Chiral Media Characterization using both Linear and Circular Polarized Waves

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***Abstract*—A new free-wave experimental method for chiral media characterization is presented. The setup is based on a pair of dual horn antennas and a diplexer 90° phase shifter, which allows to produce both circularly and linearly polarized waves and to measure the co- and cross-polar components of the transmission and reflection coefficients of a chiral slab under normal illumination. Then, six experimental measurements are available to retrieve three constitutive parameters. Furthermore, by including the reflection coefficient of the backed metal sample an over determined system of equations is obtained, which leads to increase the accuracy of the material characterization.**

I. Introduction

During recent decades, a great variety of novel and complex materials have been designed with promising practical applications. These new materials represent a challenge to the standard characterization techniques that must be able to deal with, for example, extreme values for the constitutive parameters, abrupt changes in magnitude or negative refractive index. Furthermore, in same cases the relations between electric and magnetic fields cannot be described by the standard constitutive equations and additional parameters must be considered. This is the case of chiral media (CM), whose macroscopic behavior can be described by including the chirality κ parameter into the constitutive equations [1]:

$$\vec{D} = \epsilon_r \epsilon_0 \vec{E} - j\sqrt{\epsilon_0 \mu_0}\kappa \vec{H}, \quad (1)$$

$$\vec{B} = \mu_r \mu_0 \vec{H} + j\sqrt{\epsilon_0 \mu_0}\kappa \vec{E}, \quad (2)$$

where ϵ_r and μ_r are the relative permittivity and permeability, respectively. The electromagnetic behavior of CM presents two effects: electromagnetic rotatory dispersion, which causes a rotation of the polarization direction for a linearly polarized (LP) wave, and circular dichroism (a change in the polarization from linear to elliptical) due to the different absorption coefficients of right and left circularly polarized (CP) waves. A large variety of resonant structures has been designed and analyzed in order to produce high values of optical activity, circular dichroism and negative refractive indices [2]–[3].

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The presence of the chirality parameter in the constitutive equations do necessary to measure, at least, three independent quantities in order to retrieve ϵ_r, μ_r and κ [4]. The standard free-wave setup consists of a pair of transmitting and receiving antennas to determine both the reflection coefficient and the co- and cross-polar components of the transmission coefficients when the slab sample is irradiated with LP waves [5]. The inherent errors associated to the measurements and the strong variations in the constitutive parameters make, in some cases, quite complicated to retrieve the constitutive parameters. Recently, CP waves were used to determine the rotation angle, ellipticity and chirality parameter, without being necessary any normalization process [6].

We present a new method to characterize CM, based on the reflection and transmission coefficients of both CP and LP waves and the reflection coefficient of a metal backed sample. The technique allows two independent measurements of the transmission coefficients. Thus, a over determine system of equations is obtain, increasing the accuracy of the retrieved parameters. The experimental validation are under development.

II. Method and Discussion

The experimental setup consists of a pair of transmitting and receiving dual polarized horn antennas, a sample holder and a vector network analyzer, Fig. 1. The two ports of the polarized receiving antenna allow the measurement of the transmission coefficient in two perpendicular directions. In the CP waves configuration, the two ports of the transmitting antenna are fed by a diplexer 90° phase shifter, which is removed for LP waves. The reflection and transmission coefficients for LP waves are given by [1]:

$$T_{xx} = \frac{2\eta_r \cos(\kappa k_0 d)}{2\eta_r \cos(nk_0 d) + j(\eta_r^2+1)\sin(nk_0 d)}, \quad (3)$$

$$T_{yx} = \frac{-2\eta_r \sin(\kappa k_0 d)}{2\eta_r \cos(nk_0 d) + j(\eta_r^2+1)\sin(nk_0 d)}, \quad (4)$$

$$R_{xx} = \frac{j\sin(nk_0 d)(\eta_r^2-1)}{2\eta_r \cos(nk_0 d) + j(\eta_r^2+1)\sin(nk_0 d)}, \quad (5)$$

$$R_{yx} = 0, \quad (6)$$

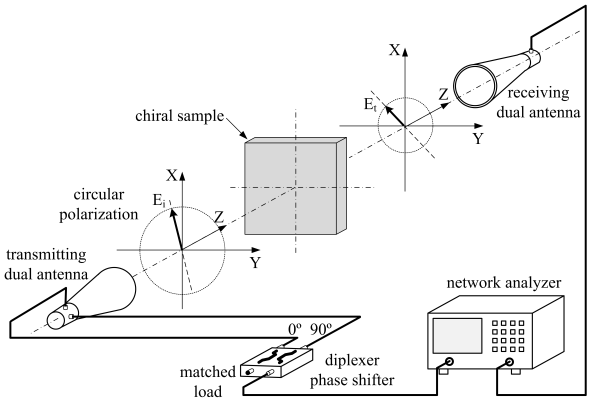

Fig. 1. Setup for CM characterization in the circular polarized waves configuration. For linear polarization, the diplexer is removed.

where d is the sample thickness, $\eta_r = \sqrt{\mu_r/\epsilon_r}$ the normalized impedance, $k_0 = \omega\sqrt{\epsilon_0\mu_0}$ and $n = \sqrt{\mu_r\epsilon_r}$ the refractive index.

