

AGH UNIVERSITY OF SCIENCE AND TECHNOLOGY

Faculty of Computer Science , Electronics and Telecommunications

FINAL PROJECT

Department of Telecommunications

Optimization of antenna coverage in telecommunication systems.

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*A final project submitted in fulfillment of the requirements
for the degree of Telecommunication Technologies Engineering
in the*

Department of Electronics and Telecommunications

June 28, 2018

Declaration of Authorship

I, Pablo Alonso Gonzalez, declare that this Final Project titled, "Optimization of antenna coverage in telecommunication systems." and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this final project has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this final project is entirely my own work.
- I have acknowledged all main sources of help.
- Where the final project is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

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Abstract

Faculty of Science and Technology
Department of Electronics and Telecommunications

Telecommunication Technologies Engineering

Optimization of antenna coverage in telecommunication systems.

by Pablo Alonso Gonzalez

This project main objective is to optimize antenna coverage in telecommunication systems. All steps that were following to achieve the final result will be explained. The cover of one road with only one type of antenna is optimized for 2-dimensional case. Special algorithm using fitness function is developed. The case was generalized to 3 dimensions so various types of antennas are used and their load was monitored.

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Chapter 1

Parametrization of Antenna Topology for 2 Dimensional Road System

1.1 Introduction

In this first chapter two dimensional road will be generated with aim of maximum coverage of road with certain strength of signal from antenna and the scheme for optimal antenna coverage will be presented trying to cover the total length of the roads. MATLAB environment will be used as the simulation tool during this project. All the steps necessary for obtaining results will be explained.

The roads can be generated randomly but the parameters of the roads will be set manually, so the same plots will be obtained for the whole project.

Trying to reduce the cost of workforce and be as realistic as possible all the antennas will be placed in the roads. By placing the antennas in the road, the workforce cost will be reduce.

1.2 Scheme for Generating Random Topology of Roads

The first part is the generation of the roads for that polar coordinates are going to be used (Figure 1.1). The parameter r can be defined with dependence of α .

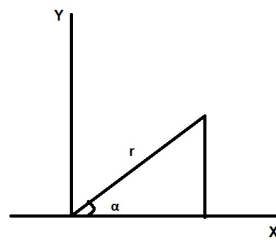


FIGURE 1.1: Point in the Polar Coordinates System.

In order to generate various shapes of roads we use $r(\alpha)$ function given as:

$$r(\alpha) = a_0 + a_1\alpha + a_2\alpha^2 + a_3\alpha^3 + a_4\alpha^4 \dots + a_n\alpha^n \quad (1.1)$$

Here $r(\alpha)$ is formed by N coefficients; that can be set manually or randomly. In this project we set 5 coefficients that will be set manually and kept constant. Each of the α coefficients is an integer.

The angle α is fixed and is different for each road. For each value of α a value of r is obtained and with both $(\alpha, r(\alpha))$ a point in polar coordinates is generated. Varying α in continuous way one can represent a road from point A to B.

1.2.1 Generating the Family of Topologies for 3 Roads

In situations with more than one road there occurs the intersections between roads. After generation of random roads with random topologies there occurs various intersections what is implemented by increasing α by small $\Delta\alpha$ at every proceeding point. One example is defined in Figure 1.2 with following values of a coefficient:

- Road 1: $a = [11 \ 20 \ -14 \ 6 \ -6]$; $\alpha = 0:0.01:12$
- Road 2: $a = [3 \ 1 \ -7 \ 15 \ -19]$; $\alpha = -3:0.01:9$
- Road 3: $a = [-5 \ 17 \ 4 \ -9 \ 8]$; $\alpha = -10:0.01:-2$

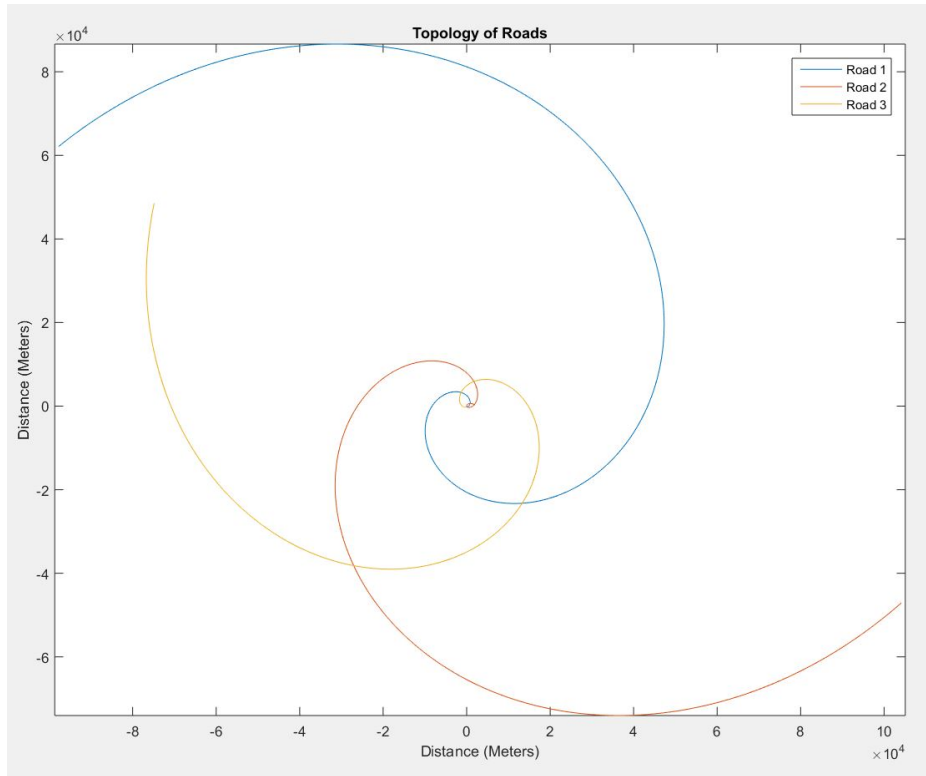


FIGURE 1.2: Example of Random Topology of 3 Roads.

1.3 Optimization of Antenna Position

Three different roads are generated initially. There are a lot of different ways of positioning the antennas, they can be placed near the road or far away, simulation will optimize antenna coverage. Positioning the antennas close to the road will reduce the cost of installation and this is one of the objectives of this project. Hence in the simulation the antennas will appear to be in the road proximity as it will be described in detail in the proceeding sections.

1.3.1 Optimization of Antenna Position for One Road of Arbitrary Shape

Steps that were followed during this project will be explained. Initially positioning of antennas was done only in single road, starting from relatively easy situation. The result obtained the first attempt is depicted in the Figure 1.3.

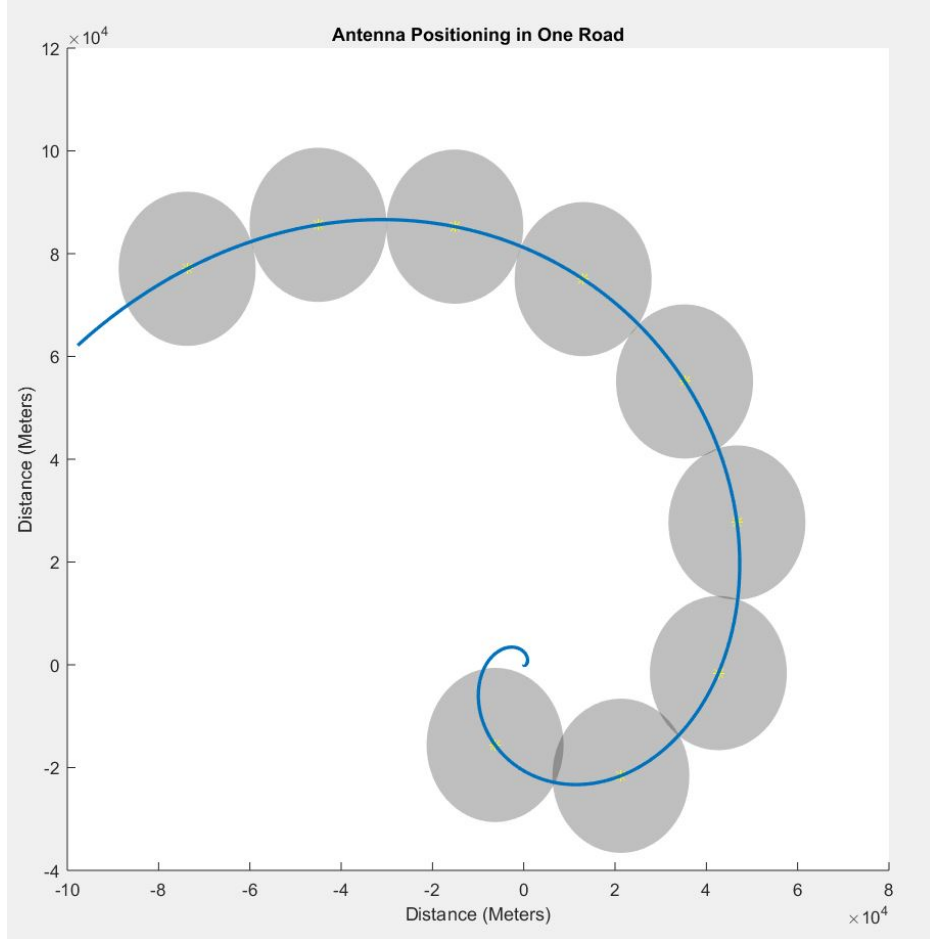


FIGURE 1.3: Optimization of Antenna Positions for One Road of arbitrary shape.

