

Modeling Nonconfined Density Currents Using 3D Hydrodynamic Models

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Abstract: Density currents generated by marine brine discharges, e.g., from desalination plants, can have a negative impact on marine ecosystems. It is therefore important to accurately predict their behavior. Predictions are often made using computational hydrodynamic models, which should be validated using field or laboratory measurements. This paper focuses on the setup and validation of three-dimensional (3D) models for estimating the transport and mixing processes that occur in these types of flows. Through a comprehensive sensitivity analysis based on the reproduction of several laboratory-generated density currents, a set of recommendations are made regarding the modeling aspects, including the domain discretization, the treatment of momentum at the density current source, the hydrostatic hypothesis and the selection of turbulence closure models. Finally, the proposed numerical model setup is validated using different experimental data showing good agreement in terms of the main variables considered: errors of less than 1.3% for dilution and of 6% for velocity. This study serves as a first step toward the full validation of these 3D hydrodynamic models for the simulation of field-scale density currents. DOI: 10.1061/(ASCE)HY.1943-7900.0001563. © 2018 American Society of Civil Engineers.

Introduction

Bottom density-driven flows, which are generally referred to as density or gravity currents, are continuous underflows that travel downslope due to their negatively buoyant characteristics, i.e., because they are heavier than the surrounding fluid. This phenomenon occurs widely in natural environments and is caused by either human activities or natural processes (Simpson 1997; Huppert 2006). Currently, in coastal and marine environments, some of the most common density currents are those generated by brine discharge from desalination plants. Hodges et al. (2011) make an analogy between the behavior of a natural salt wedge and such brine discharges into shallow waters, both of which are governed by the density difference, by the hydrodynamics of the surrounding area (Shao et al. 2008), and by the bottom slope. Due to the potentially negative impacts of these human-induced currents on the environment (Lattemann and Höpner 2008; Sánchez-Lizaso et al. 2008; Laspidou et al. 2010; Dawoud and Al Mulla 2012) there is a growing interest in obtaining accurate predictions of their behavior.

Dense underflows have been widely investigated in laboratory experiments (Alavian 1986; Garcia 1993; Gerber et al. 2011; Ottolenghi et al. 2017b) and field studies (Hebbert et al. 1979; Dallimore et al. 2001; Fernandez and Imberger 2006;

Hodges et al. 2011). Major efforts have also been made to predict the behavior of these currents from different modeling techniques and through comparisons with previous laboratory experiments (Choi 1999; La Rocca et al. 2008; Lombardi et al. 2015; Sciortino et al. 2018). As a broad classification, two modeling techniques are available for studying these flows numerically using the hydrodynamic equations (i.e., continuity, momentum, and transport equations): integral models and those that determine the vertical structure of the flow. The integral model for density currents was first introduced by Ellison and Turner (1959) and was further developed by Alavian (1986) and Parker et al. (1986) among others, primarily focused on turbidity currents (Akiyama and Stefan 1985; Parker et al. 1986; Garcia 1993; Choi and Garcia 1995; Bradford et al. 1997; Imran et al. 1998; Choi 1999). In general, these integral models assume a hydrostatic pressure distribution within the density current and use vertical depth-integrated equations. They have been designated single- or double-layer shallow water models depending on whether they consider only the heavier layer (e.g., Ungarish 2007a, b; La Rocca et al. 2008; Lombardi et al. 2018) or divide the entire depth into two layers (e.g., Ungarish 2008; La Rocca and Pinzon 2010; La Rocca et al. 2012). Note that although it has been demonstrated that these integral models are capable of providing good results under laboratory controlled conditions, they are not capable of taking into account the complexity of real environmental conditions that may occur in nature and may affect the evolution of the density current flow.

Conversely, there are numerical studies that have used models based on the resolution of three-dimensional (3D) Navier-Stokes (N-S) equations with differing degrees of simplification to solve the vertical structure of density current flows. Specifically, the vertical distribution of the main variables of density currents has been numerically analyzed from hydrodynamic models that solve N-S equations taking into account the Reynolds approximation (Reynolds-averaged Navier-Stokes equations or RANS). In these applications, a turbulence closure model (TCM) estimates the Reynolds stress in conjunction with wall functions. Stacey and Bowen (1988a, b) solved the vertical distribution of one-dimensional turbidity currents using a mixing length model as the TCM. Other authors have employed the κ - ϵ model (Rodi 1984) as

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76 the TCM (e.g., Eidsvik and Brørs 1989; Brørs and Eidsvik 1992; 77 Choi and Garcia 2002). The κ - ϵ model has also been applied to 78 density currents plunging into reservoirs by Farrell and Stefan 79 (1988) and Bournet et al. (1999). In recent years, a number of direct 80 numerical simulations (DNSs) of density currents have been 81 reported in the literature (e.g., Härtel et al. 2000; Lowe et al. 82 2005; Birman et al. 2005; Cantero et al. 2006, 2007). These more 83 sophisticated simulations are capable of capturing interfacial vortex 84 dynamics such as Kelvin-Helmholtz instabilities and the formation 85 of lobe-cleft structures at the current head. Other authors such 86 as Patterson et al. (2005, 2006) have conducted simulations of 87 axisymmetric density currents using implicit large eddy simulations 88 (LESs) (Almgren et al. 1996) relying on the use of subgrid 89 scale modeling (SGS). Nowadays, there are several remarkable 90 studies focused on LESs of different kinds of density currents 91 (e.g., Ottolenghi et al. 2016a, b, 2017a, 2018). However, DNS and 92 LES are still prohibitively expensive in terms of computational time 93 and especially when considering field-scale simulations.

94 An alternative to these models based on the resolution of the 95 hydrodynamic equations is given by the lattice Boltzmann method 96 (LBM), defined in the framework of the kinetic theory, which 97 describes the flow in terms of probability density functions. The 98 simplicity and versatility of the LBM has encouraged its development 99 in the computations fluids dynamics within the last decade 100 (e.g., Aidun and Clausen 2010). Regarding the reproduction of 101 complex flows such as density currents, Rocca et al. (2012) developed 102 a LBM for two-layered shallow-water flows by considering 103 two separate sets of LBM equations, one for each layer, obtaining 104 good agreement between the LBM numerical results and the experimental 105 results when the evolution of the flow does not depend 106 on the viscosity. Recently, Ottolenghi et al. (2018) reveals that this 107 alternative can be also applied for three-dimensional numerical 108 simulations of density currents for different Reynolds number by 109 implementing an equivalent large eddy simulation model in the 110 LBM framework. Nevertheless, although LBM has been successfully 111 applied to simulate density currents generated by laboratory 112 experiments, both considering regular and complex geometries 113 (e.g., Prestininzi et al. 2016), significant research still needs to be 114 done to strengthen the LBM for simulating real field-scale density 115 currents.

116 Focusing on the most developed hydrodynamic equations-based 117 models mentioned and their application, integral models can be 118 considered adequate for field-scale practical studies of water 119 resources management where a coarse approximation of the 120 characteristic flow quantities at an equilibrium state may be sufficient. 121 Conversely, the most complex numerical approximations (e.g., DNS and 122 LES) are highly time-demanding computationally and are typically 123 applied at the laboratory scale under controlled conditions. However, 124 intermediate complexity 3D hydrodynamic models can be used to 125 solve field-scale applications; as an example, Bombardelli and Garca 126 (2002) assessed the potential development of density currents in the 127 Chicago River while capturing their spatial variability. Kulis and 128 Hodges (2006) carried out a layer-number sensitivity test to 129 numerically simulate density currents of the Corpus Christi Bay in 130 Texas using a sigma-coordinate 3D hydrodynamic model based on 131 RANS equations and while taking into account the Boussinesq 132 approximation and the hydrostatic hypothesis. Applying similar 133 3D hydrodynamic models, Firoozabadi et al. (2009) and Mahgoub 134 et al. (2015) simulated density currents and validated their results 135 against some laboratory measurements. Nevertheless, to the authors' 136 knowledge, none of the reviewed studies have been fully validated, 137 i.e., considering both horizontal spreading and the vertical structure 138 of the main flow variables (velocity and concentration). In addition, 139 none of these studies provide

140 recommendations to accurately reproduce these kind of flows. This 141 will be very useful in practical purposes such as the design of brine 142 discharges into seawater, which have to meet strict water quality 143 criteria regarding the salinity concentration far from the discharge 144 point (e.g., Sánchez-Lizaso et al. 2008).

145 The current paper focuses on establishing a suitable setup of a 146 3D hydrodynamic model based on RANS equations, while taking 147 into account the Boussinesq approximation and the hydrostatic or 148 nonhydrostatic pressure hypothesis, for simulating nonconfined 149 density currents. The study numerically reproduces a set of laboratory 150 experiments carried out in the Environmental Hydraulics Institute 151 (IH Cantabria) by applying advanced optical techniques 152 (Pérez-Díaz et al. 2016, 2018), as well as reproducing other 153 experiments under different flow conditions presented by Choi and 154 Garcia (2001). The numerical simulations are fully calibrated and 155 validated against laboratory measurements by comparing the main 156 flow and mixing characteristics. Therefore, the present paper outlines 157 an optimum modeling setup that predicts the behavior of these 158 types of flows using 3D hydrodynamic models. In addition, taking 159 into account that these models are also capable of simulating real 160 environmental conditions that may affect field-scale density currents 161 (Shao et al. 2008), the findings of this study are also presented 162 as a starting point for future field-scale studies.

163 The paper is presented as follows. First, the methods used are 164 introduced, then the calibration results obtained through a comprehensive 165 sensitivity analysis are presented and discussed. Third, the 166 validation results are presented, and finally conclusions are drawn.

167 Methods

168 Numerical Model

169 The numerical model used in this study is TELEMAC-3D. It is an 170 open-source 3D hydrodynamic model (Hervouet 2007; LNHE 171 2007) that solves the three-dimensional hydrodynamic equations 172 (i.e., continuity, momentum, and transport equations) considering 173 the Reynolds and Boussinesq approximations. This model was 174 adopted for this study because it combines a number of suitable 175 characteristics to simulate these types of flows. These include a 176 large number of subroutines based on a large volume of scientific 177 literature that reproduces processes at different scales; a clearly 178 structured Fortran90 source code that allows for simple user 179 programming and modification of subroutines; the option of using 180 both hydrostatic and nonhydrostatic pressure formulations; the 181 variable sigma-layer coordinate (i.e., terrain-following) for vertical 182 domain discretization; and unstructured horizontal domain 183 discretization that allows computationally efficient high resolution 184 results for specific areas (e.g., near sources and sinks of water, or 185 around complex geometric features). Brief descriptions of such model 186 features that play an important role in reproducing the behavior of 187 density currents are listed subsequently.

188 Regarding turbulence modeling, TELEMAC-3D uses the eddy 189 viscosity and diffusivity concepts (ν_t and Γ coefficients, respectively) 190 of the Boussinesq approach. To estimate the values of these 191 turbulence coefficients, several TCMs can be used, such as zero- 192 equation models (based on an algebraic relation), single equation 193 models (based on a combination of an algebraic relation and an 194 equation), two-equation models (based on two transport-diffusion 195 equations), and even more complex models (e.g., the Reynolds 196 stress model). In this study, zero-equation models (including 197 constant, Smagorinsky, Prandtl mixing length, and Nezu and 198 Nakagawa mixing-length model) and a two-equation model (κ - ϵ) 199 were used. The simplest TCM, the constant model, defines constant

200 eddy viscosities and diffusivities according to the grid resolution
 201 and characteristic velocity of the type of flow motion studied
 202 (Madsen et al. 1988). Mixing length and Smagorinsky TCMs are
 203 based on the mixing-length concept proposed by Prandtl. While
 204 mixing-length TCMs such as the standard Prandtl model (Rodi
 205 1984) and the Nezu and Nakagawa model (Nezu and Nakagawa
 206 1993) are only applied as vertical TCMs, the Smagorinsky model
 207 (Smagorinsky 1963) is a subgrid turbulence model wherein mixing
 208 length is dependent on the grid and on a dimensionless coefficient
 209 according to the type of flow involved (anisotropic or isotropic
 210 flow). Finally, the most complex TCM used in this study is the
 211 two-equation κ - ε model. In this paper, eddy viscosities are evalu-
 212 ated by applying the following Kolmogorov-Prandtl relationship
 213 per direction:

$$\nu_t = c_\mu \frac{\kappa^2}{\varepsilon} \quad (1)$$

214 where κ = turbulent kinetic energy; ε = turbulent kinetic energy
 215 dissipation rate; and c_μ is an empirical constant. This TCM adds
 216 two more equations to the system, which are (in conservative form
 217 and with Einstein tensor notation)

$$\frac{\partial \kappa}{\partial t} + U_i \frac{\partial \kappa}{\partial x_i} = \frac{\partial}{\partial x_i} \left(\frac{\nu_t}{\sigma_\kappa} \frac{\partial \kappa}{\partial x_i} \right) + P - G - \varepsilon \quad (2)$$

$$\frac{\partial \varepsilon}{\partial t} + U_i \frac{\partial \varepsilon}{\partial x_i} = \frac{\partial}{\partial x_i} \left(\frac{\nu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial x_i} \right) + c_{1\varepsilon} \frac{\varepsilon}{\kappa} [P + (1 - c_{3\varepsilon})G] - c_{2\varepsilon} \frac{\varepsilon^2}{\kappa} \quad (3)$$

