R&E-SOURCE http://journal.ph-noe.ac.at
Open Online Journal for Research and Education
Special Issue #6, March 2017, ISSN: 2313-1640

PÄDAGOGISCHE HOCHSCHULE NIEDERÖSTERREICH

Deciding Geometric Properties Symbolically in GeoGebra

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Abstract

It is well known that Dynamic Geometry (DGS) software systems can be useful tools in the teaching/learning of reasoning and proof. GeoGebra 5.0 was recently extended by an Automated Theorem Prover (ATP) subsystem that is able to compute proofs of Euclidean geometry statements. Free availability and portability of GeoGebra has made it possible to harness these novel techniques on tablets, smartphones and computers. Then, we think it is urgently necessary to address the new challenges posed by the availability of geometric ATP's to millions of students worldwide.

Keywords:	*Schlüsselwörter:*
Dynamic geometry	Dynamische Geometrie
Automated reasoning	Automatisches Beweisen
GeoGebra	GeoGebra

1 Introduction

It is well known that *Dynamic Geometry* software (DGS) systems can be useful (as well as challenging) tools in the teaching/learning of reasoning and proof. In fact, DGS allow the student to formulate certain geometric facts (e.g. as intermediate steps towards establishing the proof of a given statement) by drawing auxiliary diagrams, and, then, getting convinced of the truth/falsity of the conjectured assertion by checking its validity in many instances, after randomly dragging some elements of the figure.

This has already raised some concerns:

> …increased availability in school mathematics instruction of … dynamic geometry systems… raised the concern that such programmes would make the boundaries between conjecturing and proving even less clear for students… [They] allow students to check easily and quickly a very large number of cases, thus helping students "see" mathematical properties more easily and potentially "killing" any need for students to engage in actual proving. (Lin & al., 2012)

Indeed, "dragging" is a characteristic feature of DGS systems and, therefore, the above expressed worries apply to all DGS. On the other hand[§], only a few DGS currently include another feature closely related to mathematics reasoning: that of having implemented an *Automated Theorem Proving (ATP)* algorithm, yielding the ability to confirm/deny the mathematical (i.e. not probabilistic) truth of a geometric statement.

DGS systems, such as *GeoGebra* (see Hohenwarter, 2002), including this *ATP* feature, can be considered as a kind of "geometry calculators" and, as such, they add, to the above expressed concerns about dragging, those—already well known but not yet well solved in mathematics education—related to the classroom use of arithmetic or scientific calculators. Can students be intellectually attracted to compute 23456769×98765432, once they know there is an algorithm that yields the correct answer 2316717923609208 and that it has been

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Partially supported by the Spanish Ministerio de Economa y Competitividad and by the European Regional Development Fund (ERDF), under the Project MTM2014-54141-P.
[§] https://en.wikipedia.org/wiki/List_of_interactive_geometry_software

implemented in their personal, say, tablet or phone? Likewise, will they be interested in finding whether the three heights of a triangle meet always at one point, if their pocket phone is able to guarantee, with a mathematical algorithm, that they do?

The answer is unclear to us; for example, maybe the two contexts (arithmetic, geometry) cannot be considered that parallel. Anyway we think there is a need to address urgently this issue, given the recent, large expansion of *GeoGebra* in the classrooms worldwide, with over 20 million users already in 2013 (Houghton, 2014) and the fact that *ATP* features have just been included in the most recent, version 5, of this software tool (see Kovács, 2015a).

The purpose of this communication is to contribute with a very small step forward to the better understanding of this involved issue. Namely, we would like to summarily describe how *ATP* is implemented in *GeoGebra* as a symbolic extension of the previous existing, numerically oriented, *Relation Tool* (see Kovács, 2015b) and, in particular, how we have designed the user interface (for example, so that the user can easily perceive the difference between a conjectural statement with a mere visual check and a fully proven theorem).

2 The Relation Tool in GeoGebra 5

GeoGebra (see Hohenwarter, 2002) is educational mathematics software that primarily focuses on facilitating student experiments in Euclidean geometry. Since *Automated Theorem Proving (ATP)* in geometry has reached a rather mature stage, some *ATP* experts agreed on starting a project of incorporating and testing a number of different automated provers for geometry in *GeoGebra*. This collaboration was started in 2010 by the authors and other participants (see Botana & al., 2015). This work was built upon previous approaches and achievements of a large community of researches, involving different techniques from algebraic geometry, formal logic and computer algebra. Moreover, various symbolic computation, open source, packages have been used, most importantly the *Singular* (Decker & al., 2012) and the *Giac* (Parisse, 2013) computer algebra systems. See Kovács, 2015a and Kovács, 2015b for a more detailed overview.
The result is a sophisticated, extended version of the *Relation Tool*, a command that was already available in the first versions of GeoGebra (back in 2002), but with *numerical* checks only. The *Relation Tool,* in its original form, allows selecting two geometrical objects in a construction, and then to check for typical relations among them, including[**] *perpendicularity*, *parallelism*, *equality* or *incidence*. Finally, it shows a message box with the obtained information (yes/no the relation holds). *GeoGebra* version 5 now displays an extra button in the message box with the caption “More...” which results in some *symbolic* computations when pressed. That is, by pressing the “More...” button, *GeoGebra*'s *ATP* subsystem starts and selects (by some heuristics) an appropriate prover method to decide if the numerically obtained property is indeed absolutely true in general. Current version of *GeoGebra* (5.0.152) is capable of choosing a) *the Gröbner basis method*, b) *Wu's characteristic method*, c) *the area method*, or d) *Recio's exact check method* as the underlying *ATP* technique.
Moreover, if the conjectured relation does not (mathematically speaking) hold, the first two methods can determine some geometrical extra-conditions, which need to hold true in order to make the given statement generally correct. These are the so-called *non-degeneracy conditions,* which usually prescribe that some of the input objects (for example, a freely defined triangle) should not be degenerate for the relation to hold true.
The *Relation Tool* is already available not only on the classic desktop platform of *GeoGebra* (which is written in Java), but also in the web version (which is mostly a machine compilation from *Java* to *JavaScript*, see Ancsin, Hohenwarter & Kovács, 2013). Newest improvements in *GeoGebra* also provide the students with a tablet version (Fig. 1a and 1b). An early version of the prover subsystem for smartphones is shown in Fig. 2. A benchmark suite with 187 test constructions is available at http://bit.ly/1OVca7q in files jar-paper.html (without non-degeneracy conditions) and ndg.html (with conditions).
In the current version of *GeoGebra* the user cannot obtain a visible or readable proof of the proven theorem by merely using the *Relation Tool*. Nevertheless, the reported result is based on a mathematically correct proof, which is philosophically a completely different, higher level result than a collection of instances obtained by dragging, that is, the classic DGS way.

