Available online at www.sciencedirect.com

ScienceDirect

Procedia Economics and Finance 5 (2013) 361 – 367

Procedia Economics and Finance

www.elsevier.com/locate/procedia

International Conference on Applied Economics (ICOAE) 2013

Assessing global inequality in well-being using generalized entropy measures

Vanesa Jordá*[a], Carmen Trueba[a], José María Sarabia[a]

[a]Universidad de Cantabria, Av. de los Castros s/n, 39005, Santander, España

Abstract

In this paper we study global inequality in well-being taking as a reference the Human Development Index. One of the main changes applied to this indicator was the substitution of the arithmetic mean for a geometric mean in the construction of the index, thus assuming different substitution grades between dimensions. In order to determine the impact of the aggregation formula in the distribution of the HDI, we propose the use of multidimensional generalized entropy measures, which allow us to study the evolution of well-being inequality based on different aggregation schemes. The results obtained suggest a reduction in global inequality in human development over the period 1980- 2011. However different inequality patterns are observed depending on the substitutability parameter, thus pointing out the crucial role played by the aggregation formula of composite indices of well-being in the evolution of global inequality.

***Keywords*: Human Development Index; well-being; multidimensional inequality; generalized entropy.**

1. Introduction

The evolution of global inequality has received large amount of attention from the academic world. In the middle of last decade, the focus of the literature changed from income perspective to multidimensional conception of well-being, which also considers non-income dimensions. In this context the Human Development Index (HDI) emerges as an indicator conceived to provide more comprehensive information than income alone. In particular, it assesses achieved human development levels through the quantification of three dimensions: income, health and education.

*Corresponding author. Tel.: +34 942202275; fax: +34 942201603.
E-mail address: jordav@unican.es

doi:10.1016/S2212-5671(13)00043-9

It should be recalled that, the construction of the HDI has attracted many criticisms throughout its history and some of them have been incorporated into the index. Accordingly, this indicator has suffered several changes in the variables used, maximum and minimum levels as well as in its construction. One of the main modifications has been applied in the 2010 Human Development Report which stated that the geometric mean would be replaced by the arithmetic mean in de construction of the index (UNDP, 2010). The traditional simple average is not considered representative since the components of the index are regarded as perfect substitutes, thus implying that the marginal rate of substitution is constant. This axiom can lead to incongruent results, in the sense that the maximization of the HDI in a society may lead to corner solutions, promoting one dimension and disregarding others (Klugman et. al, 2011). The formula introduced in 2010 marks a conceptual change concerning the relationship between dimensions which seems to represent the concept of human well-being more appropriately.

The HDI as a measure of human development has been subject of several studies of inequality, which conclude that inequality in human development has continuously declined over time (McGillivray and Markova, 2010; McGillivray and Pillarisetti, 2004; Pillarisetti, 1997). Martínez (2012) also studies the evolution of well-being inequality based on the new construction of the HDI and its prior version, concluding different patterns for both indicators, especially when the Kolm index is used. Thus, the evolution of the well-being inequality is strongly influenced by the aggregation formula.

Consequently, in this paper we study inequality in human development using a class of multivariate inequality measures which allow us to state different substitutability grades between dimensions.

2. Methodology

Consider that human well-being is characterised by m dimensions assessed in n countries. These values are included in the nxm distribution matrix **X** in $\Re_{++}^{n \times m}$, where each element x_{ik} is the value of the attribute k in the country i. Since we are using the HDI as a benchmark, x_{ik} are standardised by its observed maximum and the so called level of subsistence[1]. Thus we define $g_k(x_{ik})$ as the transformed elements which are integrated in the standardised matrix **G**.