Only the interest road is display and as can be shown there are some problems because there are some points without coverage. Omnidirectional antennas are used so the antennas are placed antenna every X meter, where X is the double of the antenna range. The first problem that appears is that in the beginning and end of the road there is no antenna coverage. This problem is easy to solve at the start of the road the position of antennas was not set and at the end of the road there is no antenna because the distance with the previous one is less than X meters. The positioning of one antenna at the end of the road will be done if this point is not coverage, does not matter if the distance with the previous antenna is less than X meters. After fixing this small problems the result obtained is display in the Figure 1.4.

After improvement first road was fully covered. After deeper analysis we find that antenna positions are not fully optimal since some points of the road are covered by signal from more than one antenna. In such case we are dealing with system redundancy. The achieved solution was placing the first antenna at $X/2$ from the start of the road. The next ones should be placed as it was done in the previous step

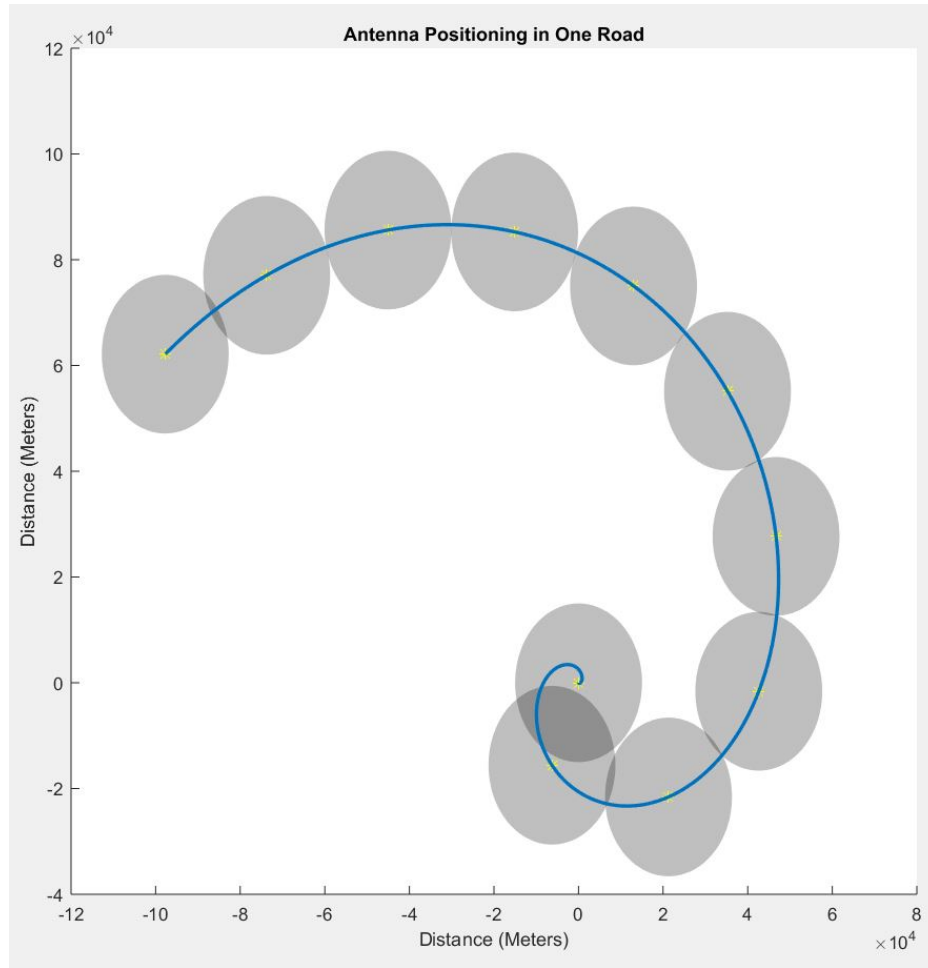


FIGURE 1.4: Optimization of Antenna Positions for One Road Improved.

(X meters from the previous antenna). In such way we obtain results given in Figure 1.5.

Now the optimal solution for one road coverage is achieved, the 100% of the road is covered with the minimum number of antennas. In the next sections the optimal solution for the 3 roads coverage will be obtained.

1.3.2 Optimization of Antenna Position for Complicated Road Topology

Now that the optimal solution for a single road signal coverage is obtained such approach will be useful for a more complicated road topology. The created algorithm in the previous step will be used in each road and the result will be presented. In the Figure 1.6 the result from applying the previous algorithm to each one of the roads is given.

As it is seen the 100% of the roads are covered but this solution is not optimal. The problems come from the crossing between roads, in such case more than one antenna covers few roads at once, and is possible to cover the hole map with less antennas. In the next example 25 antennas are used and one antenna provides signal coverage for more than one road. As result 100% road coverage with signal is obtained

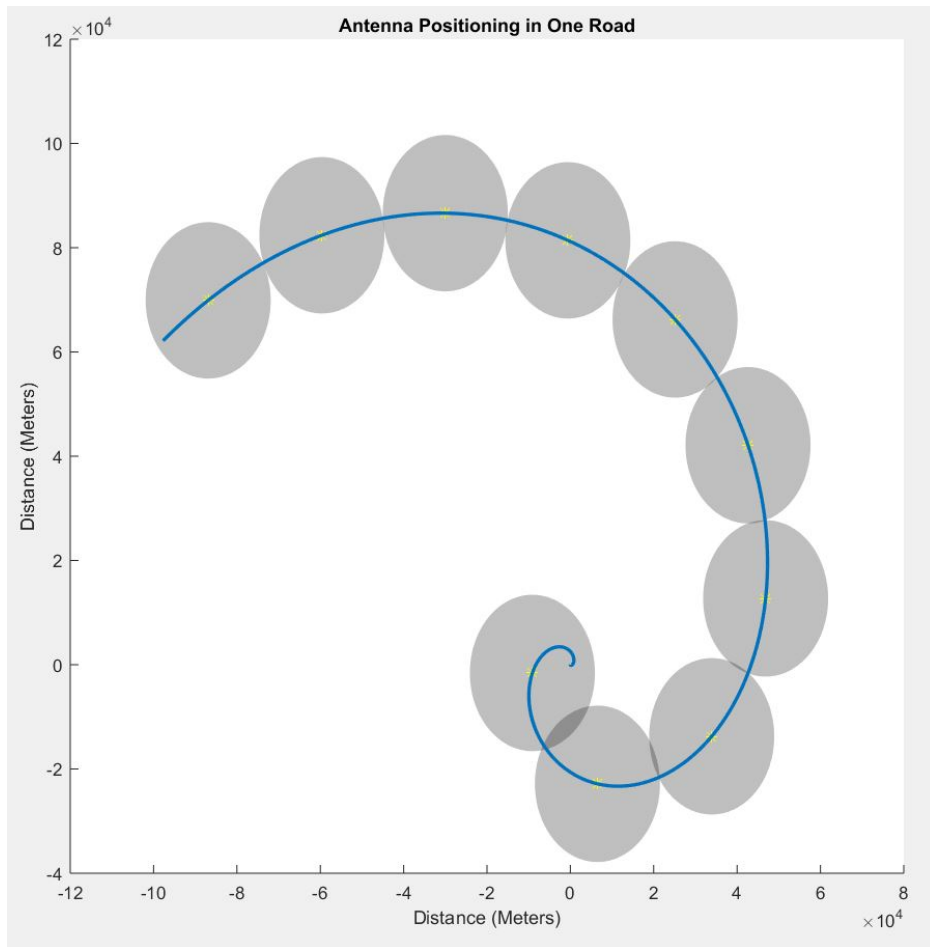


FIGURE 1.5: Optimization of Antenna Position for One Road.

1.4 Optimization of Antenna for Roads with Intersections

The accurate of intersections brings potential problems. In Figure 1.7 the intersection points between the roads are calculated.

Placement of antennas in all intersections is not optimal. Antennas will be placed in the intersections but if there is an antenna closer than X , that is the double of the range, then there will not be an antenna in the next intersection point. This was implemented and is visualized in Figure 1.8.

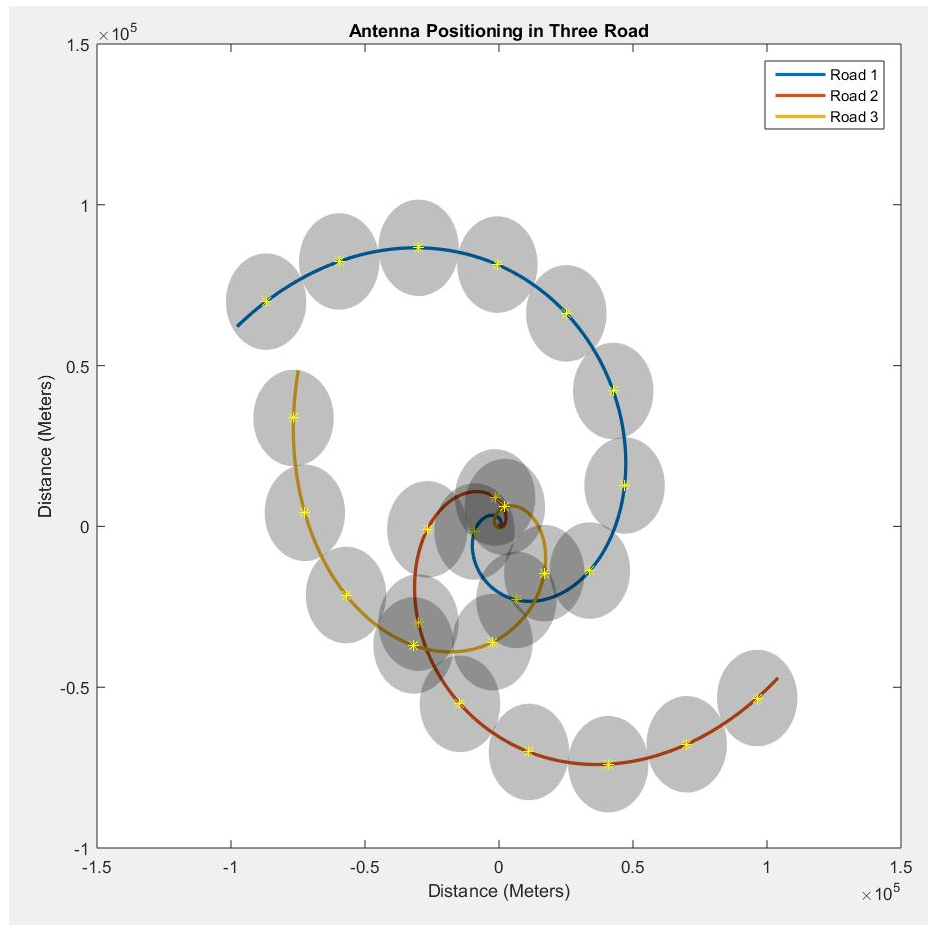


FIGURE 1.6: Optimization of Antenna Positions for Three Road System with Arbitrary Topologies in 2 Dimensions.

