operator that propagates iteratively one or more copies of the best particles in the swarm, making diversity vanish.

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