To assess multidimensional well-being inequality, we use the two step procedure proposed by Maasoumi (1986). First, different dimensions are aggregated for each individual using a generalised mean of order $-\beta$, which was interpreted by Maasoumi as the well-being indicator for person i:

[1] The HDI is made up of three intermediate indices, using country-level data on income (r), health (s) and two variables of education ($e1$, $e2$), which reflect achievements in each dimension respect to the subsistence level and the historical maximum value observed. Intermediate indices of income, health and education are expressed, respectively, as follows:

$$g_s(s_i) = \frac{s_i - s_{\min}}{s_{\max} - s_{\min}} \;,\; g_r(r_i) = \frac{\ln r_i - \ln(r_{\min})}{\ln(r_{\max}) - \ln(r_{\min})} \;,\; g_e(e1_i, e2_i) = \left[\left(\frac{e1_i - e1_{\min}}{e1_{\max} - e1_{\min}}\right)\cdot\left(\frac{e2_i - e2_{\min}}{e2_{\max} - e2_{\min}}\right)\right]^{1/2} .$$

Subscript min refers to the subsistence level which is fixed in zero for education variables, 100$ for income and 26 years for life expectancy. Conversely, subscript max denotes the maximum values observed which are 13.1 and 18 years for e1and e2 respectively, 107721$ for income and 85 years in the case of health dimension.

$$s_i = \left(\sum_{k=1}^{K} \delta_k \left(g_k(x_{ik}) \right)^{-\beta} \right)^{-1/\beta} \qquad i = 1, \ldots, N, \tag{1}$$

where δ_k is the weight assigned to each dimension in the construction of the index. Taking the HDI as a benchmark, we apply an equal weighting scheme hence this parameter is fixed in 1/3.

It is worth pausing for a moment to analyse the role played by the parameter β which determines the degree of substitution between dimensions. It can take values from $-\infty$ to ∞ with different interpretations of the relationship between dimensions and global well-being. As extreme cases, we emphasise $\beta = -1$ implying that the dimensions are treated as perfect substitutes, corresponding with the old HDI. In contrast, when β tends to $-\infty$, the components are viewed as perfect complementary, thus only homogeneous achievements in all components counts. Between the previous two situations we have infinity of intermediate options. It must be highlighted that if the value of the parameter is set to 0, s_i corresponds with the new HDI constructed using a geometric mean, thus expressing Equation (1) as:

$$s_i = \prod_{k=1}^{K} \delta_k g_k(x_{ik}).$$

Furthermore, if this parameter takes positive values, a good performance in any dimension would mask even extremely low achievements in the other ones. It is clear that this assumption is absolutely incongruent with the theoretical framework of human development. Notwithstanding this controversy, we have also considered positive values for this parameter in order to study the sensibility of the multidimensional inequality to different aggregation schemes.

For the second step, inequality indices of the Generalised Entropy (GE) class are used to quantify well-being inequality:

$$GEM_\gamma(\mathbf{G}) = \frac{1}{\gamma(1+\gamma)} \frac{1}{N} \sum_{i=1}^{N} \left[\left(\frac{s_i}{\bar{s}} \right)^{1+\gamma} - 1 \right], \quad \gamma \neq -1, 0,$$

where $\bar{S}$ is the arithmetic mean of the s_i values and γ is a sensitivity parameter indicating the weight assigned to distances between well-being values in specific parts of the distribution. In this study the parameter value is set to -1, 0 and -1.5. For the parameter values 0 and -1, the generalised entropy measures are represented, respectively, by the following expressions:

$$GEM_{-1}(\mathbf{G}) = \frac{1}{N} \sum_{i=1}^{N} \log \left(\frac{\bar{s}}{s_i} \right),$$

$$GEM_0(\mathbf{G}) = \frac{1}{N} \sum_{i=1}^{N} \frac{s_i}{\bar{s}} \log \left(\frac{s_i}{\bar{s}} \right).$$

3. Data and Results

In this section we use the methodology described above to investigate global inequality patterns of well-being as well as the role played by the aggregation formula in explaining the evolution of its distribution. We use the most recent available data from International Human Development Indicators (UNDP, 2012) on the three components of the HDI. We cover the period 1980 to 2010 with different frequency of the data. For the period 1980 to 2005 we have 5-years intervals and from 2005 to 2011 the data have annual frequency.

As we inferred previously, since different parameter values assume diverse relationships between dimensions, different well-being distributions would be observed. This theoretical implication is supported by Fig. 1, which present kernel density estimates of s_i distributions computed using a Gaussian kernel with optimal bandwidth (Silverman, 1986). It is observed that the distributions of the old HDI and its new version are really similar thus implying that the changes applied to this indicator in 2010 have not affected its distribution significantly. However, for extreme parameter values, different features are observed. Note that when complementary relationship gains force (β = -10), human development distribution is more concentrated around two modes, whereas a three-mode-distribution is observed when a large positive values are set to the β parameter.