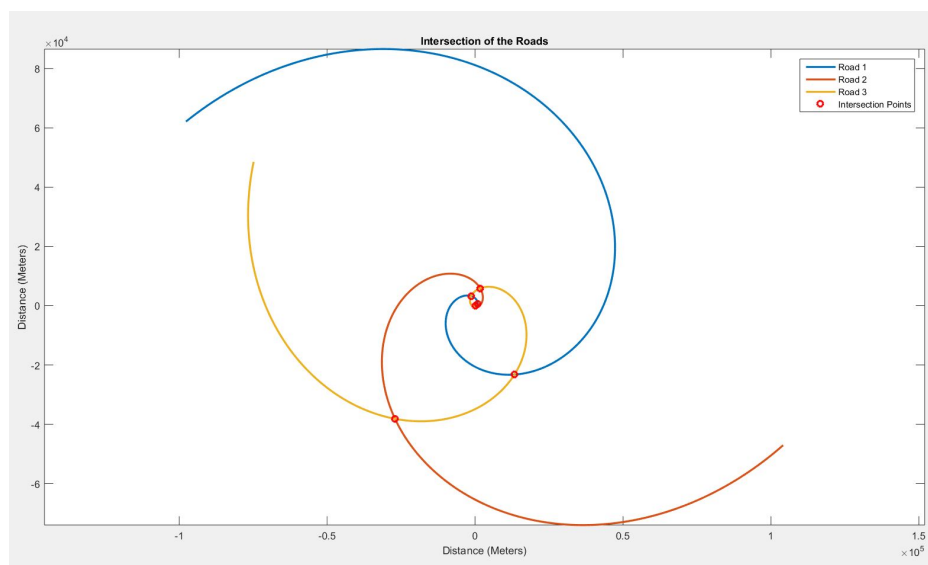


FIGURE 1.7: Intersection Points Between the Three Roads in Reference to Fig 1.6.

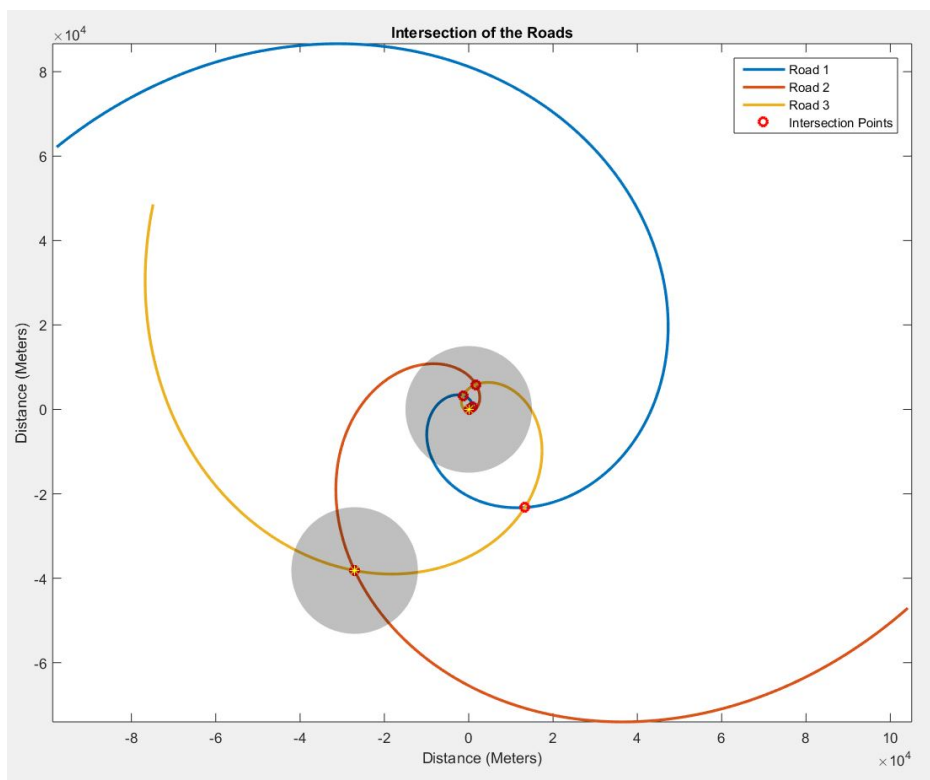


FIGURE 1.8: Example of Antennas Placement with Intersections Between Roads.

Chapter 2

Fitness Function Algorithm for Optimization of Antenna Coverage in Telecommunication System

2.1 Introduction

In second chapter the genetic algorithms for efficient roads coverage is described. The fitness function is going to be used during this chapter. In the previous chapter there was only one kind of antenna but in the presented artificial evolution 3 different types of antennas are going to be used. Each antenna has a different cost and range (Example is given in Table 2.1).

Antenna 1	Long-range (Radio = r)	Cost = c
Antenna 2	Long-range (Radio = 2r)	Cost = 3c
Antenna 3	Long-range (Radio = 3r)	Cost = 6c

TABLE 2.1: Range And Cost for Different Antennas.

The simple and initial case of fitness function can be given as:

$$FitnessFunction = \frac{coverage}{cost} k \quad (2.1)$$

Objective is to maximize the coverage and minimize the antenna cost in telecommunication systems. We aim maximize the value of fitness function. The k parameter is a coefficient that can be changed to improve the whole procedure.

2.2 Initial Conditions for Random Antenna Coverage

The first step is to place the 3 different types of antennas randomly and calculate the fitness function. Then we can search for antenna distributions that maximize fitness function and this approach most optimal antenna coverage for given road system. The obtained results were not fully satisfactory the problem is illustrated in Figure 2.1.

The signal road coverage was 54.95% and it is problematic. This problem was also indicated by fitness function values. there are proper solutions to this problems that needs to be found.

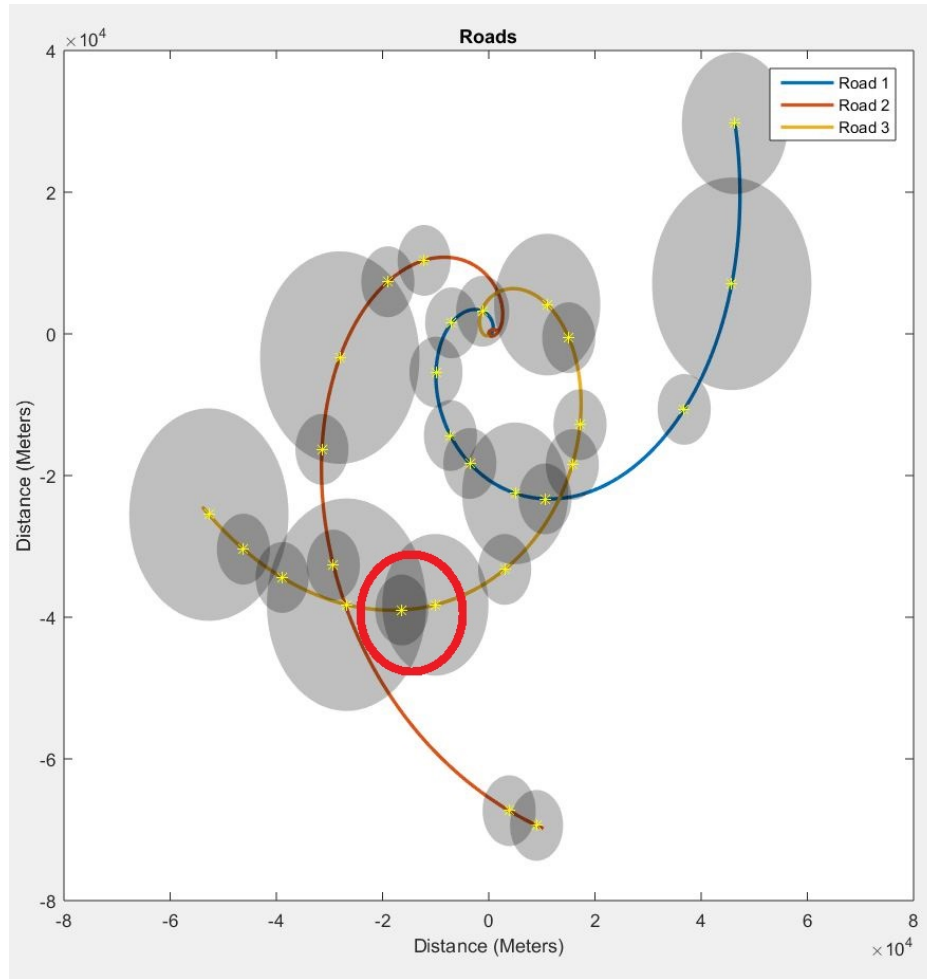


FIGURE 2.1: Case of Antenna Coverage with Big Redundance as Obtained by Random Initial Conditions.

2.3 Algorithm for Placing the Antennas in Coverage Radio Order

One idea for solving the problem of useless antennas is by placing them in coverage radio order. This means antennas with lower range are placed in first position and the ones with higher range are those placed in last stage. This solution solves the problem of the useless antennas as is given in Figure 2.3.