218 where production terms denoted by the shear P and buoyancy G
 219 values are estimated by

$$P = \nu_t \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \frac{\partial U_i}{\partial x_j} = 2\nu_t D_{ij} D_{ij} \quad (4)$$

$$G = \beta g \frac{\nu_t}{\sigma_c} \frac{\partial T}{\partial x_i} \quad (5)$$

220 where indices i and j vary from 1 to 3 according to the direction
 221 involved; and β = fractional density, i.e., the volume expansion.
 222 The κ - ε model contains several empirical constants obtained from
 223 comprehensive data-fitting for a broad range of turbulent flows.
 224 Rodi (1984) compiled the following standard values:

$$\begin{aligned} c_\mu &= 0.09; & \sigma_\varepsilon &= 1.00; & \sigma_\kappa &= 1.30; \\ c_{1\varepsilon} &= 1.44; & c_{2\varepsilon} &= 1.92; & c_{3\varepsilon} &\approx 0-1 \end{aligned} \quad (6)$$

225 Among the empirical constants, $c_{3\varepsilon}$, which is associated with the
 226 buoyancy term G in Eq. (3), is originally established as equal to 1
 227 for stable situations (i.e., when G is negative) and equal to 0 for
 228 unstable stratifications (Launder and Spalding 1974; Viollet 1988).
 229 However, definition of the empirical coefficients $c_{3\varepsilon}$ and c_μ is not
 230 straightforward. Discussions and numerical tests on these constants
 231 are presented in subsequent sections.

232 TELEMAC-3D uses several solution methods, including a
 233 semi-implicit finite-element method, to solve the full set of equations.
 234 Different procedures can be applied to each solution variable
 235 (i.e., velocities, depth, tracers, turbulent kinetic energy, and the dis-
 236 sipation rate). While the method of characteristics (Hervouet 2007)
 237 is generally used for the velocity calculations, more conservative
 238 and monotonic schemes are used for the depth and the tracers
 239 (LNHE 2007). In focusing on the advection scheme for tracers
 240 (in our case the salinity), the present study considers a second-order

central-upwind scheme originally based on the Kurganov and
 Petrova (2007) scheme and adapted by Bourban (2013). For κ - ε
 variables, the default advection scheme established by TELEMAC-
 3D is the method of characteristics. However, a sensitivity test
 based on the advection scheme for these variables was carried out
 in this study. Finally, note that relative accuracies, number of iter-
 ations, and preconditionings for each variable required for the rec-
 ommended iterative solver (conjugate gradient) are established in
 accordance with the recommendations made in the TELEMAC-3D
 user manual (LNHE 2007).

Experimental Databases

Two experimental databases were used to establish the validated
 setup of the TELEMAC-3D model for predicting the behavior of
 nonconfined density currents. The main and largest database was
 generated from a set of laboratory-generated density currents tested
 at IHCantabria's facilities. These laboratory experiments, presented
 in Pérez-Díaz et al. (2018), consisted of saline density currents
 that evolved over a gentle-slope (α) plastic-material base within
 a $3 \times 3 \times 1 \text{ m}^3$ test tank filled with freshwater to simulate the re-
 ceiving body (both fluids at the same temperature but with different
 saline concentration). The constant-flux saline effluent was dis-
 charged through a rectangular height-adjustable slot ($b_o \times h_o$) at
 the base [Fig. 1(b)], simulating the start of far field region of mixing
 that commonly forms when brine is discharged by submerged jets
 (Papakonstantis and Christodoulou 2010; Palomar 2014). Fig. 1(a)
 shows a schematic diagram of the experimental setup outlined. The
 main initial characteristics of this set of laboratory-generated den-
 sity currents are listed in Table 1 and are outlined in Fig. 2. Note
 that Case C1 was selected as the reference case on which all other
 cases were modified by one of the initial parameters (thickness h_o ,
 flow rate Q_o , slope α , or density difference $\rho_a - \rho_o$). This way, a
 comprehensive set of laboratory-generated density currents was
 carried out to experimentally characterize these kinds of flows
 under different flow-expected conditions.

To obtain high quality velocity and concentration measure-
 ments in the longitudinal profile of the density currents tested, the
 mentioned set of experiments used nonintrusive laser optical tech-
 niques, namely particle image velocimetry (PIV), and planar laser
 induced fluorescence (PLIF). The PIV technique consists of captur-
 ing the movements of small seeding particles within the flow be-
 tween consecutive laser pulses that illuminate the flow, while PLIF
 consists of an indirect way to measure concentration based on cap-
 turing the light re-emission of a fluorescent dye once it is illumina-
 ted by the laser at determined wavelength. In the mentioned labora-
 tory study, to capture the maximum covering area (1,400 mm), two
 LaVision Imager ProX 4 (CCD) cameras with a resolution of
 2048×2048 pixels were located adjacently [Fig. 1(a)] with an
 overlapping zone. This configuration together with the appropriate
 selection of PIV and PLIF calibration parameters enabled the ac-
 curate measuring of concentrations and velocities. In addition, to
 study the lateral and front spreading [Fig. 2(b)], a set of photos were
 taken with a camera located at the upper part of the test tank that
 was able to capture the whole plan view. Further information on the
 experimental setup and on the advanced PIV-PLIF measurement
 techniques used to obtain this experimental database can be found
 in Pérez-Daíz et al. (2018).

The other experimental database used in this study was devel-
 oped by Choi and Garcia (2001), who published the results of a
 set of density current experiments carried out in a $2.44 \times 3.66 \times$
 1.22 m^3 tank. These experiments were chosen for this study be-
 cause they generated nonconfined saline density currents with
 different initial conditions and because the authors provided all

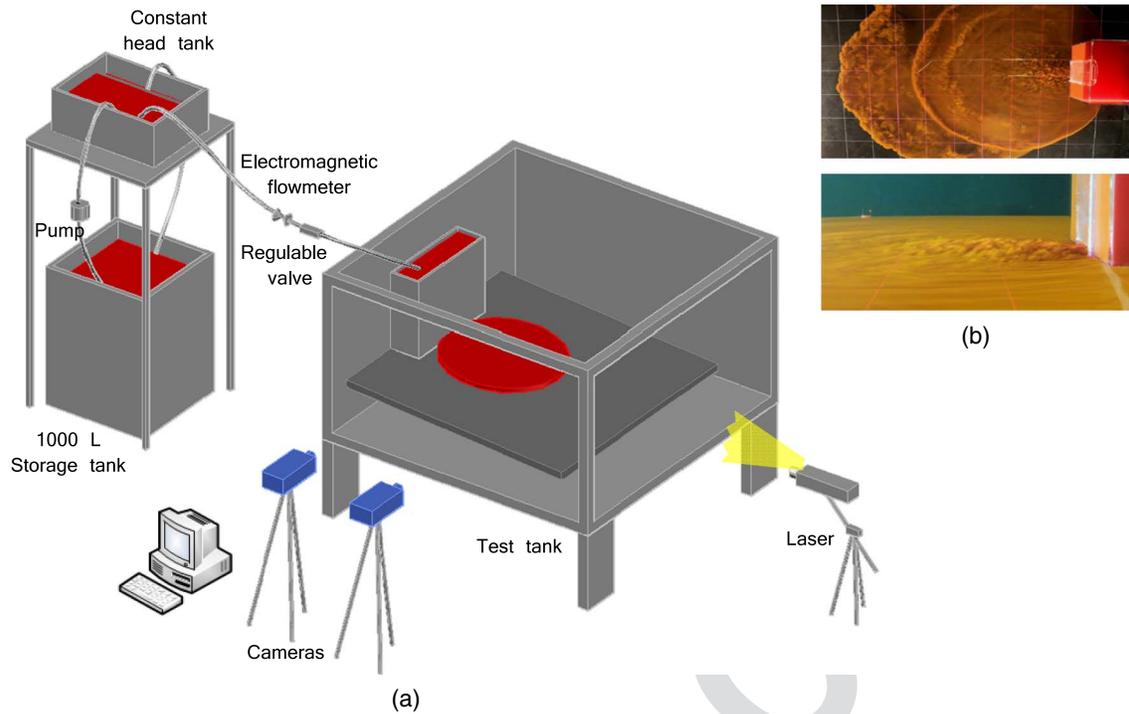


Fig. 1. (a) Schematic diagram of the PIV-LIF experimental setup; and (b) photographs of the discharge device.

Table 1. Main characteristics of laboratory configurations

Cases	Slot dimensions, $b_0 \times h_0$ (m)	Water depth, Ha_0 (m)	Slope, α (%)	Density difference,	Discharge flow-rate,	Buoyancy flux, Bf_0	Reynolds number, R_0 ($u_0 h_0 / \nu$)	Froude number, F_0 ($u_0 / \sqrt{h_0 g_0'$)	
				$\Delta\rho$ ($\rho_a - \rho_0$) (kg/m^3)	Q_0 ($u_0 b_0 h_0$) (L/min)	$(Q_0 g \Delta\rho / \rho_a)$ (cm^4/s^3)			
T1:1	C1	0.1×0.026	0.46	1.0	3.14	14.60	749	2,424	3.37
T1:2	C2	0.1×0.016	0.46	1.0	3.10	15.10	765	2,515	7.67
T1:3	C3	0.1×0.026	0.46	1.0	3.13	19.20	984	3,197	4.47
T1:4	C4	0.1×0.026	0.42	2.5	3.07	14.98	753	2,494	3.49
T1:5	C5	0.1×0.026	0.36	4.5	3.14	14.09	775	2,512	3.49
T1:6	C6	0.1×0.026	0.46	1.0	11.08	14.89	2,700	2,499	1.84

Source: Adapted from Pérez-Díaz et al. (2016).

Note: b_0 = slot width; h_0 = slot height; ρ_0 = effluent density; ρ_a = ambient density; u_0 = discharge velocity; g = gravity; and ν = fluid viscosity.

information necessary to reproduce them numerically. Choi and Garcia (2001) studied the spreading rates of nonconfined density currents on sloping beds while varying the density difference and slope angle. The sloping bed used was composed of fiberglass, i.e., with a roughness equivalent to that of glass or Plexiglass. The initial flow parameters of these experiments are clearly detailed in Table 1 of Choi and Garcia (2001). Specifically, experiments from DEN1 to DEN9 (see Table 1 of Choi and Garcia 2001) were used in the present study.

312 Methodology

The present study involved of a series of more than 90 numerical simulations that reproduced the aforementioned laboratory-generated density currents. Simultaneously, data collected from the experiments were used to calibrate and validate the numerical predictions. The simulations of the calibration stage were used to create a numerical setup for predicting the behavior of these types of flows. This calibration stage consisted of a sensitivity analysis of key numerical aspects that may influence the numerical prediction

of density currents such as the domain discretization, the treatment of momentum at the density current source, the hydrostatic hypothesis, and the selection of the TCM. Once the sensitivity analysis was conducted, the proposed setup was validated from a final set of simulations that reproduced the complete set of laboratory-generated density currents.

The specific methodology of the sensitivity analysis involved several simulations of the base application Case C1 (Table 1) by varying by one numerical aspect while keeping the remainder unchanged. Table 2 summarizes the numerical aspects and the variations considered in this study. The significance of the numerical aspect considered for the prediction of density current behavior was analyzed by comparing the numerical results of characteristic magnitudes such as horizontal density current spreading (b), and velocity (U) and dilution (S) evolution within the density current. Dilution was calculated using the following expression:

$$S = \frac{C_0 - C_a}{C - C_a} \quad (7)$$

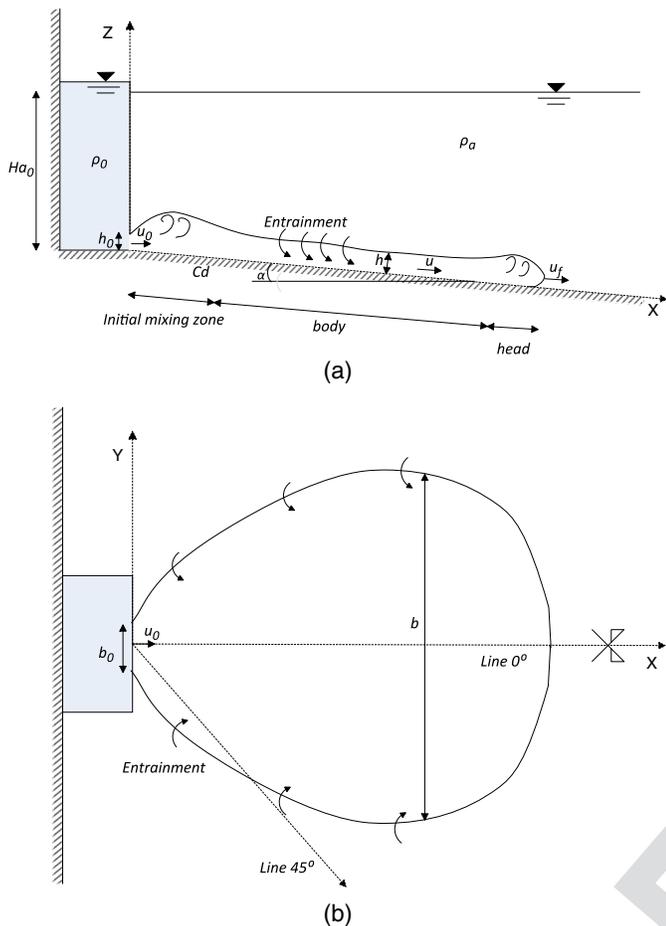


Fig. 2. Scheme of a nonconfined density current: (a) longitudinal profile view; and (b) plan view.