[**] See https://wiki.geogebra.org/en/Relation_Command for a full list.

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3 Conclusion

As a conclusion we think it is enough to quote here the early message contained in the premonitory ICMI Study (Howson & Wilson, 1986):

...even if the students will not have to deal with computers till they leave school, it will be necessary to rethink the curriculum, because of the changes in interests that computer have brought.

Actually, it is more than necessary: it is urgently needed to address the new challenges posed by *DGS* programs with geometric *ATP* features, now freely available on multiple platforms to millions of students worldwide. *GeoGebra* is a good example, as we have described in this note.

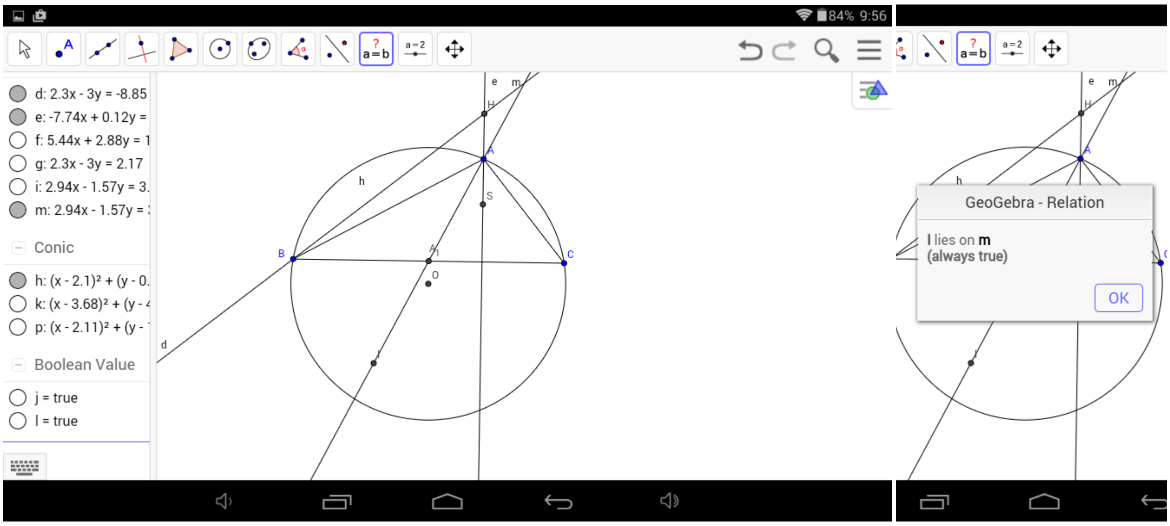

Fig. 1a and 1b: Chou's *Example 230* (Chou, 1987) in the Android tablet version of GeoGebra: *Show that the symmetric (S) of the orthocenter (H) of a triangle (ABC) with respect to a vertex (A), and the symmetric (I) of that vertex with respect to the midpoint of the opposite side (A_1), are collinear with the circumcenter (O) of the triangle.* Note that the Gröbner basis method gives a simpler output for the non-degeneracy conditions than originally reported by Chou.

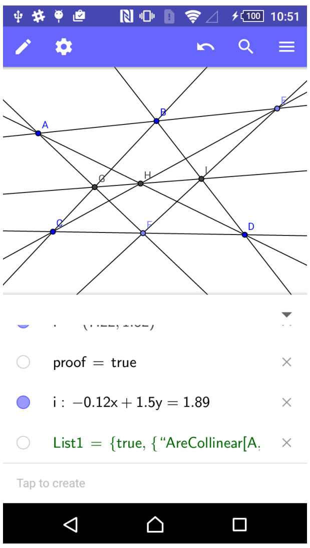

Fig. 2: Investigating Pappus' hexagon theorem on an Android phone.
GeoGebra's **Prove[AreCollinear[G,H,I]]** and **ProveDetails[AreCollinear[G,H,I]]** commands return **proof=true** and **List1={true, {"AreCollinear[A,B,C]", "AreCollinear[A,C,D]"}}** which mean that the theorem is true if both the triangles ABC and ACD are non-degenerate.

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