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An Accurate Method to Estimate Complex Permittivity of Dielectric Materials at X-band Frequencies

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Abstract- **In this paper, a measurement method is presented to estimate the complex permittivity of a dielectric material even if its length is higher than the half wavelength in the waveguide. The S_{ij}-parameters at reference planes in the rectangular waveguide loaded by material sample are measured by Network Analyzer. First of all, the expression of the complex permittivity as a function of S_{ij}-parameters are calculated by applying the transmission lines theory. Further, a comparison of the estimated values of the complex permittivity obtained from the presented method and the Nicholson-Ross method is presented. Finally, the results for complex permittivity of Teflon, Nylon and Verde measured at the X-band frequencies are presented.**

Index Terms- **characterization, complex permittivity, Nicholson-Ross method, microwave, waveguide.**

I. INTRODUCTION

The monolayer dielectric materials are currently used in microwave integrated circuits and monolithic microwave integrated circuits [1-2]. Many measurement techniques have been developed and used in recent years to estimate the complex permittivity of monolayer dielectric materials [3-4-7]. These techniques include free space methods, cavity resonators, and transmission line or waveguide techniques. Each technique has its distinct advantages and disadvantages. The free space methods are less accurate because of the unwanted reflection from surrounding objects [4]. The resonant cavity measurement technique is more accurate, but it gives narrow band [5]. The Nicholson-Ross technique is widely used [6-8] to determine accurately the complex permittivity of dielectric material over a wide-band of frequencies. This method has the disadvantage of having inaccuracy peaks for low loss materials at frequencies where the length of the sample is a multiple of half wavelength of the waveguide [8]. To solve this problem, several researchers have used iterative optimization techniques [8-9]. But these iterative techniques work well when the initial guesses are well chosen. In addition, the initial guesses require knowledge of the length of the sample of the material to be characterized. By applying the transmission lines theory, the expression of the complex permittivity as a function of the S_{ij} parameters are calculated. A comparison of the estimated values of the complex permittivity obtained from the presented method and the Nicholson-Ross method is presented. The results for complex permittivity of Teflon, Nylon and Verde measured at the X-band frequencies are presented.

II. THEORY

A. Direct Problem

This section presents the calculation of the Sij-parameters of a rectangular waveguide loaded with a monolayer dielectric material as shown in Fig. 1. The mono-layer dielectric material has complex permittivity $\varepsilon_r = \varepsilon'_r - j\,\varepsilon''_r$ and is located between transverse planes z = 0 and z = L.

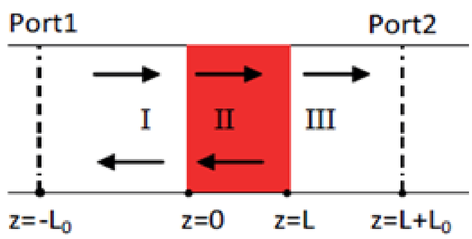

Fig.1. Rectangular waveguide loaded with a mono-layer dielectric material.

INTERNATIONAL JOURNAL OF MICROWAVE AND OPTICAL TECHNOLOGY,
VOL.15, NO.1, JANUARY 2020

The Sij-parameter is found by analyzing the electric field at the sample interfaces. Assuming that only the dominant TE_{10} mode propagates in the loaded waveguide Fig. 1, the formulation of the Sij-parameters can be expressed as a function of complex permittivity using transmission lines theory. In the regions I, II and III, we can write the spatial distribution of the electric field for an incident field normalized to 1 in the region I:

$$E_I = 1\exp(-j\gamma_0 z) + A_1 \exp(j\gamma_0 z) \quad (1)$$

$$E_{II} = A_2 \exp(-j\gamma z) + A_3 \exp(j\gamma z) \quad (2)$$

$$E_{III} = A_4 \exp(-j\gamma_0 z) \quad (3)$$

Where $\gamma_0 = \sqrt{\frac{\omega^2}{c^2} - \frac{\pi^2}{a^2}}$ and $\gamma = \sqrt{\frac{\varepsilon_r \omega^2}{c^2} - \frac{\pi^2}{a^2}}$ are the propagation constants in vacuum and in layer respectively, (ω) is the angular frequency, (*a*) is the dimension of the waveguide and (*c*) is the speed of light in vacuum. The constants A_i are determined from the boundary conditions. Tangential component of the electric field is continuous at sample interfaces:

$$E_I(z=0) = E_{II}(z=0) \quad (4)$$

$$E_{II}(z=L) = E_{III}(z=L) \quad (5)$$

Tangential component of the magnetic field is continuous at the sample interfaces:

$$\frac{\partial E_I}{\partial z}(z=0) = \frac{\partial E_{II}}{\partial z}(z=0) \quad (6)$$

$$\frac{\partial E_{II}}{\partial z}(z=L) = \frac{\partial E_{III}}{\partial z}(z=L) \quad (7)$$

By application of these boundary conditions, we obtain the following matrix system:

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ \gamma_0 & \gamma & -\gamma & 0 \\ 0 & e^{-j\gamma L} & e^{j\gamma L} & -e^{-j\gamma_0 L} \\ 0 & -\gamma e^{-j\gamma L} & \gamma e^{j\gamma L} & \gamma_0 e^{-j\gamma_0 L} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} = \begin{bmatrix} 1 \\ \gamma_0 \\ 0 \\ 0 \end{bmatrix} \quad (8)$$

By solving this matrix system, we obtain the equation of the S_{11} and S_{21} parameters as a function of the complex permittivity of the layer material:

$$S_{11}(\varepsilon', \varepsilon'') = A_1 e^{-2j\gamma_0 L_0} = e^{-2j\gamma_0 L_0} \frac{j(\gamma_0^2 - \gamma^2)\sin(\gamma L)}{2\gamma_0\gamma\cos(\gamma L) + j(\gamma_0^2 + \gamma^2)\sin(\gamma L)} \quad (9)$$

$$S_{21}(\varepsilon', \varepsilon'') = A_4 e^{-j\gamma_0(2L_0+L)} = e^{-2j\gamma_0 L_0} \frac{2\gamma_0\gamma}{2\gamma_0\gamma\cos(\gamma L) + j(\gamma_0^2 + \gamma^2)\sin(\gamma L)} \quad (10)$$

To calculate S_{12} and S_{22} parameters as a function of complex permittivity we assume that the fundamental mode TE_{10} is incident in the region III and we follow the same procedure used to calculate the S_{11} and S_{21}.

$$S_{12}(\varepsilon', \varepsilon'') = S_{21}(\varepsilon', \varepsilon'') \quad (11)$$

$$S_{22}(\varepsilon', \varepsilon'') = S_{11}(\varepsilon', \varepsilon'') \quad (12)$$

We simplify the phase term by taking $L_0 = 0$ then:

$$S_{11}(\varepsilon', \varepsilon'') = S_{22}(\varepsilon', \varepsilon'') = \frac{j(\gamma_0^2 - \gamma^2)\sin(\gamma L)}{2\gamma_0\gamma\cos(\gamma L) + j(\gamma_0^2 + \gamma^2)\sin(\gamma L)} \quad (13)$$

$$S_{21}(\varepsilon', \varepsilon'') = S_{12}(\varepsilon', \varepsilon'') = \frac{2\gamma_0\gamma}{2\gamma_0\gamma\cos(\gamma L) + j(\gamma_0^2 + \gamma^2)\sin(\gamma L)} \quad (14)$$

B. Inverse Problem

1. NICHOLSON-ROSS Method

The Nicholson-Ross method gives the permittivity complex expression as [8]:

$$\varepsilon_r = \varepsilon'_r - j\varepsilon''_r = -\left(\frac{c}{2\pi f L} Ln(T)\right)^2 \quad (15)$$