Table 2. Numerical aspects and their alternatives

T2:1	Numerical aspect	Abbreviation	Options
T2:2	Horizontal discretization	Δx	Wide range of Δx functions of b_0
T2:3	Vertical discretization	Δz	Wide range of Δz functions of h_0
T2:4	Source input	Sc_e	Information of Q or Q and V^a
T2:5	Hydrostatic hypothesis	Hyd	With or without hypothesis
T2:6	Horizontal TCM	$TCMh$	Cst^b , $Smag^c$, $\kappa-\epsilon$
T2:7	Vertical TCM	$TCMv$	Cst , ML^d , $\kappa-\epsilon$
T2:8	Advection scheme $\kappa-\epsilon$	$AdSch_{\kappa\epsilon}$	$Charcs^e$, 2nd O-KP ^f

^a V = injection velocity.

^bConstant model.

^cSmagorinsky model.

^dMixing-length models.

^eMethod of characteristics.

^fSecond-order Kurganov and Petrova scheme.

where C_0 = initial salinity concentration of the source; C_a = surrounding fluid salinity concentration; and C = salinity concentration at the study point within the density current.

340 Initial Model Setup

The initial and boundary conditions of the TELEMAC-3D applications were defined with the aim of numerically simulating the

real conditions of the experimental setup. Accordingly, nodes across the domain were initialized with constant corresponding elevation values and with zero velocities (stagnant receiving water). For the boundary conditions, the free surface, open boundaries (i.e., liquid boundaries), and base and rigid walls were taken into account. At the free-surface boundary, a rigid-lid approximation that considers zero gradients and zero fluxes perpendicular to the boundary was applied (i.e., $\partial U_i / \partial z = \partial \kappa / \partial z = \partial \epsilon / \partial z = \partial T / \partial z = 0$). At the open boundaries, streamwise gradients of all of the variables (i.e., velocities, tracer, and fluxes) were set to zero and a prescribed elevation was applied. Strictly speaking, boundary conditions for velocities subjected to rigid wall reflect a no-slip condition (i.e., Dirichlet conditions $U_i = 0$). However, due to the presence of turbulence and of a boundary layer, the velocity close to the base quickly becomes non-zero, and the no-slip condition is replaced with tangential stress [i.e., $\tau = \mu(\partial \vec{U} / \partial n)$] due to friction subjected to the base. This tangential stress is replicated by a turbulence model for the bottom using the friction or shear velocity $\tau = -\rho(U^*)^2$ and the distance to the bottom z . Assuming that the flow is hydraulically rough [i.e., the characteristic roughness size of the base is greater than the thickness of the viscous sublayer (Hervouet 2007)], the velocity profile close to the base was defined by a logarithmic law function of the Nikuradse coefficient k_s representing the roughness size. As the base material used for the experiments was plastic, a Nikuradse coefficient of 10^{-5} m was set. For the rigid vertical walls, slip conditions were assumed (i.e., without friction). Tracer concentration gradients were also set to zero for the rigid walls (base and vertical rigid walls). For the turbulent kinetic energy κ and its dissipation rate ϵ , the boundary conditions defined by Rodi (1984) for rigid walls were applied.

Note that while in other numerical experiments (e.g., Firoozabadi et al. 2009) dense fluid enters the domain through an open-liquid boundary with the slot dimensions, in this study the saline flow rate was determined based on a series of discrete source terms. The number of source terms was determined via the slot dimensions and the domain discretization. This way, the findings of this study can be applied to future field applications (e.g., brine discharges from desalination plants) wherein the saline outflows are typically located within the study domain rather than along boundaries.

As these types of hydrodynamic models and corresponding grid tools are designed and generally configured (e.g., accuracy levels and number of iterations) to model coastal and ocean processes, laboratory tests should be scaled up to prevent numerical problems from emerging. Froude similarity [i.e., the relevant forces are the inertial and gravity forces (Heller 2011)] and mechanical similarity (i.e., geometric, kinematic, and dynamic similarity) are expected to be achieved and thus fewer scale effects are expected to be obtained. However, as the Reynolds number, R , of the case studied was not sufficiently high (i.e., $R \gg 2000$) to directly neglect the viscous force, a previous sensitivity analysis varying the scale $Sc_F = L_{NUM} / L_{LAB}$ (considering the Froude similarity) was performed. Scale factors studied include the following: $Sc_F = 1; 10; 20; 50; 80; 100; 200; 1000$. This analysis showed that after converting all of the results to the same scale, the relative difference of main quantities (concentration and velocity) for the scale-sensitivity cases (at geometrically equivalent locations) was always lower than 2%. This negligible difference attributable to numerical and scale effects shows that Froude similarity can safely be assumed. Thereafter, a $Sc_F = 100$ scale factor was used so that the modeled density currents have similar characteristics to the far field region of brine discharges.

13 Table 3. Numerical aspects in each of the sensitivity tests considered

T3:1	Sensitivity tests	Δx	Δz_{\min}	<i>Sc</i>	<i>Hyd</i>	<i>TCMh</i>	<i>TCMv</i>	<i>AdSch_{k\varepsilon}</i>
T3:2	Domain discretization	$\Delta x_1 = [bo/20, bo]$ $\Delta x_2 = [bo/4, 3bo]$	$[ho/40, ho]$	<i>Q</i> and <i>V</i>	with	Cst	MLPrandtl	—
T3:3	Source-input hydrostatic hypothesis	$\Delta x_1 = bo/8$ $\Delta x_2 = bo$	$ho/20$	<i>Q-Q</i> and <i>V</i>	with-without	Cst	MLPrandtl	—
T3:4	Turbulence	$\Delta x_1 = bo/8$	$ho/20$	<i>Q</i> and <i>V</i>	with	Cst	Cst, ML	Chars
T3:5	Modeling	$\Delta x_2 = bo$	—	—	—	Smago	$k-\varepsilon$	2nd O-KP

406 Sensitivity Analysis Results and Discussion

407 Domain Discretization

408 Appropriate computational grid design is critical to simulate real
 409 physical processes while avoiding artificial numerical effects.
 410 As a general rule, a fine grid domain discretization (i.e., high res-
 411 olution) is needed when high spatial-temporal gradients of the
 412 variables modeled are anticipated. For negatively buoyant density
 413 current flows, high variability areas are the zone closest to the
 414 bottom (high vertical gradients) and the surroundings of the dis-
 415 charge location (high vertical and horizontal gradients). This sec-
 416 tion presents the results of our domain discretization sensitivity
 417 tests, in which the horizontal and vertical discretization were var-
 418 ied, leaving the rest of the numerical aspects constant according to
 419 Table 3. We note that sigma-layer coordinates are necessary to
 420 resolve the vertical structure of the density current. The number of
 421 sigma planes and their spacing was therefore investigated as part of
 422 the sensitivity tests.

423 First, sensitivity to the horizontal discretization was studied. For
 424 this purpose, vertical discretization close to the bottom was set as
 425 $\Delta z = ho/20$, following the recommendations of Kulis and Hodges
 426 (2006). As shown in Table 3, two horizontal discretization param-
 427 eters were defined, namely Δx_1 , the highest resolution close to the
 428 discharge location, and Δx_2 , the lowest resolution in the area fur-
 429 thest away from the discharge location. To compare corresponding
 430 patterns of horizontal spreading, Fig. 3(a) shows the front positions
 431 versus time in the plane of symmetry and in the plane at 45° relative
 432 to the symmetry plane (see lines 0° and 45° of Fig. 2). These spatial
 433 and temporal quantities are normalized by the characteristic length
 434 and time scales of plume-like behavior flows (Chu and Jirka 1987;
 435 Choi and Garcia 2001), $L_p = Q_0^{3/5}/Bf_0^{1/5}$ and $T_p = Q_0^{4/5}/Bf_0^{3/5}$
 436 (Table 1). Furthermore, Figs. 3(b and c) show the longitudinal profile
 437 of normalized maximum velocity (U_{\max}/U_0) and minimum
 438 dilution (S_{\min}), respectively, used to analyze the evolution of the
 439 density current for the approximate centerline, i.e., where the dilu-
 440 tion of the density current is the lowest.

441 Fig. 3(a) shows that for values of Δx_1 smaller than half of the
 442 horizontal slot dimension, i.e., $bo/2$, the spreading converges to
 443 similar values for all of the cases. From the longitudinal profile of
 444 the normalized maximum velocity shown in Fig. 3(b), higher differ-
 445 ences can be observed in the region close to the discharge point.
 446 Using the test with the smallest value of Δx_1 as a reference (a value
 447 close to the one), it is evident that a discretization of at least $\Delta x_1 <$
 448 $bo/4$ is needed to capture flow motion in the discharge surround-
 449 ings. Additional tests varying Δx_2 from $bo/4$ to $3bo$ show that in
 450 areas positioned far from the discharge location, the horizontal dis-
 451 cretization can be coarser and the results do not show significant
 452 differences. In this way, the computational time can be minimized,
 453 but due to the high levels of horizontal discretization variability,
 454 special attention must be paid to the *TCMh* when it is set as the
 455 constant model. In such cases, it is recommended that different
 456 eddy viscosity and diffusivity coefficient values are applied along

the study domain according the grid resolution (Madsen et al. 1988).

Second, the sensitivity to the vertical discretization was ana-
 lyzed. Following Kulis and Hodges' (2006) methodology to find
 the optimal vertical discretization, several tests were conducted
 both with and without the *TCMv* for a wide range of vertical
 discretizations (i.e., wide range of number of layers): $\Delta z_{\min} =$
 $[ho/40, ho]$. To give computationally efficient simulations, the
 highest resolution for each case (i.e., the lowest values of Δz ,
 Δz_{\min}) was established for the region twice the height h_0 from

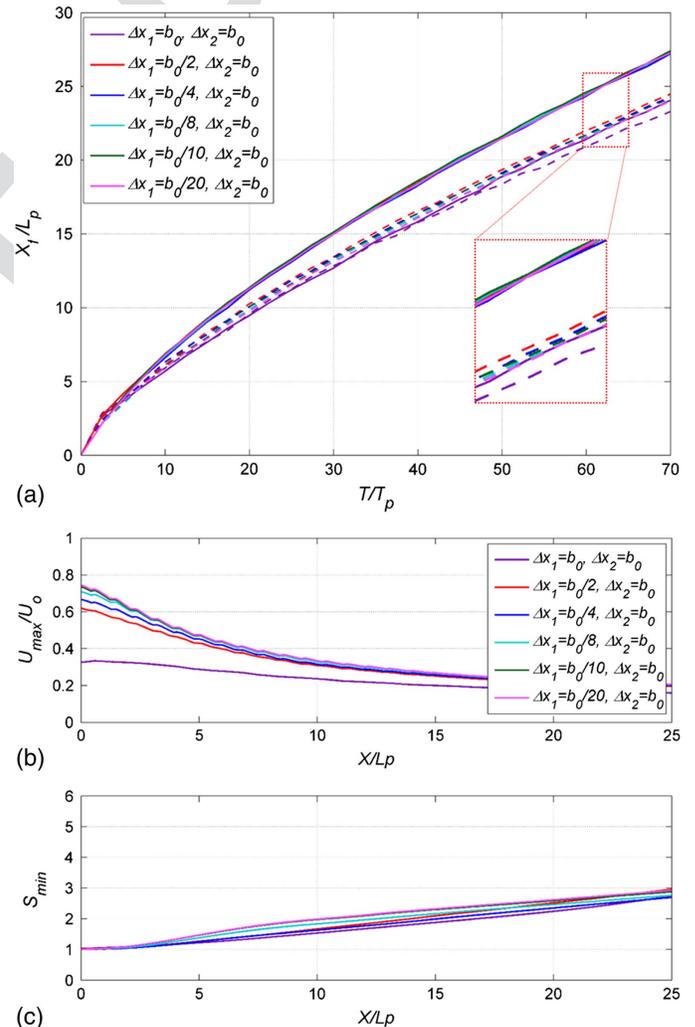


Fig. 3. Horizontal discretization sensitivity tests: (a) dimensionless front position versus dimensionless time for lines 0° (continuous line) and in line 45° (dashed line); (b) longitudinal profile of normalized maximum velocity; and (c) longitudinal profile of minimum dilution.