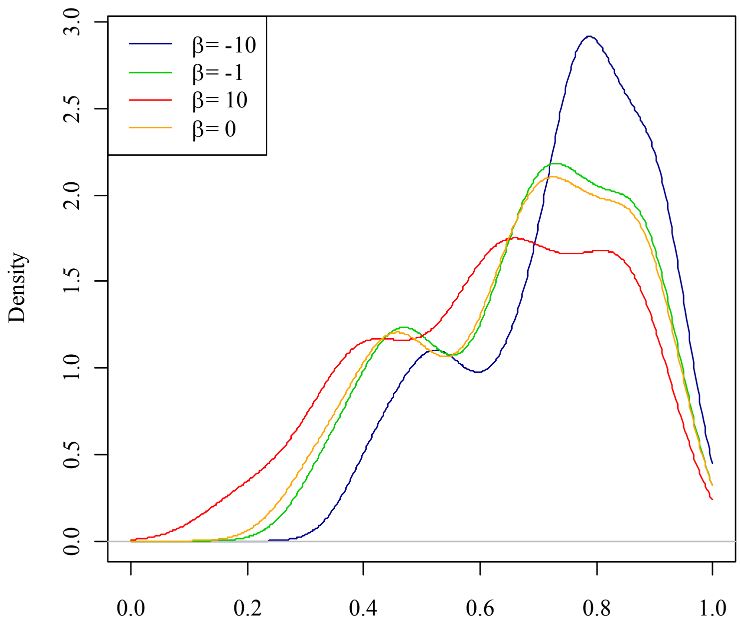
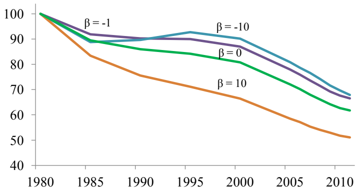
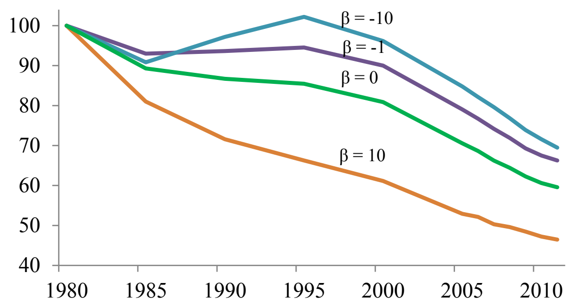


Fig. 1. World HDI distribution assuming different substitutability rates

The multidimensional GE measures are then computed to study the evolution of global inequality according to different aggregation schemes. The results are summarized in Fig. 2 which shows the evolution of well-being inequality for the years 1980 to 2011. To facilitate the comparison of results, inequality has been normalised to be 100 in 1980. Different values of the sensitivity parameter have been considered in order to test the robustness of the results, particularly, $\gamma = 1.5, -1, 0$. Taking the study period as a whole, our results reveal that human development levels have converged significantly. Concretely, GEM values have decreased between 30% and 50% over the last three decades, depending on the aggregation scheme considered, that is conditioned to degree of substitution between dimensions determined by the β parameter. Fig. 2 also points out that as the degree of substitution decreases, the fall in inequality becomes more pronounced. It is worth noting that the evolution of global inequality has only been stable when β is equal to 10, that is, when it is assumed that a good performance in any dimension eclipses deficiencies in other aspects of well-being.

(a)


(b)


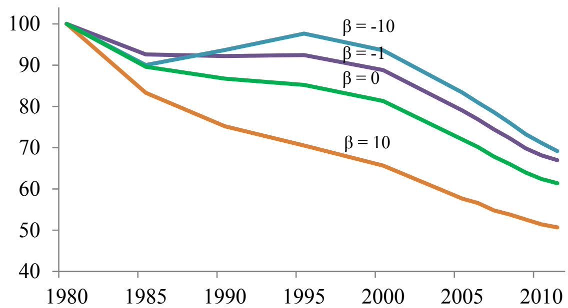
(c)


Fig. 2. Evolution of inequality in human development using multidimensional generalized entropy measures. The sensitivity parameter has been set to -1.5 (a), -1 (b) and 0 (c).