Moreover there is a stage when the program cannot continue because it cannot place more antennas of maximum range because the other antennas are in close proximity.

2.4 Evaluate the Fitness Function With Placement of New Antenna

In the previous sections of this chapter random number the antennas were generated, and they were placed randomly and after placement of all of them the fitness function is calculated. In this section the antennas are going to be placed one by one and the fitness function recalculated each time. If the fitness function gets better the

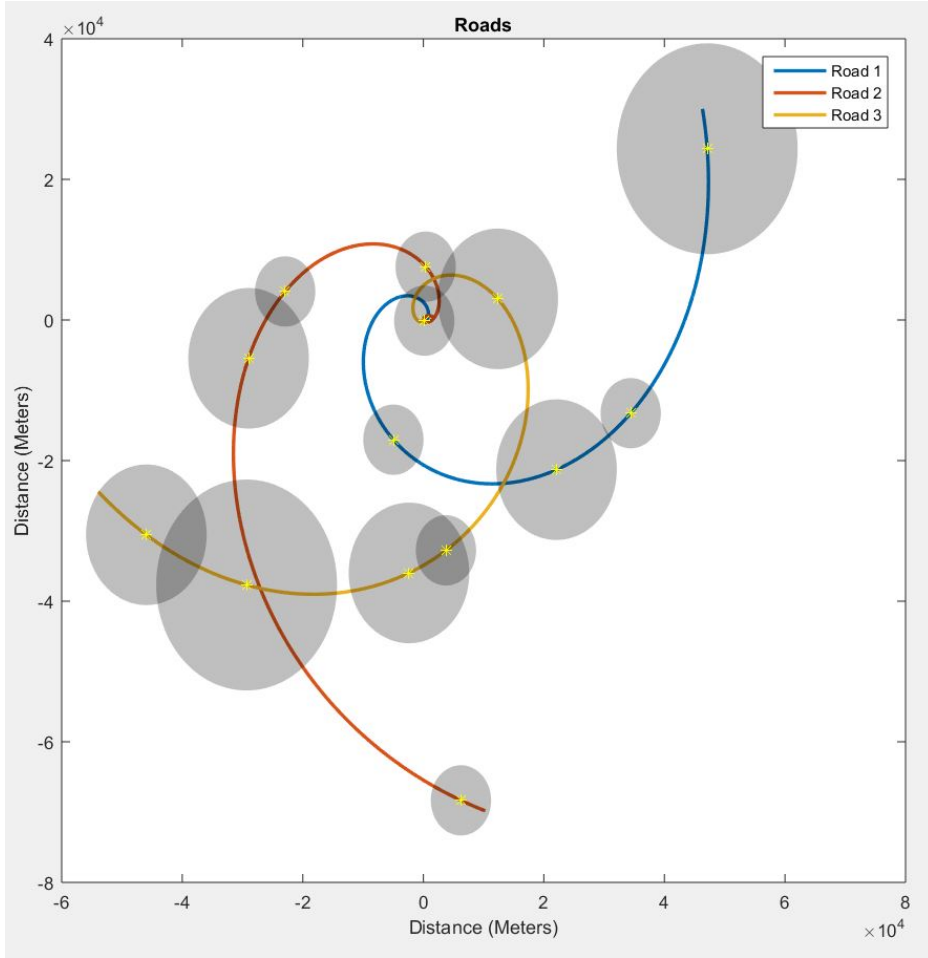


FIGURE 2.2: Coverage Map with Fitness Function Initial Random Conditions.

new antenna is going to be placed and if the fitness functions is not improved another random antenna is going to be generated. After enginery with fitness function results in Figure 2.4 and Figure 2.5 were obtained.

However there is a feeling that results can be further improved.

2.5 Evolution of Fitness Function for a Better System Performance

The program from the previous section is going to be used but in order to obtain better results the fitness function will be modified. The new one is changed to take more into account the coverage area and less the cost, but the cost still have an important point anyway. The proposed fitness function formula is:

$$FitnessFunction = \frac{coverage^8}{cost} \quad (2.2)$$

After one thousand simulation steps the result obtained is the one shown in Figure 2.6. There are a lot of the small antennas placed since the low cost of this type of antenna is taken into account.

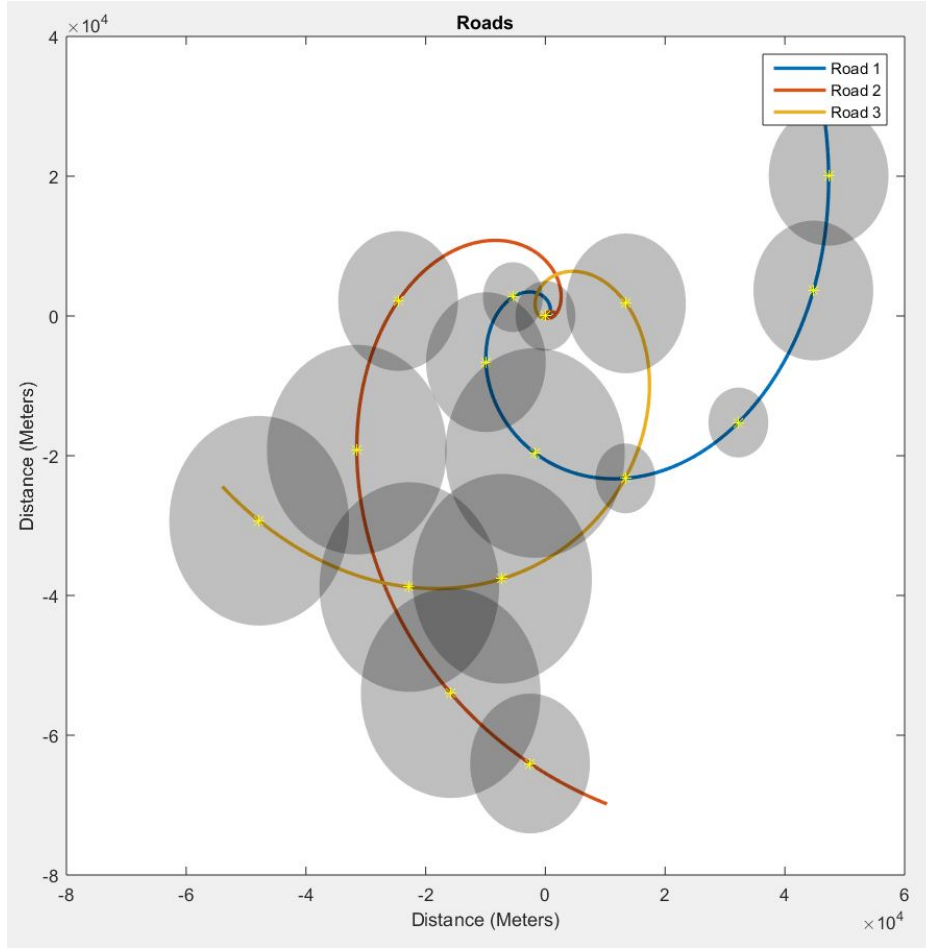


FIGURE 2.3: Placement of Antennas Minimizing Antenna Redundancy.

Another important plot that can be obtained is how the fitness function during one thousand iterations as can be seen in the Figure 2.7.

2.6 Case of Evolution of Fitness Function Targeted at 99% Road Coverage

The next step for improving the program is just by executing of simulations until the 99% of the roads are covered. This is simple to achieve by just adding a break in the loop that jumps when the % is equal or higher than 99. This results are display in Figure 2.8 and Figure 2.9. The number of iterations needed in order to obtain 99% coverage were 2032 iterations in this example.

2.7 New Adjustable Fitness Function

The previously used fitness function was simple and there is a need to change into a more complicated. The fitness function is the one given below:

$$FitnessFunction = \frac{a_0 + a_1coverage^2 + a_2coverage^3}{b_0 + b_1cost + b_2cost^2} \quad (2.3)$$

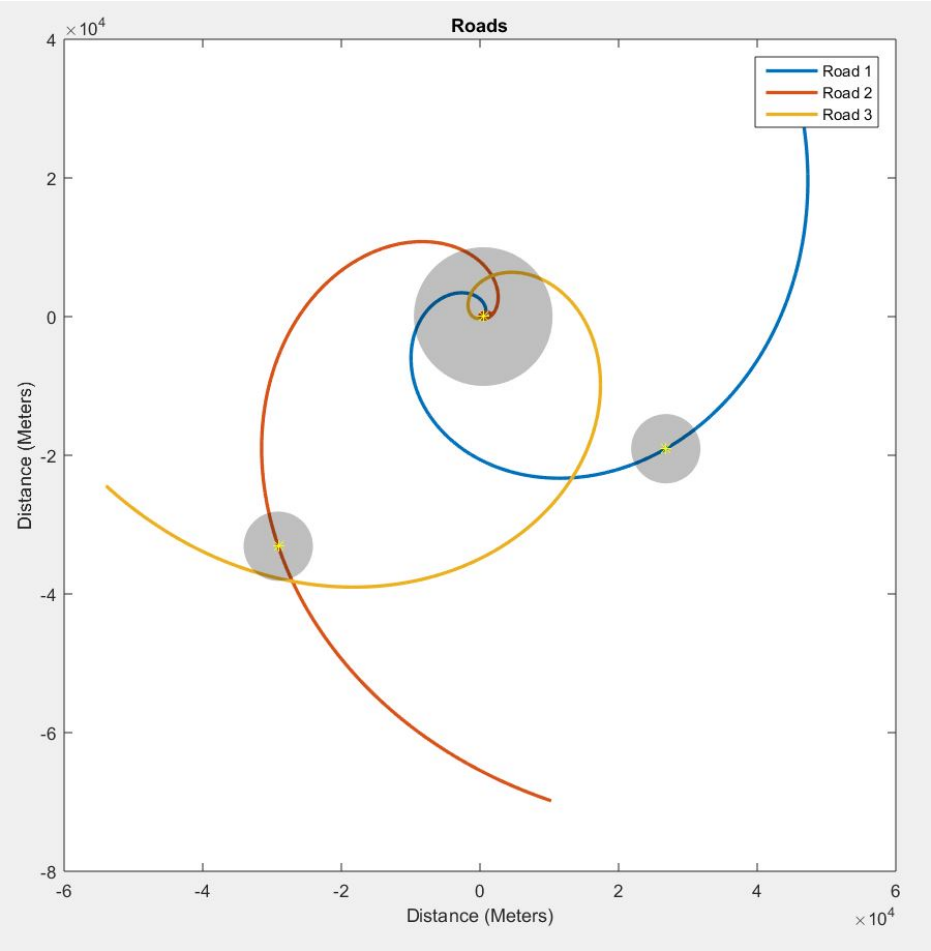


FIGURE 2.4: Distribution Coverage of Antenna Minimizing Antenna Redundancy and Cross-Section.

