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CORRIGENDUM: CRITICAL CONES FOR SUFFICIENT SECOND ORDER CONDITIONS IN PDE CONSTRAINED OPTIMIZATION*

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Abstract. We correct an error in the proof of Theorem 3.1 in [E. Casas and M. Mateos, *Critical cones for sufficient second order conditions in PDE constrained optimization*, SIAM J. Optim., 30 (2020), pp. 585–603]. With this correction, all results in that paper remain true.

Key words. optimal control, semilinear partial differential equations, optimality conditions, sparse controls

AMS subject classifications. 35K59, 35J61, 49K20

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All equation and lemma numbers refer to those of the paper being corrected [1].

To obtain the inequality after (3.16), we use the fact that $\|z_w\|_{L^\infty(Q)} \le 2\varepsilon_0$ if $\|y_u - \bar y\|_{L^\infty(Q)} < \varepsilon_0$ and refer the reader to the similar inequality (3.7) obtained for $z_{u-\bar u}$. But the proof of the estimate of the norm of $z_{u-\bar u}$ in $L^\infty(Q)$ in terms of ε when $\|y_u - \bar y\|_{L^\infty(Q)} < \varepsilon$ relies on Lemma 2.4, which is not applicable to z_w. To obtain the estimate for $\|z_w\|_{L^\infty(Q)}$, we proceed as follows.

First, we define the constants

$$K_{Q,3} = 2C_{Q,\infty}^3(\beta-\alpha)^2(T+\nu_\Omega)|\Omega| \text{ and } \varepsilon_5 = \min\left\{\varepsilon_2, 8\frac{\varepsilon_0^3}{K_{Q,3}}\right\},$$

where $C_{Q,\infty}$ is given as in Lemma 2.3. Next, we take $u \in U_{\rm ad}$ such that $u - \bar u \in G_{\bar u}^\tau$ and $\|y_u - \bar y\|_{L^\infty(Q)} < \varepsilon_5$. From (3.11), the fact that $u - \bar u \in G_{\bar u}^\tau$, and (2.12) and by using that $\varepsilon_5 \le \varepsilon_2$, we deduce that

$$\begin{aligned}\tau\|w\|_{L^1(Q)} &\le J'(\bar u; u-\bar u) \le \tau\left(\|z_{u-\bar u}\|_{L^1(Q)} + \nu_\Omega\|z_{u-\bar u}(\cdot,T)\|_{L^1(Q)}\right)\\ &\le 2\tau(|Q| + \nu_\Omega|\Omega|)\varepsilon_5 = 2\tau(T+\nu_\Omega)|\Omega|\varepsilon_5.\end{aligned}$$

Since $\|w\|_{L^\infty(Q)} \le \beta - \alpha$, with the above inequality and $\varepsilon_5^{1/3} \le 2\varepsilon_0/K_{Q,3}^{1/3}$ we infer that

$$\begin{aligned}\|w\|_{L^3(Q)} &\le \left(\int_Q(\beta-\alpha)^2|w(x,t)|\,dxdt\right)^{1/3} = (\beta-\alpha)^{2/3}\left(\|w\|_{L^1(Q)}\right)^{1/3}\\ &\le (\beta-\alpha)^{2/3}\big(2(T+\nu_\Omega)|\Omega|)\big)^{1/3}\varepsilon_5^{1/3} \le \frac{2}{C_{Q,\infty}}\varepsilon_0.\end{aligned}$$

Finally, using Lemma 2.3, we obtain the desired estimate:

$$\|z_w\|_{L^\infty(Q)} \le C_{Q,\infty}\|w\|_{L^3(Q)} \le 2\varepsilon_0.$$

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To complete the proof, it is enough to replace the definition of ε in (3.17) by

$$\varepsilon = \min\left\{\varepsilon_0, \varepsilon_5, \frac{1}{C_{f,M_\infty} C_{Q,2}}\right\}.$$

REFERENCE

[1] E. Casas and M. Mateos, *Critical cones for sufficient second order conditions in PDE constrained optimization*, SIAM J. Optim., 30 (2020), pp. 585–603, https://doi.org/10.1137/19M1258244.