Where the transmission coefficient is [8]:

$$T = \frac{S_{11} + S_{21} - \Gamma}{1 - (S_{11} + S_{21})\Gamma} \quad (16)$$

And the reflection coefficient is [8]:

$$\Gamma = K \pm \sqrt{K^2 - 1} \quad (17)$$

With: $K = \frac{S_{11}^2 - S_{21}^2 + 1}{2S_{11}}$

INTERNATIONAL JOURNAL OF MICROWAVE AND OPTICAL TECHNOLOGY,
VOL.15, NO.1, JANUARY 2020

We see that when $S_{11} = 0$ a peak of discontinuity appears. In this case, by using the equation (13), we have: $\gamma L = n\pi$ with $n \in N^*$. That means:
$L = n\lambda_g /2$ where λ_g is the wavelength in the sample.

$$f_{peak}(n=1) = \frac{c}{2}\sqrt{\frac{1}{\varepsilon'_r}\left(\frac{1}{a^2}+\frac{1}{L^2}\right)} \qquad (18)$$

This technique has the disadvantage of having inaccuracy peak for low loss materials at f_{peak} where the length of the sample is a multiple of half wavelength in the waveguide. The Nicholson-Ross method does not allow to determine the complex permittivity of the sample starting from this peak frequency.

2. New Method

To solve this problem, we propose a new method that is based on elimination of the length L of the sample in the expression of the complex permittivity. To calculate the complex permittivity ε''_r we have to extract the propagation constant by inverting the system of equations (13) and (14) :

$$cos(\gamma L) = \frac{2j\gamma_0\gamma S_{11}}{(\gamma^2-\gamma_0^2)S_{21}} \qquad (19)$$

$$sin(\gamma L) = \frac{(\gamma^2-\gamma_0^2)+(\gamma^2+\gamma_0^2)\,S_{11}}{(\gamma^2-\gamma_0^2)S_{21}} \qquad (20)$$

We eliminate the sample length L by $\cos^2(\gamma L) + \sin^2(\gamma L) = 1$:

$$A\gamma^4 + 2B\gamma_0^2\gamma^2 + \gamma_0^2 C = 0 \qquad (21)$$

Then:

$$\gamma^2 = \pm\gamma_0^2.\left(\frac{-B\pm\sqrt{B^2-AC}}{A}\right) \qquad (22)$$

Where: $A = S_{21}^2 - (1+S_{11})^2$ and

$B = 1 - S_{21}^2 + S_{11}^2$ and $C = S_{21}^2 - (1-S_{11})^2$

Finally:

$$\varepsilon_r = \left(\frac{\pi c}{\omega a}\right)^2 \pm \left(1-\left(\frac{\pi c}{\omega a}\right)^2\right).\frac{-B\pm\sqrt{B^2-AC}}{A} \qquad (23)$$

The major advantage of this expression is the independence of the length of the sample. So even if L is higher than the half wavelength this method gives the complex permittivity. The expression giving L after knowing ε_r is:

$$L = \frac{1}{\gamma}\arccos\left(\frac{2j\gamma\gamma_0 S_{11}}{(\gamma^2-\gamma_0^2)S_{21}}\right) \qquad (24)$$

III. NUMERICAL RESULTS

For the following results, we consider the measurement system shown in Fig. 2. The S_{ij}-parameters at reference planes of a X-band rectangular waveguide WR90 loaded by a monolayer dielectric material was measured using the E8634A Network Analyzer.

Fig.2. The measurement system.

A Teflon sample with complex relative permittivity $\varepsilon_r = 2.04 - j0.002$ cited in [9] and with thickness L = 9.92 *mm* placed in a WR90 Rectangular waveguide. The S_{11} and S_{21} parameters are measured and plotted in Fig. 3. The Fig. 4 shows that the Nicholson-Ross method does not allow to determine the complex permittivity of the sample beyond the frequency

INTERNATIONAL JOURNAL OF MICROWAVE AND OPTICAL TECHNOLOGY,
VOL.15, NO.1, JANUARY 2020

f=11.7*GHz* despite the condition $\Gamma < 1$ being verified in Fig.5. This is due to the parameter S_{11} which vanishes at this frequency. Fig. 4 shows also that the complex permittivity of Teflon is found with good agreement with those determined in recent works [9]. Teflon is a low-loss material so when S_{11} vanishes S_{21} is -1 as shown in Fig. 3. The only singular point of the new method is when the coefficient A vanishes see Fig. 5 and Fig. 4. This means theoretically that $\sin(\gamma L) = 0$ at the frequency f=11.7 *GHz*.