467 the base and from that depth to the surface, spacing was gradually
 468 increased by a factor of 1.5. The results of these tests (with and
 469 without TCM v) were used to evaluate the relationship between
 470 global modeled and numerical mixing (i.e., mixing only due to
 471 numerical effects not related to the real physics of the process) at
 472 different vertical discretizations. For cases in which the TCM v was
 473 turned off (no turbulence NT tests), all mixing can be attributed to
 474 molecular Brownian motion. As in these NT tests the kinematic
 475 viscosity (i.e., molecular) coefficient was set to 10^{-9} m²/s, vertical
 476 mixing should effectively be zero. Therefore, any mixing observed
 477 in the NT tests can be attributed to numerical effects or so-called
 478 numerical mixing (or numerical diffusion). Conversely, for cases
 479 in which the TCM v was turned on, mixing can be attributed to
 480 the combination of turbulent and numerical mixing, henceforth re-
 481 ferred to as global modeled mixing. The aim of this specific analy-
 482 sis is to detect the number of layers from which numerical mixing
 483 can be considered negligible as compared with the global modeled
 484 mixing. Calculating the vertical entrainment coefficient (E) in the
 485 symmetrical longitudinal profile (line 0°) for both types of tests
 486 generates a quantitative measure for mixing. The entrainment co-
 487 efficient is calculated from the salinity longitudinal profile using
 488 the method developed by Dallimore et al. (2001). This method is
 489 based on equations for the conservation of volume (Eq. 8) and sol-
 490 ute mass (Eq. 9)

$$\frac{d(Uh)}{dx} = EU \quad (8)$$

$$U\beta h = \text{constant} \quad (9)$$

491 where U , β , and h are the mean values for velocity, fractional
 492 density [$\beta = (\rho - \rho_a)/\rho_a$] generated from saline concentrations,
 493 and the density current thickness for each location, respectively.
 494 The value of the current thickness h is defined as the distance from
 495 the base where the salinity concentration is less than 10% of
 496 the maximum concentration value at the corresponding location.
 497 Combining Eq. (8) with Eq. (9) generates equation Eq. (10) where
 498 dC/dx is the variation in concentrations when the current travels
 499 with the mean velocity. By assuming similarity of the concentration
 500 profiles (Parker et al. 1987; Pérez-Díaz et al. 2018), C can be set as
 501 proportional to C_{\max}

$$E = -\frac{h}{C} \frac{dC}{dx} = -\frac{h}{C_{\max}} \frac{dC_{\max}}{dx} \quad (10)$$

502 Fig. 4 shows the value of the average normal entrainment co-
 503 efficient E_N relative to the number of vertical layers within the den-
 504 sity current $N_{zh} = h_0/\Delta z_{\min}$ for different horizontal discretizations
 505 and for both kind of tests, namely with and without the application
 506 of the TCM v . The variable E_N represents a single average value
 507 calculated from $X/L_p = 15$ to eliminate the effects of the density
 508 current's initial adjustment on the normal flow state.

509 Fig. 4 reveals that with the exception of those for the most
 510 coarsely resolved tests, the global modeled entrainment rates
 511 (i.e., entrainment rates of cases with the TCM v set to ML) converge
 512 on the order of 10^{-2} while numerical entrainment rates (i.e., entrain-
 513 ment rates of cases with the TCM v turned off, NT) decrease nearly
 514 exponentially as the vertical resolution increases. For tests involv-
 515 ing values of $h_0/\Delta z_{\min}$ higher than 8, the difference between the
 516 two entrainment rates reaches close to or higher than an order of
 517 magnitude of 10^{-2} (specifically, for $h_0/\Delta z_{\min} = 8$, global modeled
 518 entrainment rates are less than 1.2×10^{-2} and numerical rates are
 519 higher 4×10^{-3}). Thus, for cases involving more than eight layers
 520 within the density current ($N_{zh} = h_0/\Delta z_{\min} > 8$), numerical mix-
 521 ing ceases to dominate global modeled mixing. We note that when

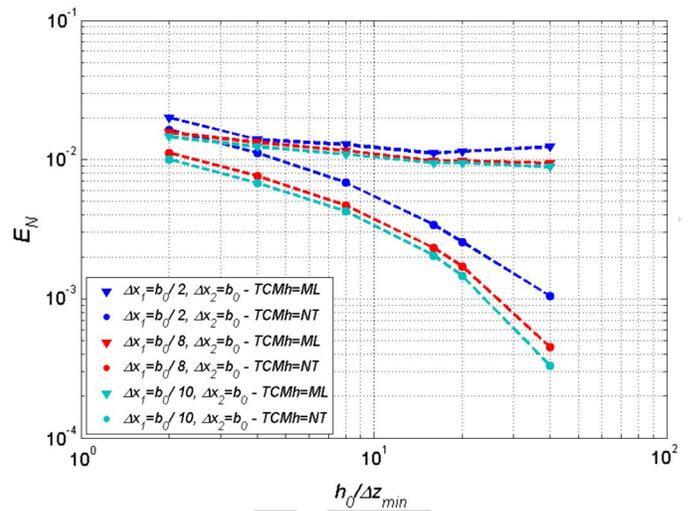


Fig. 4. Vertical discretization sensitivity tests showing normal entrainment versus vertical discretization for different horizontal discretizations.

considering the horizontal resolution sensitivity tests for the each of the corresponding number of layers, the previous pattern is maintained and always generates higher entrainment rates for tests involving coarser horizontal resolutions. Fig. 4 shows that both horizontal and vertical resolutions affect the global modeled mixing; the higher the resolution, the lower the degree of numerical diffusion. To ensure that the effects of numerical diffusion are negligible, our testing shows that $h_0/\Delta z_{\min}$ values of 16–20 are needed. As shown in Fig. 4, this ensures rates of numerical diffusion that are approximately an order of magnitude below the global modeled mixing.

Source Input and Hydrostatic Hypothesis

Having defined a suitable computational grid, numerical aspects that can also affect the initial region of the density current are studied. As cited in the numerical model description, the TELEMAC-3D model allows both hydrostatic and nonhydrostatic pressure fields to be assumed. The hydrostatic assumption is mainly valid for anisotropic flows for which scales of motion are substantially larger in the horizontal than in the vertical. As density currents are primarily horizontal flows, the hydrostatic hypothesis should be appropriate. However, as Mahgoub et al. (2015) show, vertical accelerations may not be negligible relative to gravitational acceleration for the starting region of the density current due to local effects found in the near field region. Numerically, with a nonhydrostatic pressure field, the vertical momentum equation is fully solved without simplification, and the pressure equation is split up into hydrostatic pressure and a dynamic pressure terms. TELEMAC-3D also allows the specification of a liquid flow rate Q an injection velocity V . This section presents our analysis of the effects of these numerical aspects on the density current behaviour. The numerical parameters for these simulations are presented in Table 3.

Note that while physically there is only one source of $h_0 \times b_0$ dimensions, due to domain discretization, it is numerically transformed in several discrete source terms. For instance, 160 discrete sources of a liquid flow rate of $Q_0/160$ are needed for the specifications shown in Table 3 ($\Delta x_1 = b_0/8$ and $\Delta z_{\min} = h_0/20$).

Fig. 5 shows the effects of the source specification type and pressure formulation (i.e., hydrostatic hypothesis) on the horizontal

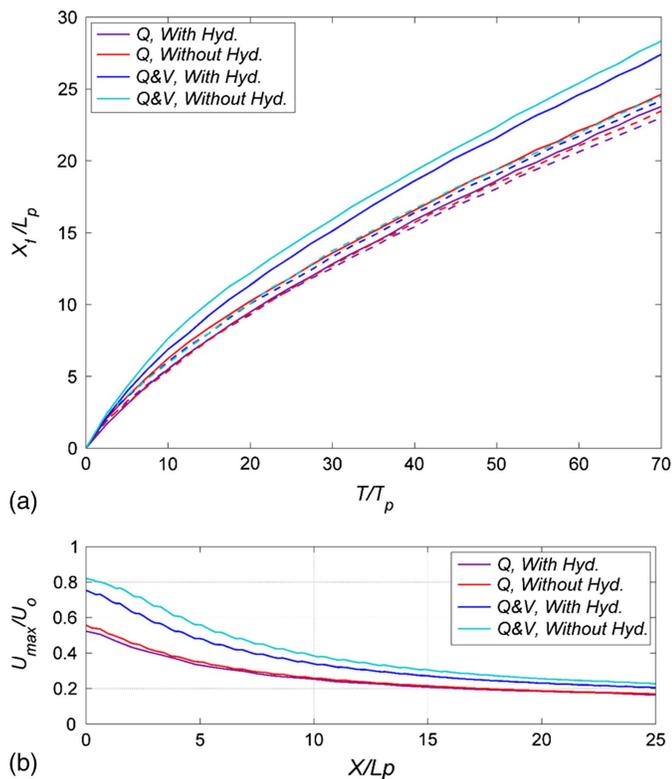


Fig. 5. Source-input and hydrostatic-hypothesis sensitivity tests: (a) dimensionless front position versus dimensionless time for lines 0° (continuous line) and 45° (dashed line); and (b) longitudinal profile of normalized maximum velocity.

spreading and the symmetrical longitudinal profiles of maximum velocity. As is shown for the source specification, Q and V information is needed to capture flow motion in the discharge surroundings, obtaining U_{max}/U_o values closer to 1. Consequently, horizontal spreading rates for cases involving injection velocity [Fig. 5(a)] are higher. The assumption of the hydrostatic pressure has a lesser impact, but tests with nonhydrostatic pressure generate values of U_{max}/U_o of closer to 1. Nevertheless, the present study found that the nonhydrostatic simulations required an additional smoothing to be applied to the free surface elevation solution to reduce oscillations. Further study would be required to optimize the nonhydrostatic model configuration, if it were required for a particular application. However, for the present application, differences between the nonhydrostatic and hydrostatic solutions were not considered to be significant. Furthermore, the solution of the more complex system of equations increased computation times by a factor of 1.5–2 for the density current simulations. Therefore, the hydrostatic pressure formulation is considered to be most appropriate, and is likely to give more acceptable simulation times for field-scale applications involving more complex environmental conditions.