For the other three aggregation schemes considered, three separated phases are roughly distinguishable. As can be observed, global inequality fell until the middle of eighties. This was followed by a decade of stagnation or slight decrease in global well-being inequality when considering the last version and the new definition of the HDI. However, an increase of inequality is observed when the complementary relationship gains force (β = -10). The last fifteen years are characterised by a strong decrease of inequality, thus being the period when almost all reduction took place.

Table 1. Multidimensional generalized entropy measures of well-being.

$GEM_{-1.5}$				
year	$\beta = 0$	$\beta = -1$	$\beta = -10$	$\beta = 10$
1980	0.0549	0.0465	0.0295	0.0978
1985	0.0491	0.0428	0.0262	0.0816
1990	0.0472	0.0420	0.0264	0.0739
1995	0.0462	0.0419	0.0273	0.0696
2000	0.0444	0.0405	0.0266	0.0650
2005	0.0396	0.0363	0.0238	0.0572
2011	0.0339	0.0309	0.0200	0.0499
GEM_{-1}				
year	$\beta = 0$	$\beta = -1$	$\beta = -10$	$\beta = 10$
1980	0.0298	0.0241	0.0148	0.0612
1985	0.0266	0.0224	0.0135	0.0495
1990	0.0258	0.0225	0.0144	0.0438
1995	0.0255	0.0227	0.0152	0.0405
2000	0.0241	0.0216	0.0143	0.0374
2005	0.0210	0.0190	0.0126	0.0324
2011	0.0177	0.0159	0.0103	0.0284
GEM_{0}				
year	$\beta = 0$	$\beta = -1$	$\beta = -10$	$\beta = 10$
1980	0.0262	0.0218	0.0137	0.0481
1985	0.0235	0.0202	0.0124	0.0401
1990	0.0227	0.0201	0.0129	0.0362
1995	0.0223	0.0201	0.0134	0.0339
2000	0.0213	0.0193	0.0128	0.0316
2005	0.0189	0.0172	0.0114	0.0277
2011	0.0161	0.0146	0.0095	0.0244

To further complete these results, Table 1 present the value of the GEM measures over the study period. As Martínez (2012) states, different constructions of the HDI change the meaning of the indicator and thus, different inequality patterns are also expected. In this respect, it is observed that the new construction of the HDI presents the greatest levels of inequality, whatever the degree of sensitivity contemplated. Regarding the remaining aggregation schemes, it is concluded that as the level of substitutability increases, greater levels of inequality are achieved for any value of γ.

Acknowledgements

The authors thank the Ministerio de Economía y Competitividad (Project ECO2010-15455) and the Ministerio de Educación (AP2010-4907) for partial support of this work.

References

Klugman, J., Rodríguez, F., and Choi, H-J., 2011. The HDI 2010: New Controversies, Old Critiques, Journal of Economic Inequality 9, p. 249.
Maasoumi, E., 1986. The Measurement and Decomposition of Multidimensional Inequality, Econometrica 54, p.991.
Martínez, R., 2012. Inequality and the New Human Development Index, Applied Economics Letters 19, p.533.
McGillivray, M. and Markova, N., 2010. Global Inequality in Wellbeing Dimensions, Journal of Development Studies 46, p. 371.
McGillivray, M. and Pillarisetti, J.R., 2004. International Inequality in Well-Being, Journal of International Development 16, p. 563.
Pillarisetti J. R., 1997. An empirical Note on Inequality in the World Development Indicators, Applied Economics Letters 4, p.145.
Silverman, B.W., 1986. Density Estimation for Statistics and Data Analysis, Chapman and Hall, London.
United Nations Development Program, 2012. International Human Development Indicators. Retrieved from http://hdr.undp.org/en/statistics/, last accessed December 10, 2012.
United Nations Development Program, 2010. Human Development Report 2010, Oxford University Press, New York.