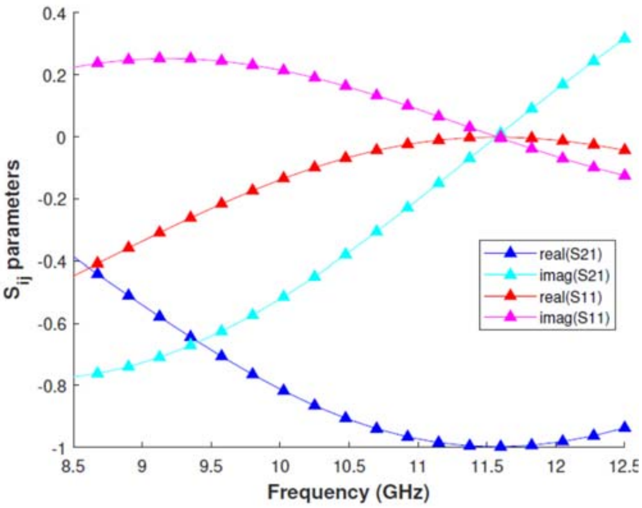

Fig.3. Measured Sij-parameters in a rectangular waveguide WR90 loaded by Teflon (L = 9.92 mm).

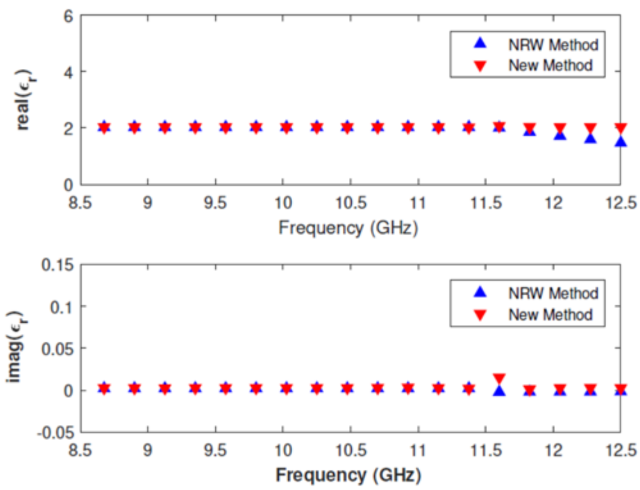

Fig.4. Complex permittivity of the mono-layer Teflon obtained from the S_{ij} measured using the Nicholson-Ross and New Method.

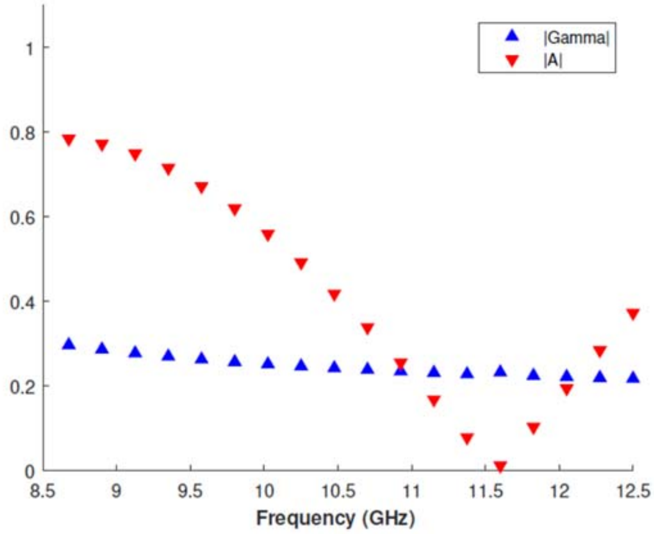

Fig.5. The reflection coefficient Γ for Nicholson-Ross method and A coefficient for the New method obtained from the Sij measured for Teflon.