Here one engineer can a or b coefficients.

2.8 Algorithm for Placement and Removal

A solution problem of useless antennas includes the possibility of adding new antennas and removal of few antennas at once. Table 2.2 describes possible values.

Add 1 Antenna	81%
Remove 1 Antenna	10%
Change 1 Antenna Position	4%
Change 2 Antenna Position	3%
Change 3 Antenna Position	2%

TABLE 2.2: Stochastic Procedure for Adding, Removal and Change of Antennas Position.

By removing one antenna with 0.1 probability useless antennas will not appear. By changing the position of one or more antennas performance is improved. The results shown in the Figure 2.10, where it was reported no useless antennas occurs.

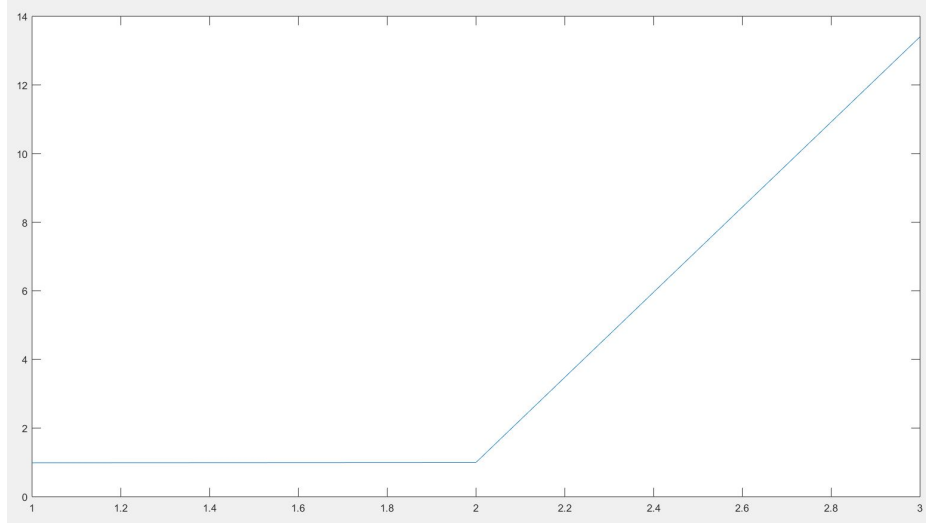


FIGURE 2.5: Evolution of Fitness Function in Case of Algorithm Described in Table 2.2.

The fitness function graph sometimes drops due to a bad antenna change or a removal of a good antenna. There is important to check the fitness function in each iteration and if the change does not improve the fitness function value it is necessary to reverse the change.

2.9 Signal Intensity Map

Electromagnetic signal intensity decrease with the distance. The phenomenological and proposed signal intensity is given by:

$$I = \frac{I_0}{d^2} \quad (2.4)$$

The result obtained is shown in Figure 2.11 and it corresponds to antenna coverage from Figure 2.12.

For improving the intensity map the intensity will be displayed in logarithmic scale because is better for visualizing the result of the intensity map. The logarithmic scale intensity map is displayed in the Figure 2.13

2.10 Generation of Random Obstacles and their Representation on the Map

In order to build more realistic problem random obstacles were generated and represented on the map. For the generation of these obstacles random polygons will be used with the number of sides set manually or randomly for each polygon. The polygons are generated in the centre of the map and later they are enlarged and displaced to a random position in the map. The result obtained looks like the Figure 2.14. In this example seven obstacles are generated.

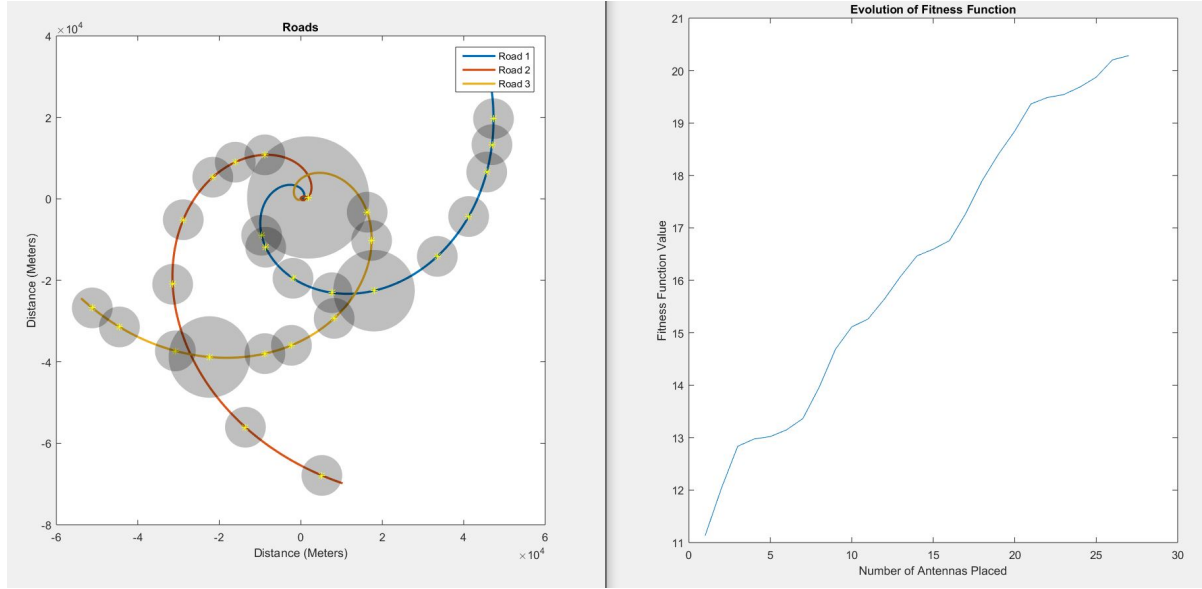


FIGURE 2.6: Antenna Placement and Fitness Function with Iterations for new Fitness Function from Formula 2.3.

2.11 Adding a Feature of Antenna Frequency Usage Into Fitness Function

The new aim is to consider the aspect of usage of each antenna. All the antennas should have the same frequency of usage in order to avoid the overload of antennas with high traffic. The length of road covered by given antenna is proportional to number of points. For making the number of points covered by each of the antennas will be calculated.

$L_1 \dots L_n$	Number of Points Covert by n Antenna
L_T	Sum off all L_n
L_a	Average Covert Points by One Antenna

TABLE 2.3: Parameters for Consider the Frequency of Usage.

$$FrequencyParameter = \frac{1}{1 + (L_1 - L_a)^2 + \dots + (L_n - L_a)^2} \quad (2.5)$$

We aim to optimize antenna frequency that is reflected in Frequency Parameter. And the new fitness function will result:

Fitness Function = Old Fitness Function \times Frequency Parameter

The final fitness function is the previous one multiplied by the parameter calculated with the equation 2.5. In similar war we can solve problems with overloading of some antennas at crossings.

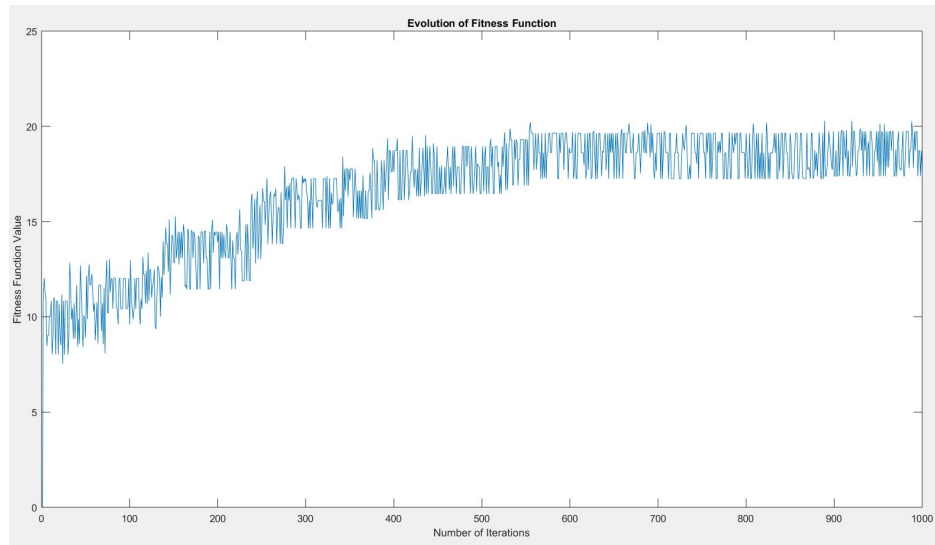


FIGURE 2.7: Evolution of the Fitness Function with Time for 1000 Iterations.

