Comments on "Undrained cylindrical cavity expansion in anisotropic critical state soils" by S.L. Chen and K. Liu

by

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The discussers congratulate the authors for their solution and the footnote citing the recent discussers' solution (Sivasithamparam & Castro 2018), which clarifies that the discussers' solution was published (strictly speaking, made available online) during the revision of the authors' manuscript. The discussers would like to briefly comment on the differences between the two solutions (rotational hardening law) and the way the results are presented (undrained shear strength and yield locus).

Rotational hardening law

Both solutions extend the isotropic solution by Chen & Abousleiman (2012) considering an anisotropic critical state model with the same rotated ellipse as the yield surface (Dafalias 1986). The main difference between the two solutions is the rotational hardening law that they consider. Sivasithamparam & Castro (2018) use the rotational hardening law proposed by Wheeler et al. (2003), i.e. S-CLAY1 model, while the authors use the original one proposal by Dafalias (1986, 1987). Dafalias & Taiebat (2013) present a detailed analysis of four rotational hardening laws, including the two above. Here, just two differences for this particular case are mentioned:

(1) The original rotational hardening law by Dafalias (1986, 1987) does not predict a unique critical state line (CSL). Dafalias & Taiebat (2013) suggest a minor modification in the rotational hardening law formulation to achieve uniqueness of the CSL. Consequently, for the studied cavity expansion problem, the authors' solution does not predict a unique inclination of the yield surface at CS (e.g. Fig. 11 of the authors' paper), nor the stress state (e.g. Fig. 10), while the discussers' solution predicts a unique inclination and stress state at CS that can be analytically obtained (Sivasithamparam & Castro 2018).

(2) Sivasithamparam & Castro (2018) show how the yield surface rotates from the initial "vertical" anisotropy (axis of the yield surface in the triaxial compression plane, which

corresponds to a Lode angle of θ=7π/6 using the authors' definition of the Lode angle, Fig. 5) towards a "radial" anisotropy (axis of yield surface in the plane strain plane, θ=10π/6). In the authors' case, minor fabric changes are predicted (Fig. 11) and rotation of the yield surface does not reach θ=10π/6. Thus, the effective vertical stress is not the intermediate stress at critical state ($\sigma'_z \neq \sigma'_r + \sigma'_\theta$), as in Li et al. (2016).

Undrained shear strength

A difference between the presentation of results in the two papers is the way the undrained shear strength (s_u) is defined to normalise the results (e.g. Figs. 7 and 8). The authors use the "isotropic" value of s_u. and consequently, the normalized values of the initial stresses for isotropic and anisotropic cases are the same in Figs. 7 and 8. The "anisotropic" value of s_u depends on the initial inclination of the yield surface. For example, Sivasithamparam & Castro (2018) present the analytical equation to get s_u for the S-CLAY1 model, and use the corresponding s_u values for each initial inclination of the yield surface (α_0). In the authors' case, this is not possible due to the lack of uniqueness of the CSL introduced by the rotation hardening law (point 1 above).

Yield locus

An isotropic yield surface may be plotted using p' and q stress invariants without loss of generality. However, for an anisotropic yield surface, it implies assuming a specific value of the Lode angle. As explained by the authors using Fig. 4, there is a continuous variation of the Lode angle during cavity expansion. So, termination of the elastic deformation is not located at the initial anisotropic yield surface corresponding to θ=π/6 and 7π/6, but rather at some other elliptical cut taken through the initial yield surface yet with a changed value of θ. That is why Sivasithamparam & Castro (2018) decided to introduce a new stress invariant $\bar{q}$ and plotted yield loci in the deviatoric plane (π-plane). Yield loci in the p'-q plane were plotted for a constant Lode angle, namely θ=7π/6. In contrast, the authors plot yield loci in the *p'*-*q* plane for different Lode angles. For example, initial yield locus in Fig. 12b corresponds to Lode angle of point C (θ≈9π/6).

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