A Nylon sample with complex relative permittivity ε_r = 2.9-j0.003 used in [9] and with thickness L = 10.2 *mm* placed in a WR90 guide. The S_{11} and S_{21} parameters are measured and plotted in Fig. 6. As shown in Fig. 7 the Nicholson-Ross method does not allow to determine the complex permittivity of the sample beyond the frequency f= 9.45 *GHz* despite the condition $\Gamma < 1$ being verified in Fig. 8. This is due to the parameter S_{11} which vanishes at this frequency. The only singular point of the new method is when the coefficient A vanishes see Fig. 8 and Fig. 7. This means theoretically that $\sin(\gamma L) = 0$ at the frequency f =9.45 *GHz*.

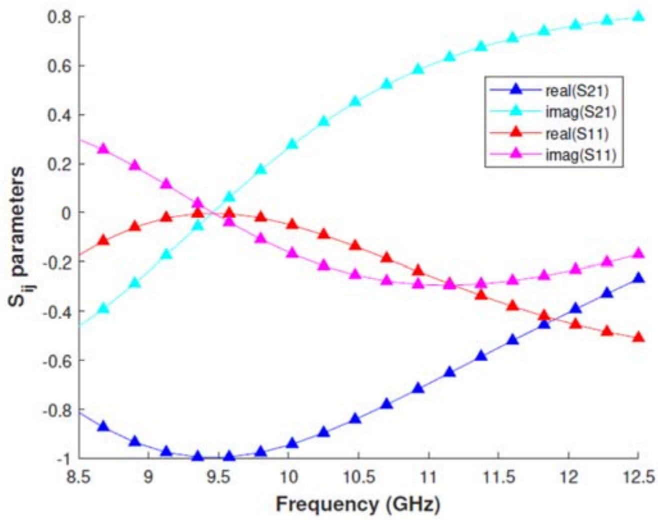

Fig.6. Measured Sij-parameters in a rectangular waveguide WR90 loaded by Nylon (L = 10.2 mm).

INTERNATIONAL JOURNAL OF MICROWAVE AND OPTICAL TECHNOLOGY,
VOL.15, NO.1, JANUARY 2020

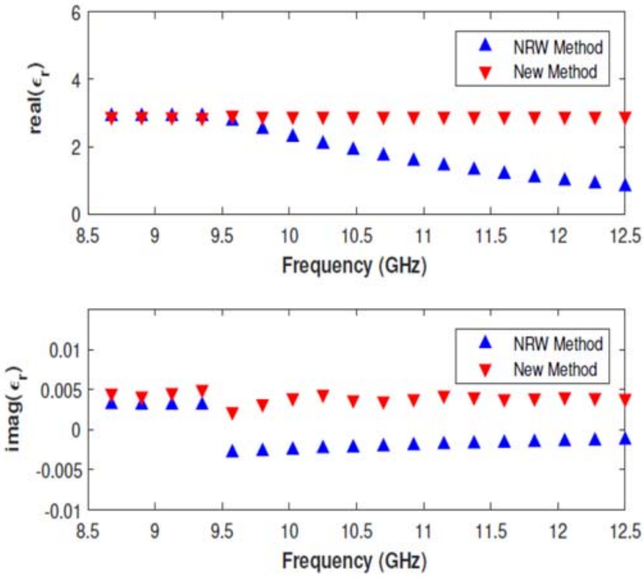

Fig.7. Complex permittivity of the mono-layer Nylon obtained from the S_{ij} measured using the Nicholson-Ross and New Method.