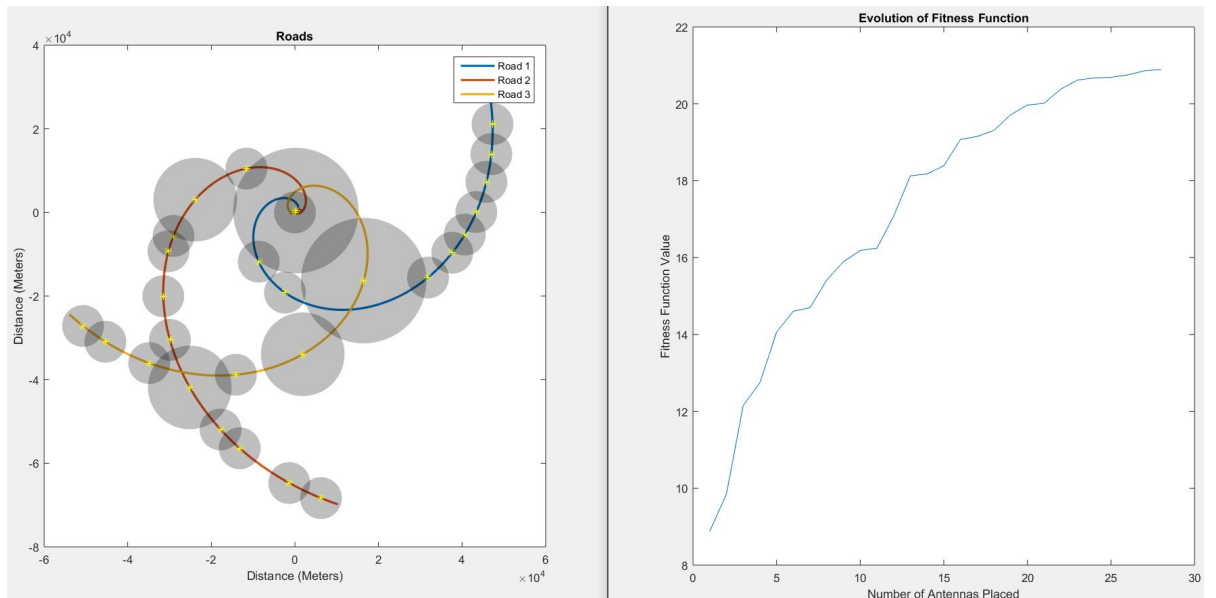


FIGURE 2.8: Antenna Placement and Fitness Function Evolution with Time

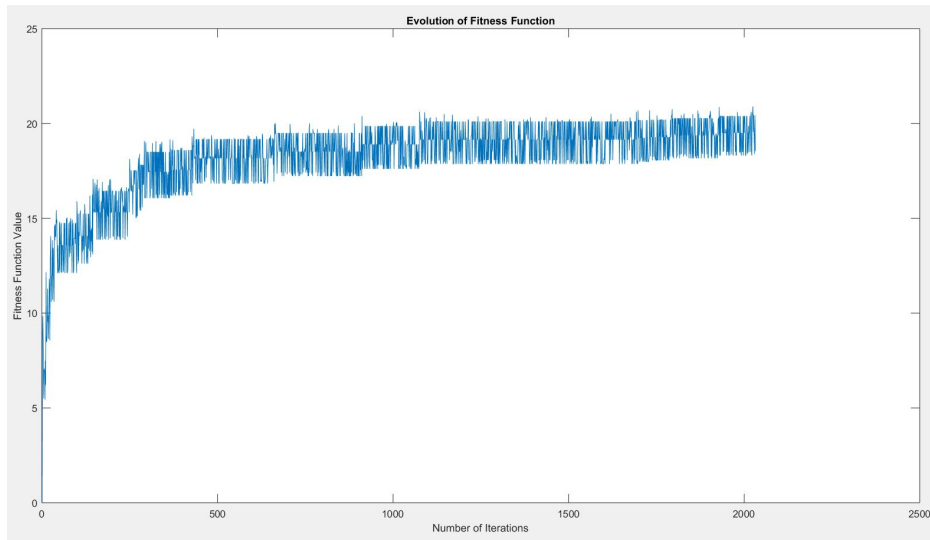


FIGURE 2.9: Evolution of Fitness Function with Time Aiming Coverage of 99% of Roads with Antenna Signal.

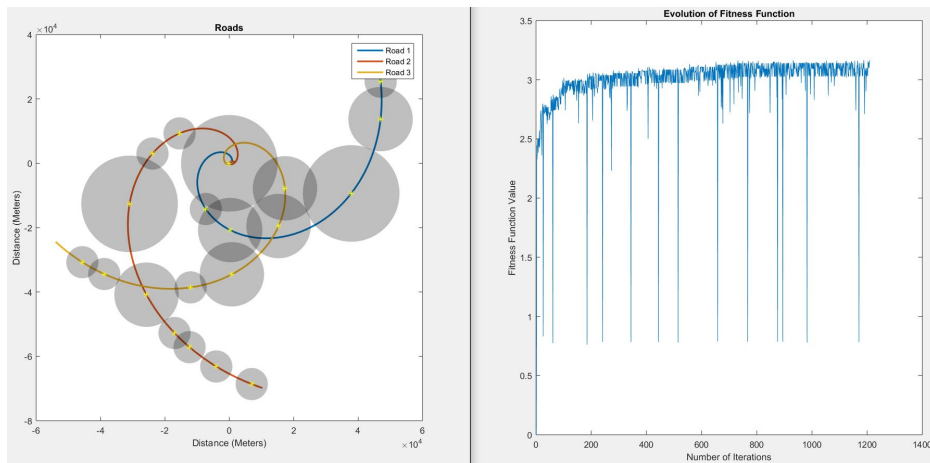


FIGURE 2.10: Antenna Placement and Fitness Function with Time With the New Improved Algorithm.

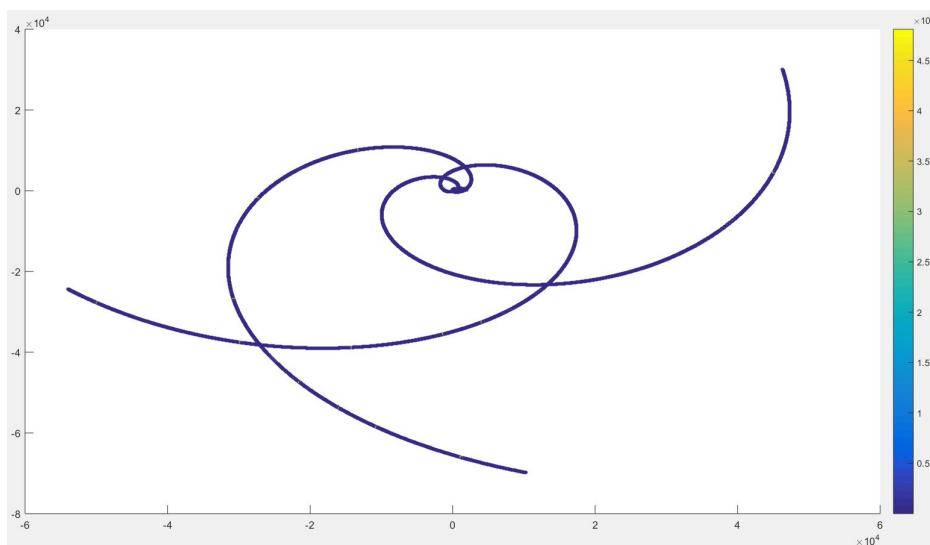


FIGURE 2.11: Intensity Signal Map Obtained from Formula 2.4.

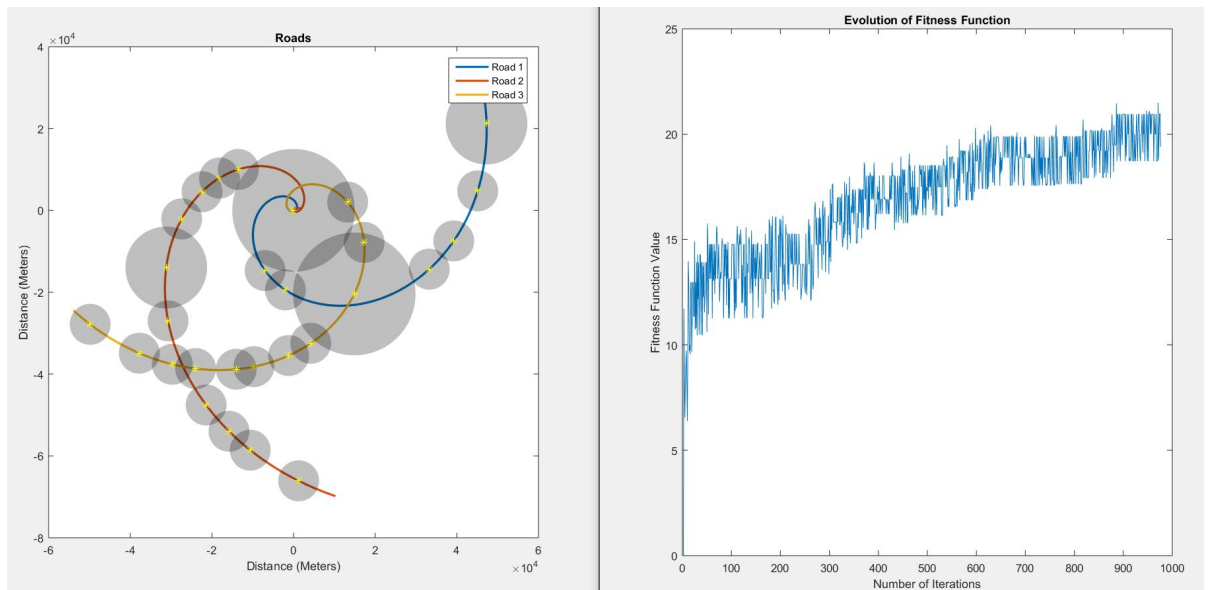


FIGURE 2.12: Final Antenna Placement Obtained as Result Fitness Function Evolution With Time.