Turbulence Modeling

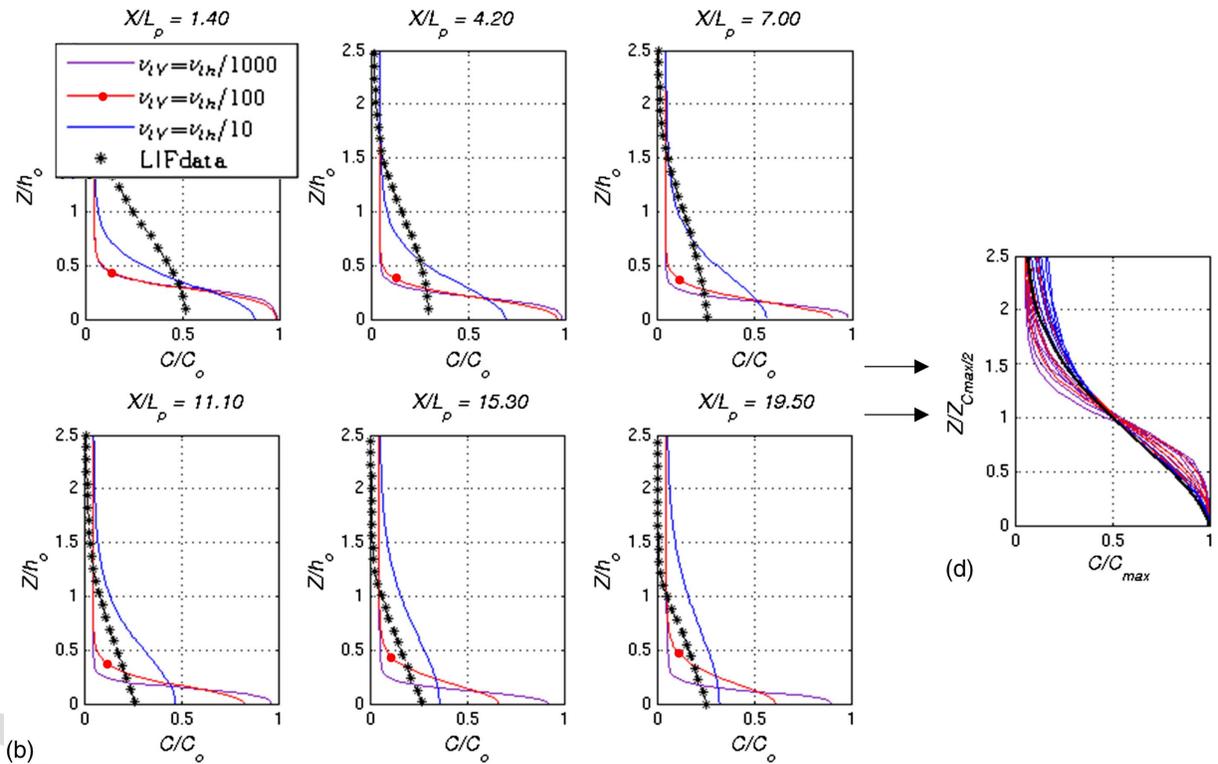
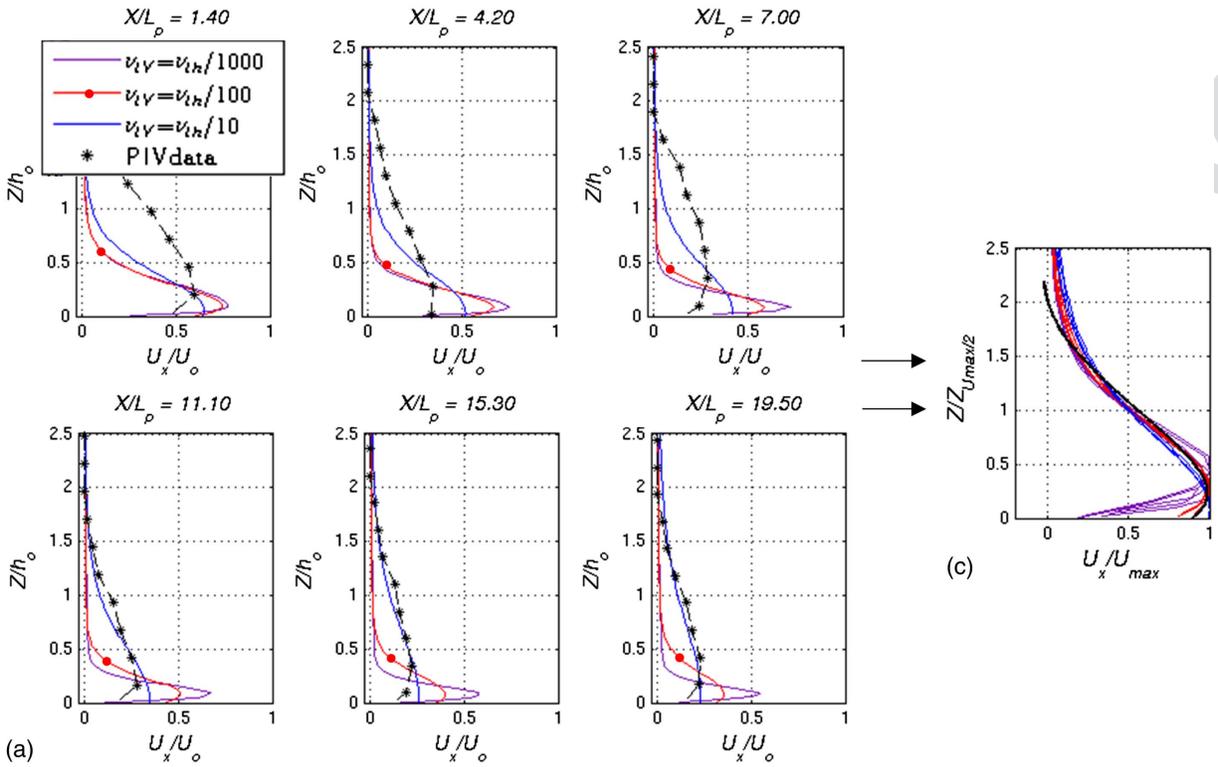
Based on the previous sensitivity tests, this section presents findings derived from the application of well-known TCMs briefly described in the previous section: constant, Smagorinsky, mixing length, and κ - ϵ . Table 3 summarizes both the fixed numerical aspects (Δx , Δz , Sce , and Hyd) and options considered for this sensitivity test (TCM h , TCM v , and $AdSche_{\kappa\epsilon}$).

For the turbulence closure model of the horizontal direction (TCM h), it is common for hydrodynamic models to use a constant turbulence model. Thus, the user must calibrate the horizontal eddy viscosity value ν_{th} depending on the particular flow being studied and depending on domain discretization (Madsen et al. 1988). However, in taking advantage of options programmed into TELEMAC-3D, simulations varying the TCM h between the constant and Smagorinsky models were compared. For the studied case, while using the Prandtl mixing length TCM v , no appreciable differences were observed in the results. For these tests, ν_{th} was defined according to the variable grid resolution (applying scaled-down absolute values of 4.7×10^{-5} to 1.6×10^{-4} m/s²), and the calibration parameter of the Smagorinsky model (C_s) was defined as 0.1, a common value for anisotropic flows (e.g., flow in a canal). The κ - ϵ model was also set as the TCM h model, but in this case the κ - ϵ model was mandatory for the TCM v , and so the influence of the TCM h could not be extracted. In addition, the simulations became so time-consuming and unstable (due to numerical problems found at open liquid boundaries based on Neumann boundary conditions for the κ and ϵ equations) that this option for the TCM h was not considered.

Due to the strong stratification associated with density currents, the turbulence closure model of the vertical direction (TCM v) is a key numerical aspect for accurately predicting their behavior. To analyze the TCM v 's influence on the vertical structures of the flow, Figs. 6–8 show downstream variations in the velocity (the main velocity component, U_x) and salinity (C) cross profiles found along the symmetry plane (see line 0° of Fig. 2) for each TCM v case in the constant, mixing length, and κ - ϵ models. For all of the simulations, the TCM h is held as constant according to the results presented previously. Furthermore, these figures present normalized cross profiles that collapse into a single profile, called a similarity profile (Ellison and Turner 1959; Parker et al. 1987; Garcia 1993). In addition to our numerical results, measured data obtained via PIV-PLIF techniques are also displayed.

Fig. 6 shows cross profiles derived from different simulations with constant TCM v based on varying ν_{tv} values of $\nu_{th}/1000$ to $\nu_{th}/10$. In these simulations the eddy diffusivity value Γ is defined by the Schmidt number ($\sigma_c = \nu_{tv}/\Gamma_v$), which is considered to take values of close to one for these types of flows. Both the velocity and salinity concentration cross profiles show that the simulation $\nu_{tv} = \nu_{th}/10$ better fits the experimental data. However, the simulation does not represent the approximate shape of the first cross sections, i.e., those with higher momentum and concentration values. For the remainder of the cross sections, the velocity agrees with the experimental data while the dilution is underestimated (i.e., higher concentrations than were expected). As is shown by the similarity cross profiles, the normalized mean horizontal velocity and mean concentration cross profiles collapse well into single profiles, which agree with the experimental similarity profiles.

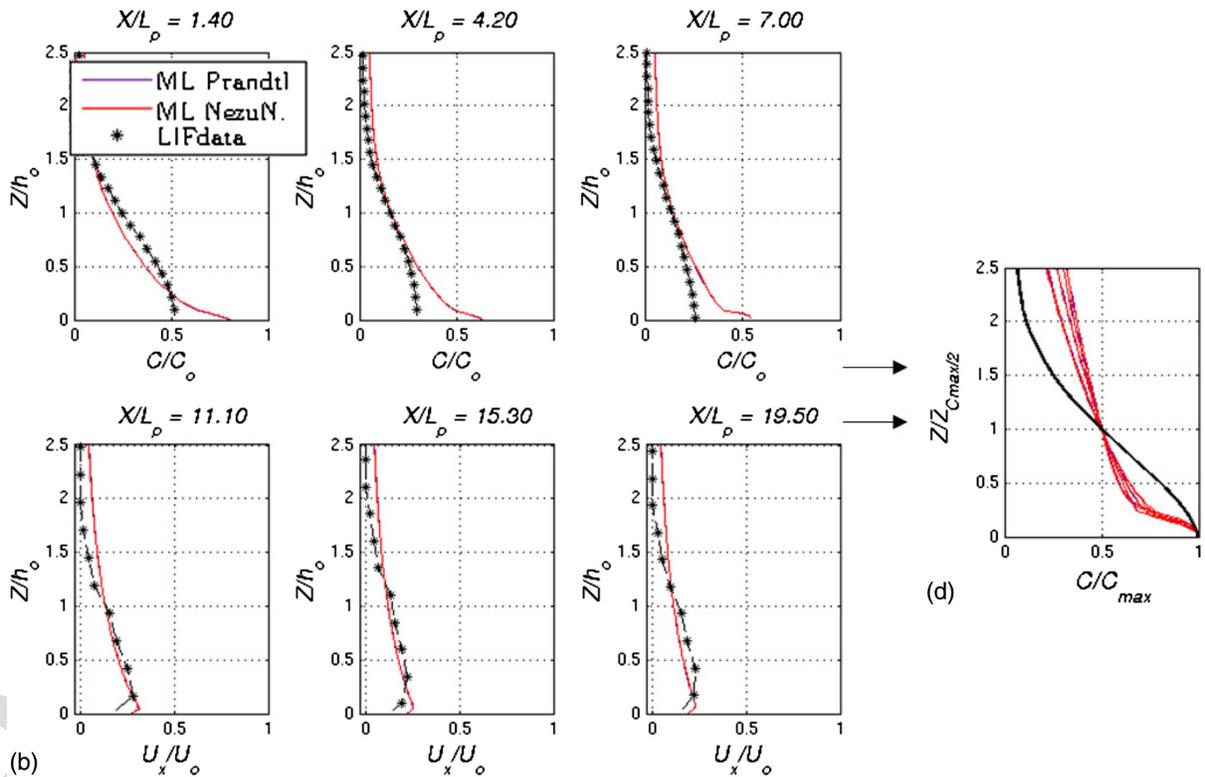
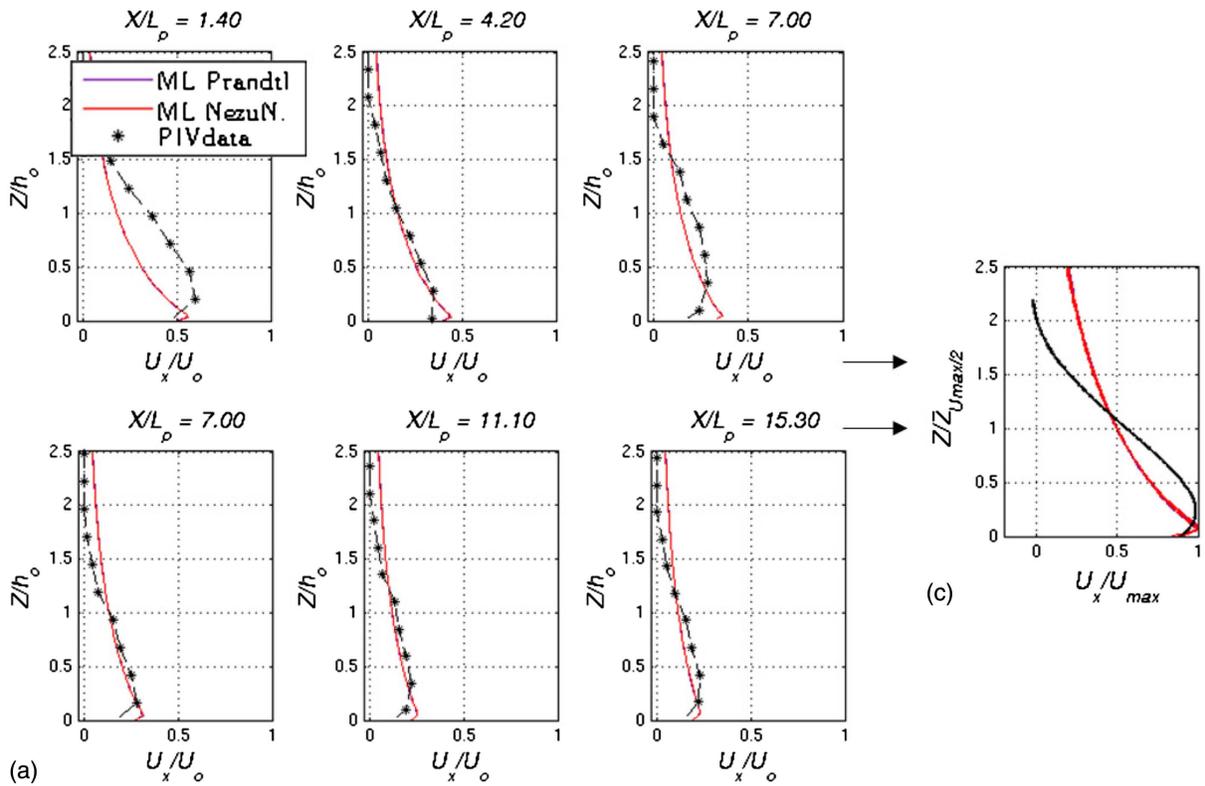
Fig. 7 shows cross profiles for the TCM v mixing length models (both the Prandtl formulation and the Nezu and Nakagawa formulation). For these cases, the damping function addressed by Munk and Anderson (1948) is used to govern vertical mass and momentum exchanges. The results of the two simulations are almost identical, showing logarithmic velocity and concentration profiles for the most parts of the density current body. In these cases, the profiles are the result of agreement between the ML model, the damping function and the turbulence model for the bottom based on the previously defined roughness size (i.e., the Nikuradse coefficient). While the velocity cross profiles match fairly well with experimental data for the whole density current body, the concentration profiles significantly underestimate dilution levels found in the region close to the bottom for areas located closer areas to the discharge.