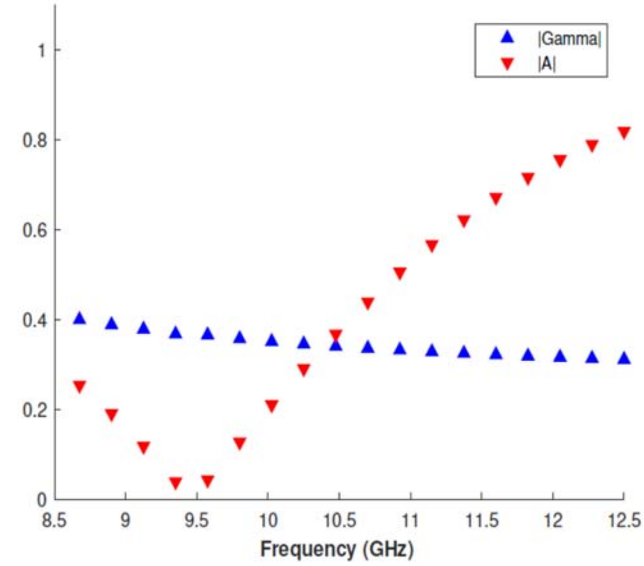

Fig.8. The reflection coefficient Γ for Nicholson-Ross method and A coefficient for the New method obtained from the S_{ij} measured for Nylon.

A Verde sample with complex relative permittivity $\varepsilon_r = 2.3\text{-}j0.002$ and thickness $L = 9.98$ *mm* placed in a WR90 guide. The S_{11} and S_{21} parameters are measured and plotted in Fig. 9. As shown in Fig. 10, the Nicholson-Ross method does not allow to determine the complex permittivity of the sample beyond the frequency f=10.9 *GHz* despite the condition Γ < 1 being verified in Fig. 11. This is due to the S_{11} parameter that vanishes at this frequency. Fig. 10 shows also that the complex permittivity of Verde is found with good agreement. Verde is too a low-loss material so when S_{11} vanishes S_{21} is -1 as shown in Fig. 9. The only singular point of the new method is when the coefficient *A* vanishes see Fig. 11 and Fig. 10. This means theoretically that $\sin(\gamma L)=0$ at the frequency f =10.9 *GHz*.

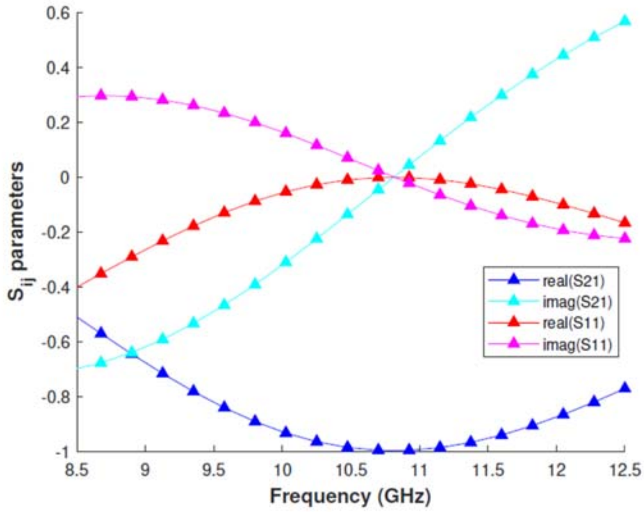

Fig.9. Measured Sij-parameters in a rectangular waveguide WR90 loaded by Verde (L = 9.98 mm).