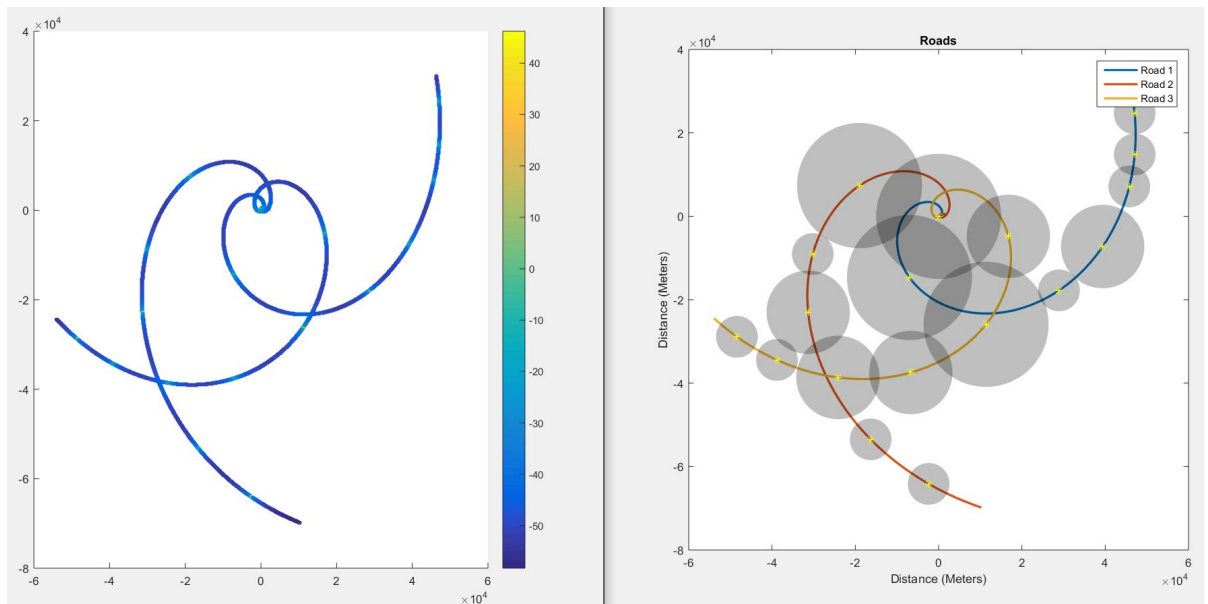


FIGURE 2.13: Antenna Placement and Intensity Map in Logarithmic Scale.

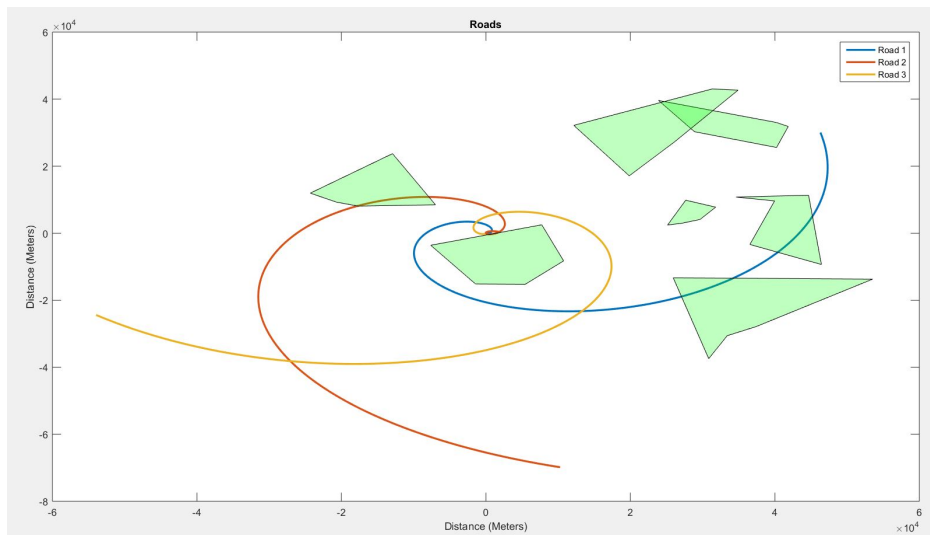


FIGURE 2.14: Representation of Random Obstacles in Telecommunication System.

Chapter 3

Parametrization of Antenna Topology for 3 Dimensional Road System and Optimization of Antenna Coverage

3.1 Introduction

The roads topology model used for simulation should be as realistic as possible, so the next step is to generate roads in three dimensions and cover them with antennas described in detail.

3.2 Parametrization of Antenna Topology for 3 Dimensional Road System

Spherical coordinates as visualized in Figure 3.1 are going to be used for the generation of roads in three dimension. The range of values of each parameter is specified in the Table 3.1.

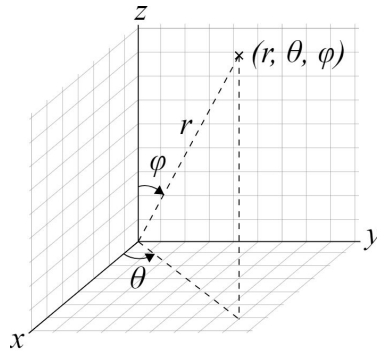


FIGURE 3.1: Spherical Coordinates System.

θ	$[0, \pi]$
φ	$[0, 2\pi]$
r	equation 3.1

TABLE 3.1: Parameters of Spherical Coordinates.

$$r(\theta) = a_0 + a_1\theta + a_2\theta^2 + a_3\theta^3 + a_4\theta^4 \dots + a_n\theta^n \quad (3.1)$$

After defining this and after setting values for the different constants results obtained are shown in Figure 3.2.

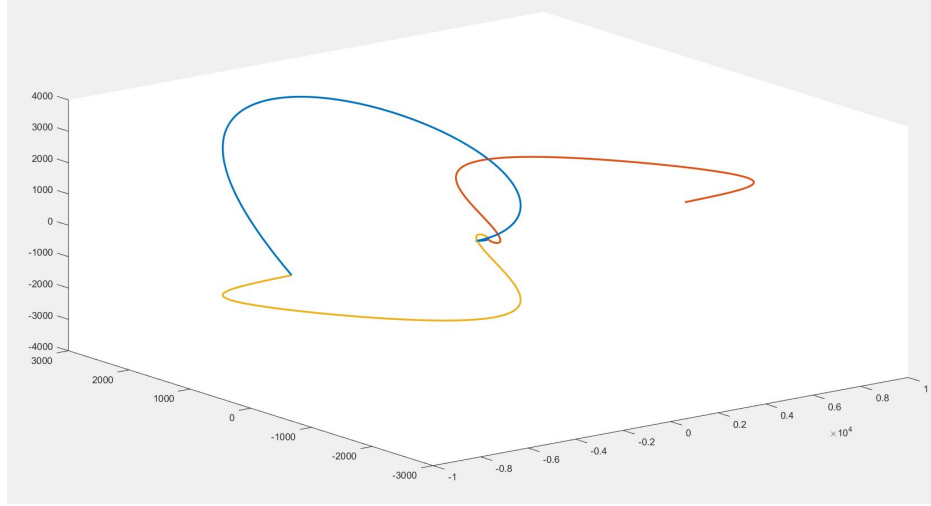


FIGURE 3.2: Topology of Roads in 3 Dimensions Obtained by Using Spherical Coordinates.

3.3 Signal Coverage After Application of Genetic Algorithm with Previous Fitness Function

The obtained results are one shown in the Figures 3.3 and 3.4, where the antennas topology, intensity map and fitness function evolution are displayed.

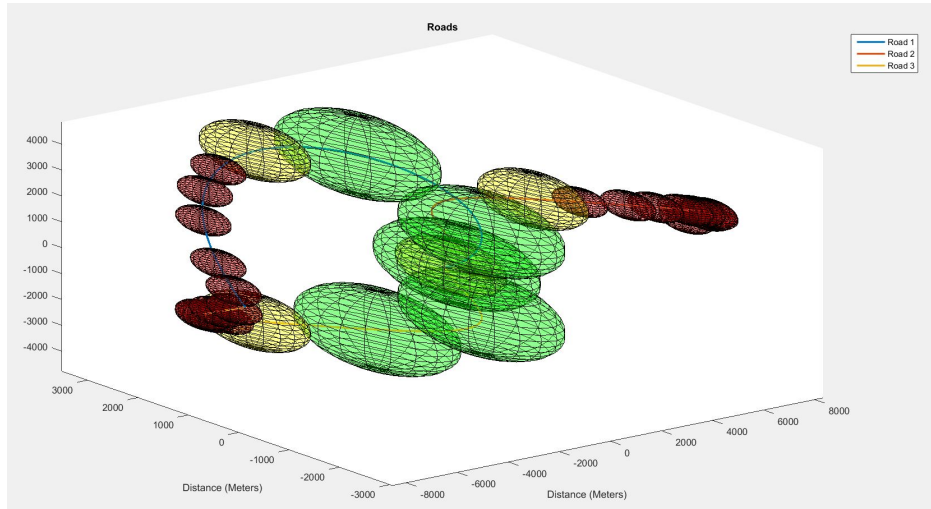


FIGURE 3.3: Antennas Topology in 3D Map.

As is shown the antenna coverage is done pretty good according with the results obtained in the chapter 2.