F6:1 **Fig. 6.** Downstream variation cross profiles for cases with $TCMv = Cst$: (a) horizontal velocity; (b) salinity concentration; and the corresponding
 F6:2 similarity cross profiles for (c) horizontal velocity; and (d) salinity concentrations.

653 In addition, the normalized mean horizontal velocity and mean
 654 concentration cross profiles collapse into single profiles but they
 655 disagree with the experimental similarity profiles. This shows that
 656 the presented $TCMv$ option does not capture the vertical shape of
 657 the analyzed density current.

Fig. 8 shows the cross profiles derived from two simulations
 with the $\kappa-\epsilon$ $TCMv$ applied. The two simulations differ in terms
 of advection scheme ($AdSch_{\kappa\epsilon}$) applied. Whereas in one case
 the method of characteristics was used, for the other case the most
 conservative scheme programmed into the TELEMAC-3D was

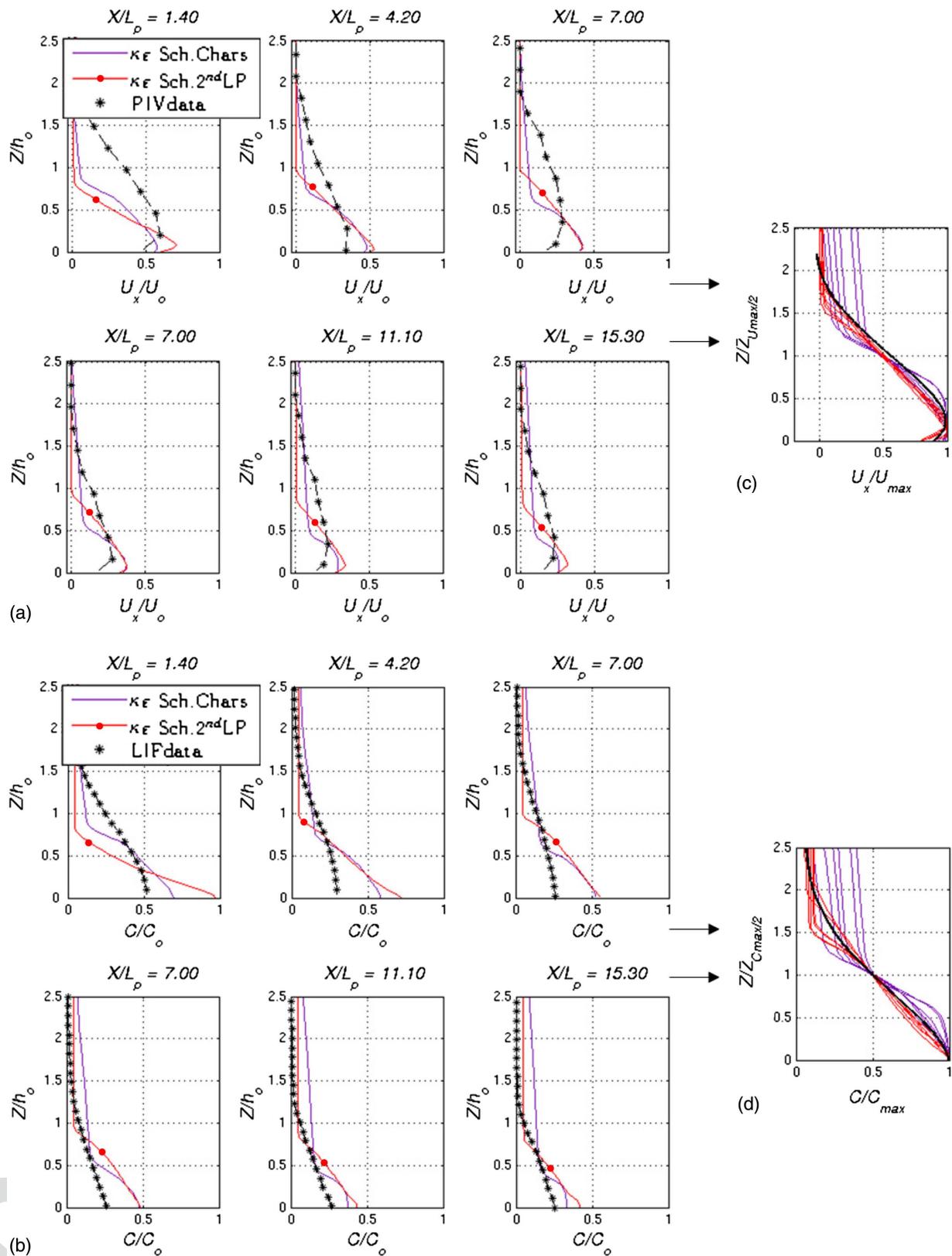


F7:1 **Fig. 7.** Downstream variation cross profiles for cases with $TCM_v = ML$: (a) horizontal velocity; (b) salinity concentration; and the corresponding
 F7:2 similarity cross profiles for (c) horizontal velocity; and (d) salinity concentrations. Note that both lines overlap in all figures.

663 used (2nd-KP). The characteristic method is recommended by
 664 LNHE (2007), as it has provided satisfactory results in many instances and is the most efficient. Nevertheless, due to the unique
 665 nature of these types of flows, where the buoyancy force is a driven
 666

force, i.e., the accurate definition of the mass-tracer quantities is
 fundamental, the authors deemed it necessary to distinguish effects
 of the advection scheme. Fig. 8 shows that both simulations over-
 estimate the velocities and concentrations close to the bottom for all

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F8:1 **Fig. 8.** Downstream variation cross profiles for cases with $TCM v = \kappa\epsilon$: (a) horizontal velocity; (b) salinity concentration; and the corresponding
 F8:2 similarity cross profiles for (c) horizontal velocity; and (d) salinity concentrations.

671 of the locations studied, although the shape of the profile obtained
 672 is noticeably different from one case to the other. Regarding the
 673 similarity profiles, from which a proper shape comparison can
 674 be undertaken, they show that the simulation applying the most

conservative advection scheme presents better agreement with the

675 experimental similarity curves.
 676 Taking the similarity cross profiles into account, i.e., the profile
 677 shape profile, the constant, and the $\kappa\epsilon$ TCM v are the models that
 678

best capture the vertical structure of the density current. As the constant model only uses ν_{tv} as a calibration parameter and as the results presented in Fig. 6 for $\nu_{tv} = \nu_{th}/10$ are its best results, we focus on the κ - ε TCMv. As described in the previous section, the κ - ε model includes several empirical constants obtained via data fitting for a broad range of flows. Of these empirical constants, the $c_{3\varepsilon}$ and c_μ affect the modeling of density currents, and their values have been the subject of debate. Hossain and Rodi (1982), Rodi (1987), and Choi and Garcia (2002) have suggested that $c_{3\varepsilon}$ values of 1–0.6 show a good agreement with experimental results for density currents. Conversely, the standard value of the other controversial empirical constant ($c_\mu = 0.09$) was used on the basis of experiments on flows for which production P and dissipation ε of the turbulence energy were in approximate balance. For weak shear flows (e.g., far-field jets and plumes for which the velocity difference across the flow represents only a small fraction of the convection velocity), P was found to be significantly different from ε , and c_μ was found to take different values (Rodi 1975). Rodi (1972) correlated experimental data and proposed a function of $c_\mu = f(\overline{P/\varepsilon})$ that is only valid for thin shear layers (similar to the density currents studied).

To study effects of these empirical constants on density currents modeling, the results of several simulations varying the values of these constants are analyzed. The range of values for $c_{3\varepsilon}$ and c_μ were chosen based on state-of-the-art findings, namely $c_{3\varepsilon}$ values of 1–0.6 and c_μ values of 0.09–2.5. Another important parameter is the Schmidt number σ_c , which contributes to the eddy diffusivity definition and which is valued at between 0.7 and 1. However, due to its lesser impact on the results compared to the $c_{3\varepsilon}$ and c_μ constants' impact, it was assumed to be equal to 0.7, a common value for heat and salinity transport. Based on the experimental results, comparisons are drawn with results obtained from different simulations using the root mean-square absolute error (RMSE) formula

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_{exp} - x_{num})^2} \quad (11)$$

where x_{exp} = experimental values of the variable studied (in this case the salinity concentration and horizontal velocity); x_{num} represents the corresponding numerical value; and N = number of pairs of comparable values. In this case, due to the large amount of data obtained from the PIV-PLIF experiments, N corresponds to the number of cross profiles multiplied by the number of data points measured within the density current thickness (which varies depending on the variable). To estimate the relative error, the normalized RMSE (NRMSE) was obtained by dividing the RMSE by the initial value of the evaluated magnitude (C_0 or U_0) and by multiplying this value by 100 to obtain the percentage. As a calibration assessment, the error obtained for each simulation while varying the $c_{3\varepsilon}$ and c_μ constants is shown in Table 4.

Table 4 shows that the results of the simulation with $c_{3\varepsilon}$ equal to 0.7 and c_μ equal to 0.2 offer the smallest errors, namely 0.135 psu for salinity and 0.005 m/s for velocity (2.8% and 4.8%, respectively). Fig. 9 shows the downstream variation and similarity cross profiles for the simulation. As is shown, the numerical and experimental data agree fairly well, both in terms of shapes and absolute values.

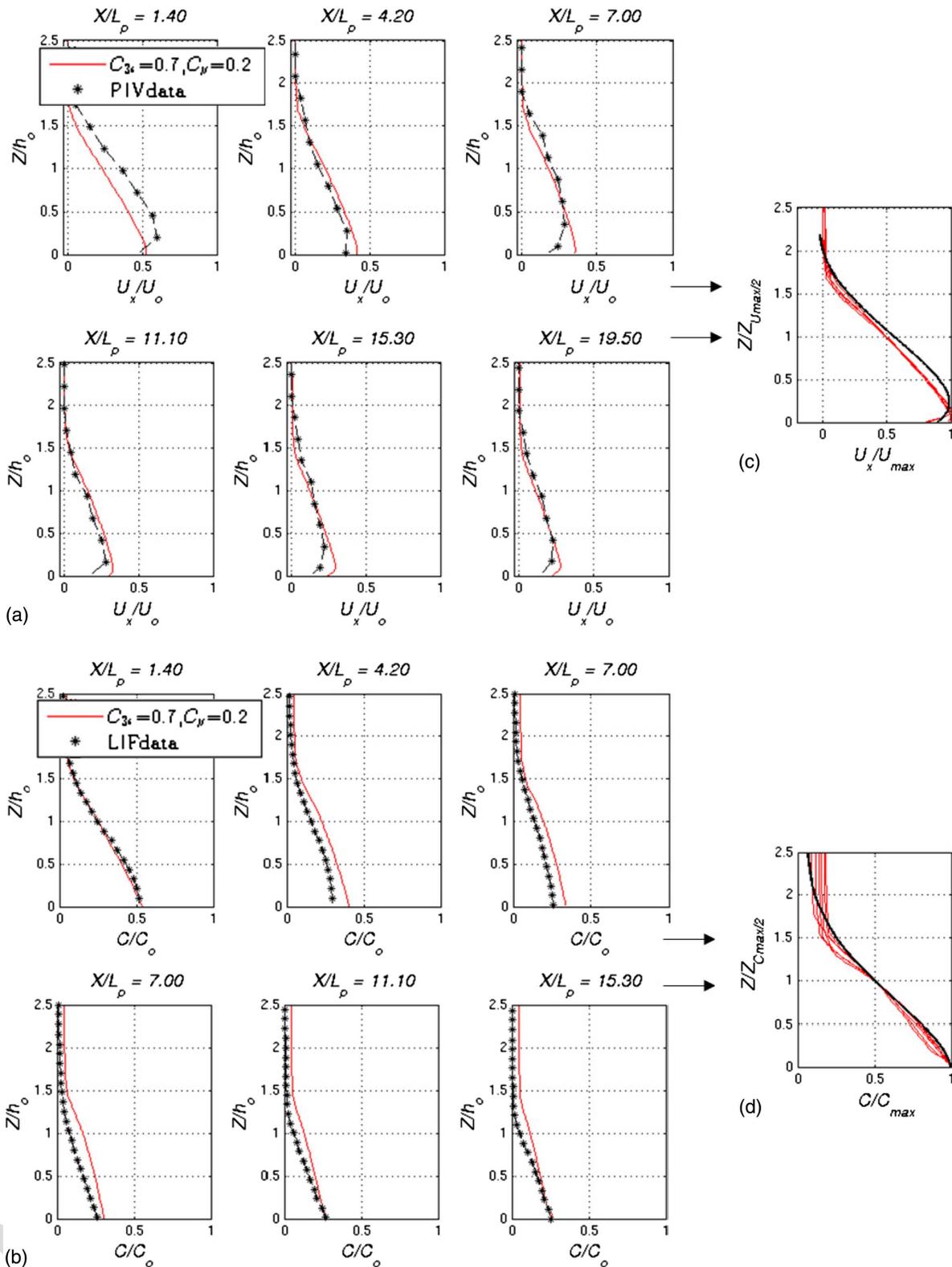
The optimal value of $c_{3\varepsilon}$ obtained from this study ($\sim[0.7 - 0.6]$) agrees with the values published in the scientific literature on density currents (e.g., Hossain and Rodi 1982; Rodi 1987; Choi and Garcia 2002). Note that certain researchers define the empirical