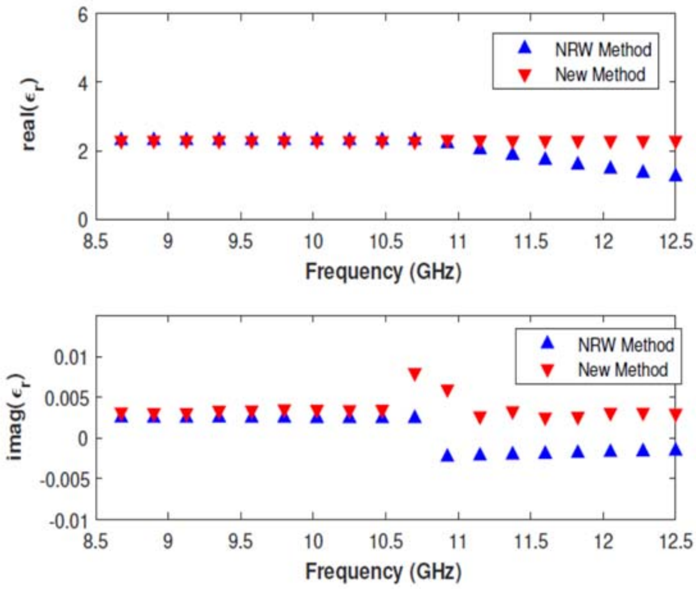

Fig.10. Complex permittivity of the mono-layer Verde obtained from the S_{ij} measured using the Nicholson-Ross and New Method.

INTERNATIONAL JOURNAL OF MICROWAVE AND OPTICAL TECHNOLOGY,
VOL.15, NO.1, JANUARY 2020

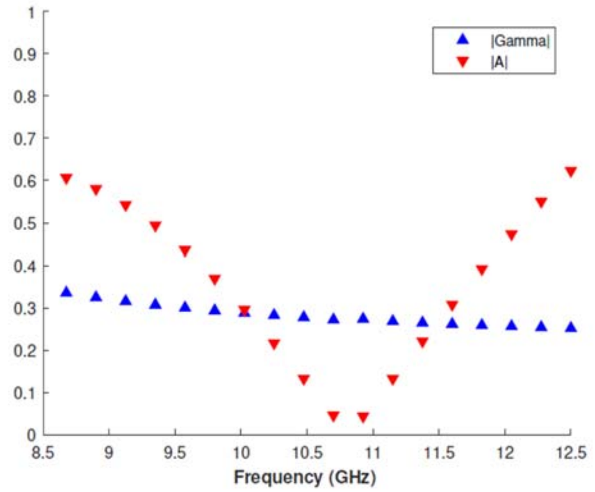

Fig.11. The reflection coefficient Γ for Nicholson-Ross method and A coefficient for the New method obtained from the S_{ij} measured for Verde.

The results presented in Table 1 shows a good agreement between the average values of the complex permittivity of each material calculated by the New method with those by the NRW method [8] for the interval f< f_{peack} in X-band. However, the NRW method for frequencies more than f_{peack} not allow to determine the same values of complex permittivity (or with a high relative error about 30% in real part and 87% in imaginary part) compared to the New method. The singular point creates 10% of relative error in imaginary part for the Verde sample. For the Nylon sample the imaginary part ε_r'' obtained by the NRW method with drawback is positive what is clearly inacceptable for a lossless material.

Table 1: Average complex permittivity and average relative error percentage on the real and imaginary parts of the complex permittivity at X-band.

Dielectric material sample	Average complex permittivity ε= ε'-j ε''			%Average errors			
	NRW method ($f < f_{peack}$) Reference method	NRW method ($f \geq f_{peack}$)	New method ($f > f_{peack}$ or $f < f_{peack}$)	New method		NRW method ($f \geq f_{peack}$)	
				ε'	ε''	ε'	ε''
Teflon	2.04-j0.002	1.96-j 0.0011	2.03-j 0.0021	0.5	5	4	45
Nylon	2.90-j0.003	2.04+j 0.0004	2.91-j 0.0029	0.3	3	30	87
Verde	2.30-j0.002	2.07-j 0.0007	2.33-j 0.0022	1.3	10	10	65

IV. CONCLUSION

A measurement method has been presented for the accurate determination of the complex permittivity of dielectric material sample from S_{ij}-parameters measured in the reference planes of the X-band rectangular waveguide. The present method makes possible the estimation of the complex permittivity of a dielectric material even if its length is unknown or is higher than half the wavelength of the rectangular waveguide. This method eliminates the drawback, when it exists, of the Nicholson-Ross method, otherwise a good agreement is between the two methods.

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