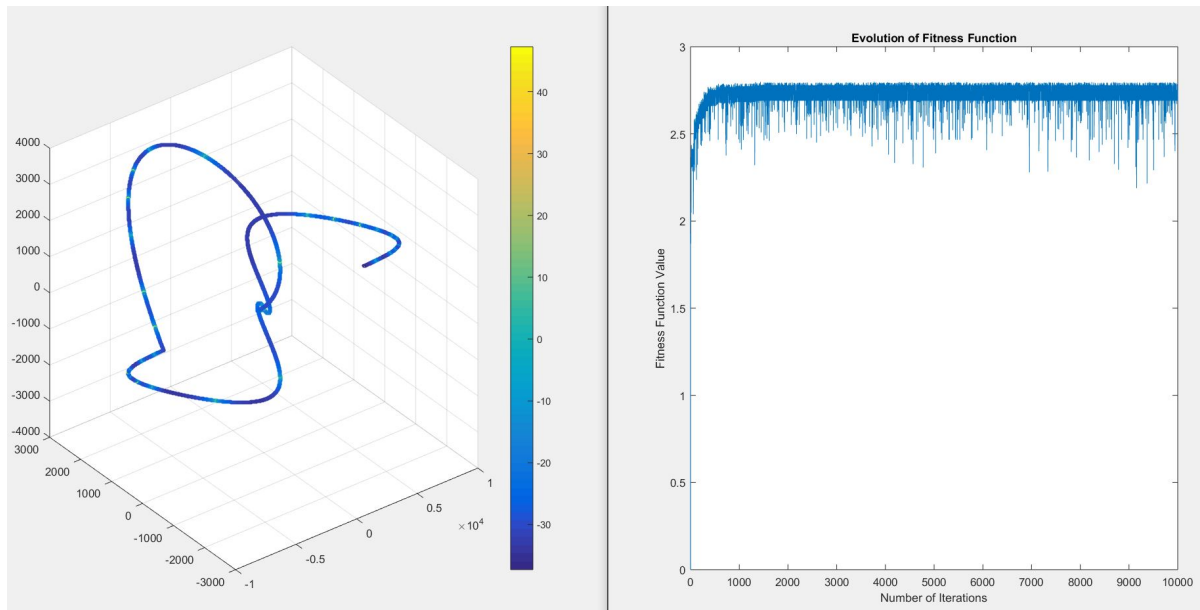


FIGURE 3.4: Example of Intensity Map and Fitness Function Evolution in 3D.

3.4 Monitoring the Antennas Generating Random Traffic

It is useful to know which antenna is going to be used more frequently. For this purpose traffic was generated. The first step is randomly deciding in which point of the road each car is going to start moving. Later we reverse order of cars movement.

We have reported that antenna number 2 is working 29.11 % of time as given in Figure 3.5.

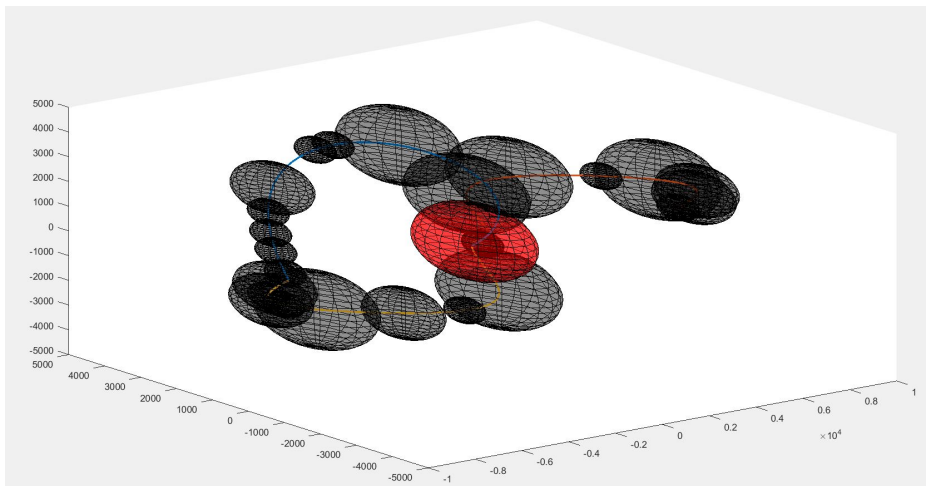


FIGURE 3.5: Antenna 2 is Highlighted in Red Color.

We have notice that antenna 27 is working only 0.39 % of time as given in Figure 3.6.

Antenna	Percentage
1	2.82%
2	29.11%
3	0.70%
4	3.61%
5	3.20%
6	0.71%
7	0.73%
8	2.91%
9	9.91%
10	3.88%
11	1.04%
12	7.20%
13	1.39%
14	0.74%
15	1.20%
16	7.49%
17	2.93%
18	2.69%
19	0.56%
20	0.51%
21	11.15%
22	0.64%
23	1.30%
24	1.14%
25	0.64%
26	1.32%
27	0.34%

TABLE 3.2: Percentage of Usage of Each Antenna Obtained in Case of Random Traffic Simulation.

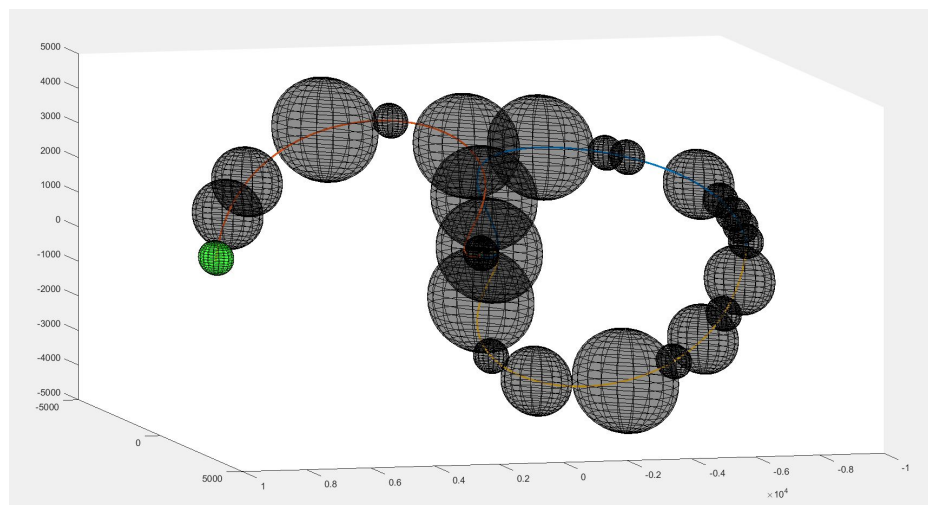


FIGURE 3.6: Antenna 27 is Highlighted in Green Color.

Bibliography

- [1] LaTeXTemplates.com *Template for a Masters Thesis*.
<https://www.overleaf.com/latex/templates/template-for-a-masters-slash-doctoral-thesis/m>
- [2] ShareLaTeX.com *Bibliography management with bibtex*.
https://es.sharelatex.com/learn/Bibliography_management_with_bibtex
- [3] MathWorks.com *Curve Intersections Function MATLAB. InterX*.
<https://www.mathworks.com/matlabcentral/fileexchange/22441-curve-intersections>
- [4] MathWorks.com *Filled Circle Function MATLAB*.
<https://www.mathworks.com/matlabcentral/fileexchange/27703-draw-a-filled-circle>
- [5] Wikipedia.org *Polar Coordinates System*.
https://en.wikipedia.org/wiki/Polar_coordinate_system
- [6] Wikipedia.org *Signal strength in telecommunications*.
https://en.wikipedia.org/wiki/Signal_strength_in_telecommunications
- [7] MathWorks.com *Transform polar to cartesian coordinates*.
<https://www.mathworks.com/help/matlab/ref/pol2cart.html>
- [8] MathWorks.com *Random Integers MATLAB*.
<https://www.mathworks.com/help/matlab/math/random-integers.html>
- [9] ShareLaTeX.com *Positioning images and tables*.
https://es.sharelatex.com/learn/Positioning_images_and_tables
- [10] WikiBooks.org *LaTeX Mathematics package*.
<https://en.wikibooks.org/wiki/LaTeX/Mathematics>
- [11] WikiBooks.org *LaTeX Labels and Cross-referencing*.
https://en.wikibooks.org/wiki/LaTeX/Labels_and_Cross-referencing
- [12] WikiBooks.org *LaTeX List Structures*.
https://en.wikibooks.org/wiki/LaTeX/List_Structures
- [13] Towards Data Science *Define a Fitness Function*.
<https://towardsdatascience.com/how-to-define-a-fitness-function-in-a-genetic-algorithm-1>
- [14] WikiBooks.org *LaTeX Tables*.
<https://en.wikibooks.org/wiki/LaTeX/Tables>
- [15] ShareLaTeX.com *Aligning Ecuations*.
https://es.sharelatex.com/learn/Aligning_equations_with_amsmath
- [16] MathWorks.com *Transform espherical to cartesian coordinates*.
<https://www.mathworks.com/help/matlab/ref/sph2cart.html>

- [17] MathWorks.com *Add Transparency*.
<https://www.mathworks.com/help/matlab/ref/alpha.html>
- [18] Stackoverflow.com *Random Polygon Function*.
<https://stackoverflow.com/questions/8997099/algorithm-to-generate-random-2d-polygon>
- [19] MathWorks.com *Color Bar*.
<https://www.mathworks.com/help/matlab/ref/colorbar.html>
- [20] MathWorks.com *Scatter Plot*.
<https://www.mathworks.com/help/matlab/ref/scatter.html>
- [21] Wikipedia.org *Spherical Coordinates System*.
https://en.wikipedia.org/wiki/Spherical_coordinate_system
- [22] MathWorks.com *Surface Plot*.
<https://www.mathworks.com/help/matlab/ref/surf.html>
- [23] MathWorks.com *FaceColor Surf Plot*.
<https://www.mathworks.com/help/matlab/ref/matlab.graphics.primitive.patch-properties>
- [24] MathWorks.com *Generate Sphere*.
<https://www.mathworks.com/help/matlab/ref/sphere.html>
- [25] MathWorks.com *Sort Array Elements*.
<https://www.mathworks.com/help/matlab/ref/sort.html>