Table 4. Calibration of empirical coefficients c_μ and $c_{3\varepsilon}$ of the κ - ε model

κ - ε coefficients		Salinity errors		Velocity errors		
$c_{3\varepsilon}$	c_μ	RMSE (psu)	NRMSE $_{C_0}$ (%)	RMSE (m/s)	NRMSE $_{U_0}$ (%)	
1	0.09	0.496	10.3	0.012	11.7	T4:1
	0.15	0.396	8.2	0.011	10.5	T4:2
	0.2	0.244	5.0	0.008	7.96	T4:3
	0.25	0.253	5.3	0.008	8.2	T4:4
0.9	0.096	0.45	9.4	0.01	10.2	T4:5
	0.15	0.358	7.5	0.01	9.8	T4:6
	0.2	0.21	4.4	0.007	7.2	T4:7
	0.25	0.219	4.6	0.007	7.5	T4:8
0.8	0.09	0.427	8.9	0.011	10.6	T4:9
	0.15	0.318	6.6	0.009	9.0	T4:10
	0.2	0.177	3.7	0.007	6.7	T4:11
	0.25	0.187	3.9	0.006	6.4	T4:12
0.7	0.09	0.318	6.6	0.008	8.3	T4:13
	0.15	0.199	4.1	0.007	6.8	T4:14
	0.2	0.135	2.8	0.005	4.8	T4:15
	0.25	0.139	2.9	0.006	5.6	T4:16
0.6	0.09	0.355	7.4	0.009	9.0	T4:17
	0.15	0.238	4.9	0.007	7.3	T4:18
	0.2	0.137	2.8	0.006	5.7	T4:19
	0.25	0.14	2.9	0.006	5.7	T4:20

constant as $(1 - c_{3\varepsilon})$ rather than as $c_{3\varepsilon}$ so that the corresponding optimum value is then ($\sim[0.3 - 0.4]$). Conversely, the optimum value of c_μ obtained (~ 0.2) is included within the range of values established by function $c_\mu = f(\overline{P/\varepsilon})$, defined by Rodi (1972). This function is only valid for thin shear layers such as density currents where the argument $\overline{P/\varepsilon}$ is the average value of P/ε across the layer. Launder et al. (1973) showed applying this function significantly improves the κ - ε model's capacity to predict such flows. Far from the standard value ($c_\mu = 0.09$), which is accepted for flows involving the production P and dissipation ε of turbulent kinetic energy in balance ($\overline{P/\varepsilon} \approx 1$), the optimum value obtained corresponds to a $\overline{P/\varepsilon}$ value of ~ 0.5 . Rather, the average production term is approximately half the average dissipation of turbulent kinetic energy. Fig. 10 shows downstream variations of terms involved in κ - ε equations, such as P and ε described previously.

For the graphs plotted in Fig. 10, the horizontal gradient of vertical velocity ($\partial U_Z/\partial X$), the vertical gradient of horizontal velocity ($\partial U_X/\partial Z$) and the vertical eddy viscosity (ν_{tv}) are extracted to obtain the production term (P) following Eq. (4). Small values of $\partial U_Z/\partial X$ were anticipated due to the horizontal nature of the flow studied, and the range of values ($[-2, 3]$ 1/s) obtained for $\partial U_X/\partial Z$ agrees well with the experimental data ($[-2, 2]$ 1/s). As was expected, the eddy viscosities reduce according to the velocity decay. Fig. 10(d) presents the production term (P), which shows very small values close to the bottom compared to the dissipation of turbulent kinetic energy (ε), plotted in Fig. 10(f). Finally, the turbulent kinetic energy (κ) is shown in Fig. 10(e). The shape of the κ cross profiles and their order of magnitude (10^{-5}) match with the experimental horizontal Reynolds stress component (the main component of turbulent kinetic energy in such horizontal flows, $\tau_{XX}/\rho U_X^2$). The κ cross profiles present a clear peak corresponding with the upper flow boundary and a zone of minimum turbulence energy at the location presenting highest velocity, which is consistent with Gray et al. (2006), Islam and Imran (2010), and Gerber et al. (2011).



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F9:2

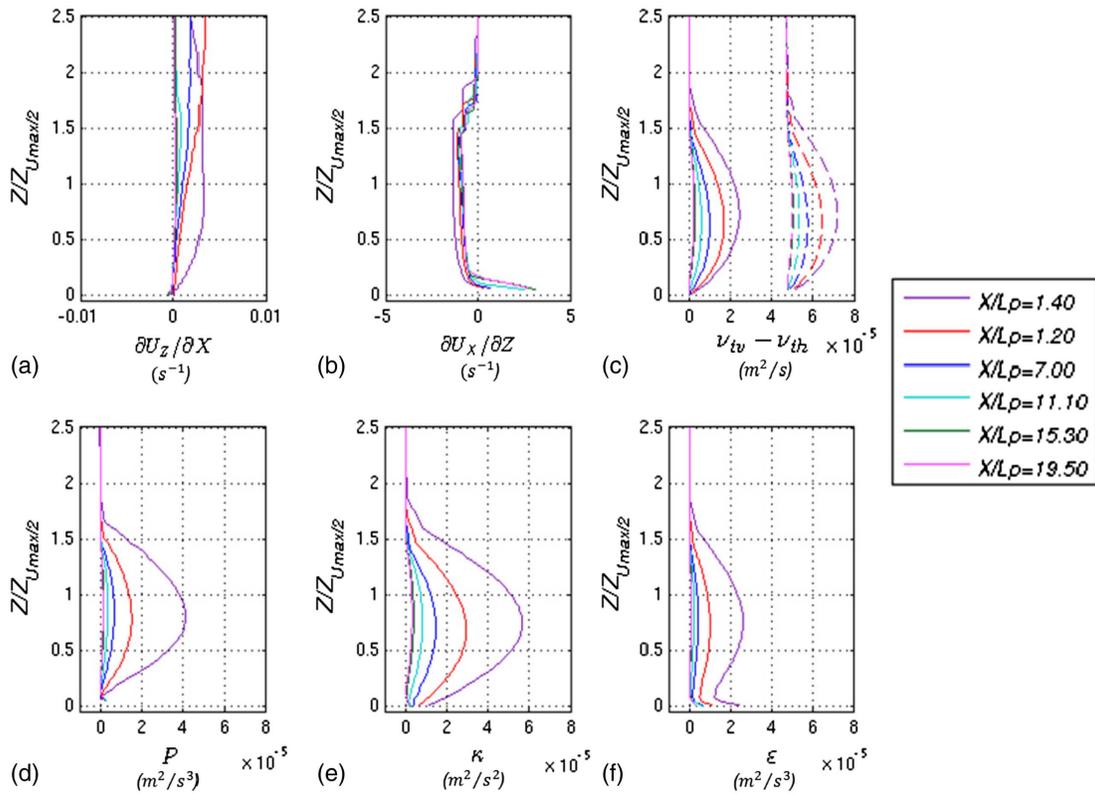
Fig. 9. Downstream variation cross profiles for cases with calibrated $TCM v = \kappa - \epsilon$: (a) horizontal velocity; (b) salinity concentration; and the corresponding similarity cross profiles for (c) horizontal velocity; and (d) salinity concentrations.

773 Validation Results

774 Based on the previous sensitivity analysis, a proposed modeling
775 setup to simulate the behaviour of density currents is shown in
776 Table 5. Apart from the previously calculated relative errors

regarding the evolution of main variables in the cross profiles,
in applying these recommendations to Case 1, a relative error of
the front position ($NRMSE_{X_f,exp}$) of less than 6% is obtained.
For the evolution of the density current in the streamwise direc-
tion, wherein the dilution of the current is the lowest (line 0°),

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F10:1 **Fig. 10.** Downstream variation of the terms involved in $\kappa\text{-}\epsilon$ Eqs. (2) and (3) for the calibrated test: (a) horizontal gradient of vertical velocity;
 F10:2 (b) vertical gradient of horizontal velocity; (c) horizontal (dashed line) and vertical (continuous line) eddy viscosities; (d) production term;
 F10:3 (e) turbulent kinetic energy; and (f) dissipation of turbulent kinetic energy.

17 Table 5. Optimal numerical aspects for predicting density currents

T5:1	Numerical parameter	Optimum
T5:2	Δx	$\Delta x_1 \leq bo/4$
T5:3	Δz	$\Delta z_{\min} \leq ho/16$
T5:4	Sc_e	Q and V
T5:5	Hyd	with
T5:6	$TCMh$	Cst
T5:7	$TCMv$	$\kappa\text{-}\epsilon$ ($c_\mu = 0.2, c_{3\epsilon} = 0.7$)
T5:8	$AdSch_{\kappa\epsilon}$	Most conserv.(p.e., 2nd O-KP)

782 the error is roughly 4% for the main velocity component
 783 (NRMSE $_{U_0}$) and is less than 1.3% for the dilution (NRMSE $_{S_{\min}}$)
 784 value. Fig. 11 illustrates the evolution of the main variables de-
 785 scribed (i.e., the front position, the dilution, and the main velocity
 786 component) and their graphical comparison with the experimental
 787 data. Moreover, Fig. 12(a) presents the good agreement between
 788 the numerical and experimental spreading results for different times
 789 using a plan-view comparison of concentration values. The longi-
 790 tudinal symmetry profile views of the numerical and experimental
 791 concentration results are shown in Figs. 12(b and c), for consis-
 792 tency with the previous results presentation.

793 To ensure that the proposed modeling setup is valid for den-
 794 sity currents generated under different flow conditions, consider-
 795 ing turbulent flow and supercritical regime (i.e., $R > 1000$ and
 796 $F > 1$), it was applied to the complete set of experimental den-
 797 sity currents (Table 1). Comparisons were made between the experi-
 798 mental and numerical results from the minimum dilution obtained
 799 from the last section of density currents ($S_{\min F}$), which represents

800 the result of the transport and mixing processes and the parameter
 801 on which the environmental regulations are commonly based.
 802 Table 6 shows the comparison between the experimental and
 803 numerical results, with subindices E and N , respectively. As is
 804 shown, a good agreement between the experimental and numerical
 805 values was obtained for all cases. Furthermore, the modeled val-
 806 ues $S_{\min F_N}$ maintain the correlation revealed by the experiments
 807 $S_{\min F_E}$. That is, establishing the value of C1 as the base case
 808 $S_{\min F_{Nb}}$ (corresponding to the case analyzed in the sensitivity
 809 analysis), the rates $S_{\min F_N}/S_{\min F_{Nb}}$ agree with the corresponding
 810 experimental ratio $S_{\min F_E}/S_{\min F_{Eb}}$, showing that steeper slopes (C4
 811 and C5) and higher initial momentum values (C2 and C3) enhance
 812 the dilution rate in contrast to the case involving higher initial
 813 buoyancy (C6).

814 Further validation was carried out using the experiments of
 815 Choi and Garcia (2001) (see description on experimental database
 816 section and on Table 1 of Choi and Garcia 2001). Fig. 13(a) shows
 817 that the predicted evolution of the dimensionless half-width ($b_{1/2}$)
 818 agrees well with the experimental data. In addition, Fig. 13(b)
 819 compares longitudinal spreading values based on front velocity
 820 evolution. In this case, velocity is fairly well reproduced when
 821 the initial high momentum region is developed. Beyond this compar-
 822 ison, conclusions regarding effects of the different densities
 823 and slopes on the described behavior can be drawn from the
 824 graphs. In agreement with previous studies (Choi and Garcia
 825 2001; Alavian 1986), steeper slopes favor rapid longitudinal
 826 spreading and hinder lateral spreading (i.e., higher front velocities
 827 and lower half-widths), and a higher density difference favors lon-
 828 gitudinal and lateral spreading (i.e., higher front velocities and
 829 higher half-widths).

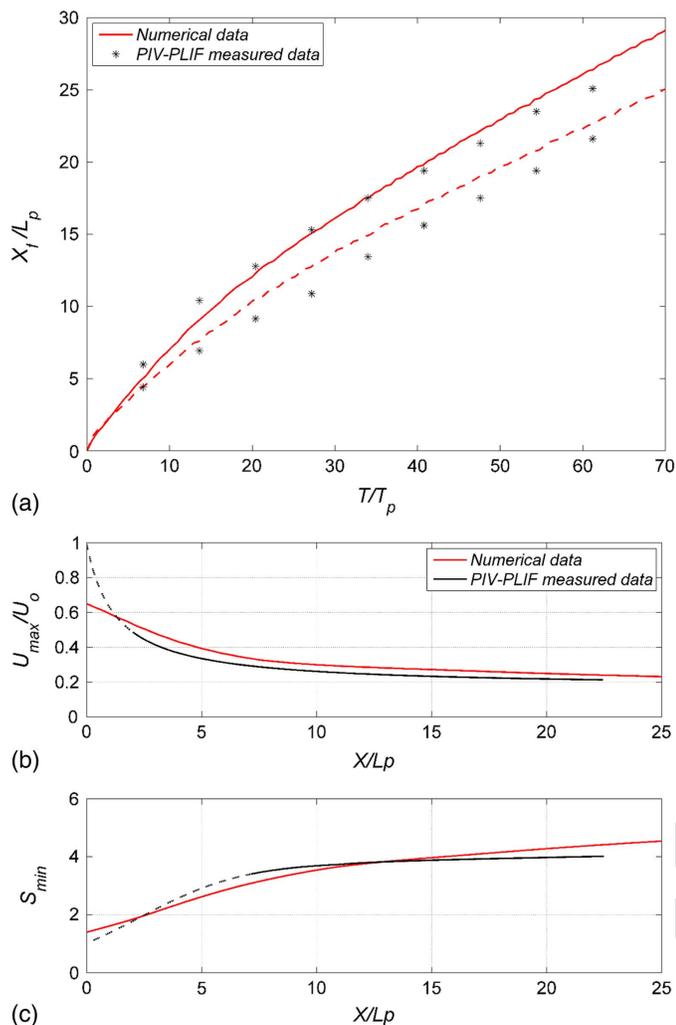


Fig. 11. Comparison between the numerical and the experimental results: (a) dimensionless front position versus dimensionless time for lines 0° (continuous line) and 45° (dashed line); (b) longitudinal profile of normalized maximum velocity; and (c) longitudinal profile of minimum dilution.

Conclusions

Through a comprehensive sensitivity and validation analysis based on the reproduction of several laboratory-generated density currents, this study shows that suitably configured 3D hydrodynamic models can simulate the behavior of saline density currents with a high level of accuracy. Moreover, as these laboratory-generated currents were scaled up to prevent numerical effects and to have similar characteristics to the far field region of brine discharges, the sensitivity and validation analysis as well as the recommendations resulted are extended to real field-scale saline current flows. The main numerical guidelines for solving such flows using these models are as follows:

- Variable spacing horizontal discretization (e.g., unstructured grids) is recommended as a means to obtain high resolutions close to the source. In this way, a lower resolution can be set to the farthest region of the source, rendering application more computationally efficient. Assuming a slot-shaped source ($b_0 \times h_0$), a common approximation for the beginning of the far field region of brine discharges, minimal spacing of at least equal to the width of the slot b_0 must be applied to ensure the momentum and mass conservation as much as possible. Specifically, for the cases analyzed in this study, a spacing of greater than $b_0/4$ is recommended.
- In the vertical direction, a high resolution must be applied to minimize vertical numerical diffusion. For the type of density currents studied in this study and considering the a slot-shaped source, vertical spacing should be at least $h_0/16$. As this fine resolution cannot be maintained along the entire water column (i.e., there would be too many numbers of layers), gradual vertical spacing with the highest resolution close to the bottom (within the density current body) is recommended. In such cases, a sigma layer coordinate (i.e., terrain following) for vertical domain discretization should be applied to keep the finest layers at the bottom.
- Full momentum source specification is advisable, i.e., both flow rate and velocity information (Q and V) should be detailed.
- The hydrostatic hypothesis is considered to be appropriately assumed, as it significantly (1.5–2 times) reduces computation time and because not applying this hypothesis does not generate significantly improved results.

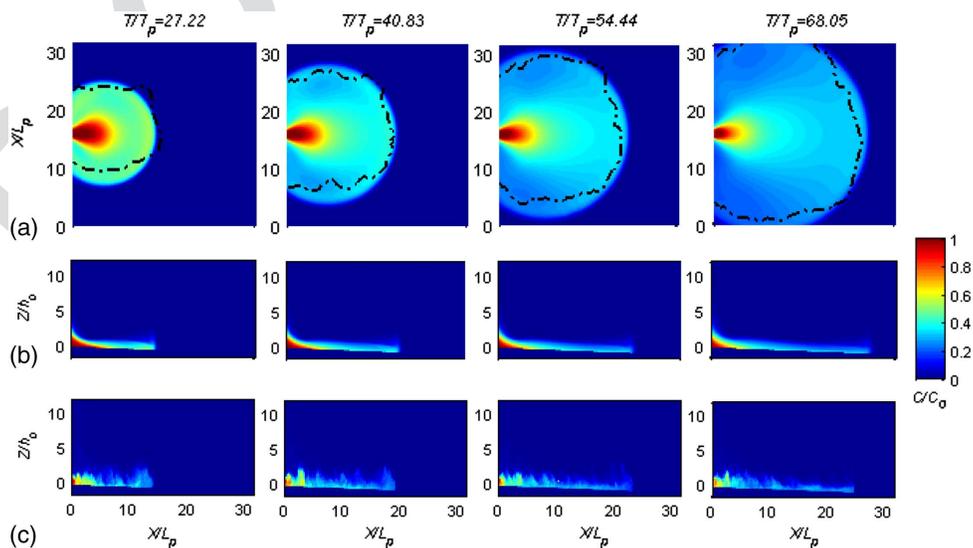


Fig. 12. Comparison between the numerical and the experimental spreading results for different times: (a) plan view (experimental results are shown as the dashed line); (b) profile view of the modelled results; and (c) profile view of the LIF experimental results.

Table 6. Minimum dilution comparison for the last section of the studied density currents

T6:1	Cases	Modified parameter	$S_{\min F_E}$	$S_{\min F_N}$	$S_{\min F_E}/S_{\min F_{Eb}}$	$S_{\min F_N}/S_{\min F_{Nb}}$
T6:2	C1	—	4.3	4.5	$S_{\min F_{Eb}}$	$S_{\min F_{Nb}}$
T6:3	C2	h_0	7.2	7.2	1.66	1.60
T6:4	C3	Q_0	6.8	5.8	1.56	1.28
T6:5	C4	α	5.3	5.9	1.23	1.31
T6:6	C5	α	6.6	6.8	1.525	1.51
T6:7	C6	$(\rho_a - \rho_0)$	4.1	3.74	0.94	0.83

- The constant model is recommended as a horizontal turbulence closure model (TCM h) varying eddy coefficient values according to the grid resolution.
- Several vertical turbulence closure models (TCM v) such as constant, mixing length, or κ - ϵ models can be successfully applied. However, given the demonstrated influential role a TCM v has on the numerical simulation of such flows, calibration should be applied to each case study. Specifically, for the cases analyzed in this study, the calibrated κ - ϵ model (empirical constants $c_{3\epsilon}$ and c_μ are equal to ~ 0.7 and ~ 0.2 , respectively) in conjunction with the most conservative advection scheme for κ - ϵ equations

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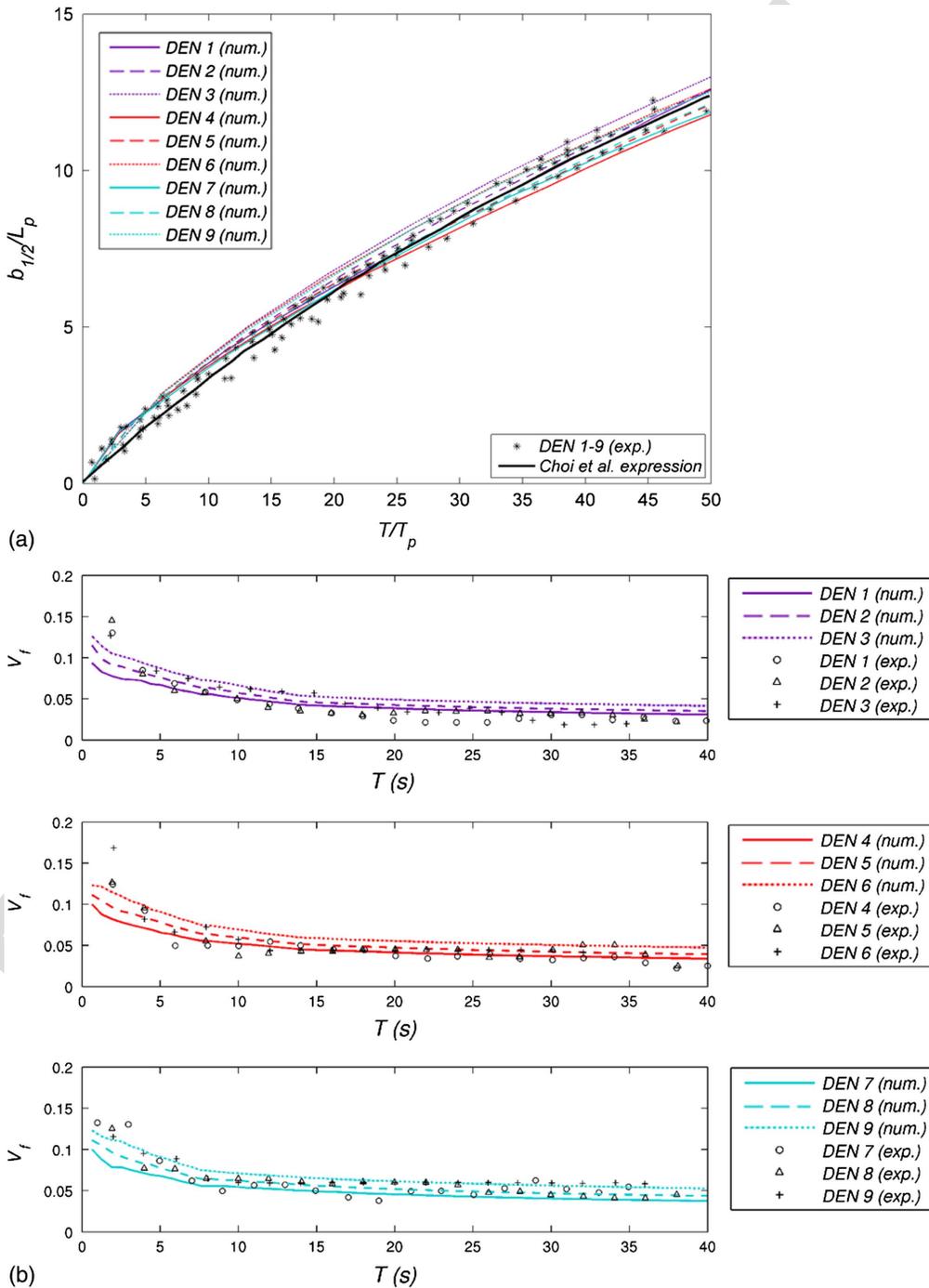


Fig. 13. Comparison between the numerical and experimental results of Choi and Garcia (2001): (a) dimensionless maximum half-width versus time; and (b) velocity front.

881 generates the best results. We recognize that applying the κ - ϵ
 882 turbulence model, which solves two more equations, is demand-
 883 ing in terms of computation time. Both the mixing length model
 884 and the constant model can generate good approximations
 885 within a more reasonable timeframe for field applications.
 886 The results obtained through this study show that by applying
 887 the previous guidelines, 3D hydrodynamic models can reproduce
 888 density current flows in stagnant receiving waters with errors of
 889 less than 1.3% for a minimum dilution line and of 6% for a maxi-
 890 mum velocity line. Due to the previous guidelines having been
 891 obtained from validations based on measurements taken under
 892 environmental controlled conditions, this contribution represents
 893 a first step toward the validation of such 3D hydrodynamic models
 894 for solving current flows under real environmental conditions.
 895 Accordingly, a next step would involve checking and validating the
 896 previous guidelines in field-scale applications where bathymetry
 897 and environmental conditions can have a significant influence on
 898 the density current evolution. Note that these models have been
 899 widely validated in terms of the reproduction of main coastal dy-
 900 namics either independently or through robust coupling with other
 901 models (e.g., atmospheric and wave models).

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