One of the advantages of the dual polarized transmitting antenna is the possibility of determining R_{yx} and to check that it must be null for CM. If this condition is not fulfilled, it means the existence of achiral effects such as a wrong sample location/orientation or anisotropic behavior in the $x-y$ plane.

The usual LP waves method makes use of R_{xx}, T_{xx} and T_{xy} and define the parameter $T^2 = T_{xx}^2 + T_{yx}^2$ in order to calculate ϵ_r and μ_r [5]. Here we propose to calculate T^2 using the transmission coefficients for both CP and LP waves:

$$T^2 = T_R T_L = T_{xx}^2 + T_{yx}^2, \quad (7)$$

where the transmission coefficients for right and left CP waves (T_R and T_L, respectively) are given by [1]:

$$T_R = \frac{2\eta_r e^{-j\kappa k_0 d}}{2\eta_r \cos(nk_0d) + j(\eta_r^2+1)\sin(nk_0d)}, \quad (8)$$

$$T_L = \frac{2\eta_r e^{+j\kappa k_0 d}}{2\eta_r \cos(nk_0d) + j(\eta_r^2+1)\sin(nk_0d)}. \quad (9)$$

As a results, two independent determinations for T^2 can be used in the standard retrieval algorithm. From T_R and T_L, the chirality parameter is obtained by using [6]:

$$\kappa = \frac{1}{2k_0d}\left(j\ln\left|\frac{T_L}{T_R}\right| + \arg\left(\frac{T_L}{T_R}\right)\right) + \frac{2\pi p}{k_0 d}, \quad (10)$$

where p is a frequency dependent integer that affects the real part of κ and can be determined by assuming a null chirality far from the resonance ($p = 0$) and both real and imaginary parts must resonate at the same frequency.

The reflection coefficients for right and left CP waves must be equal for CM and are expressed as [1]:

$$R_R = R_L = \frac{j\sin(nk_0d)(\eta_r^2-1)}{4\eta_r\cos(nk_0d) + j2(\eta_r^2+1)\sin(nk_0d)}, \quad (11)$$

being the reflection coefficients related by $R_{xx} = R_R + R_L$.

We propose a new retrieval algorithm using (11) and the reflection coefficient of the metal backed sample:

$$R_{xx}^* = \frac{1 + (\eta_r^2-1)\sin^2(nk_0d)}{1-(\eta_r^2+1)\sin^2(nk_0d) + j\eta_r\sin(2nk_0d)}, \quad (12)$$

where the coefficient was obtained by applying the continuity conditions at the sample and metal interfaces.

By doing the change of variables: $\rho = (\eta_r - 1)/(\eta_r + 1)$ and $P = e^{jnk_0d}$, (5) and (12) can be rewritten as:

$$R_R + R_L = \frac{\rho(1-P^2)}{1-\rho^2P^2}, \quad (13)$$

$$R_{xx}^* = \frac{\rho - P^2}{1-\rho P^2}. \quad (14)$$

Clearing P^2 in (13)–(14) and equaling, a three degree equation for ρ is obtained, which produces several values for n and η_r. Therefore, several values are also expected for the relative permittivity and permeability, which are obtained by:

$$\epsilon_r = \frac{n}{\eta_r}, \qquad \mu_r = n\,\eta_r. \quad (15)$$

This multi evaluation is overcome by starting the retrieval far from the resonance, when a dielectric behavior is expected, and considering continuous changes in the variables. A special care must be taken into account at the resonant frequency range, where abrupt changes can be expected in the real part of n and η_r. At these frequencies, it is necessary to consider that n and η_r must resonate at the same frequency and the imaginary part of both variables follow a continuous behavior.

III. Conclusions

A new method for CM characterization based on dual polarized horn antennas has been presented. The experimental system allows to illuminate the sample using both LP and CP waves, keeping the same setup. The proposed setup provides more experimental magnitudes than previous systems and, therefore, can help to reduce the experimental errors. Furthermore, the reflection coefficient of the sample backed with metal can be also used to retrieve material constitutive parameters.

References

[1] I. V. Lindell, A. H. Sihvola, S. A. Tretyakov, and A. J. Vitanen, *Electromagnetic Waves in Chiral Media,* Boston, USA: Artech House, 1994.

[2] T. G. Mackay, and A. Lakhtakia, "Negatively Refracting Chiral Metamaterials: A Review", *J. Photon. Energy*, Vol. 1, pp 018003(1–29), 2010.

[3] B. Wang, J. Zhou, T. Koshny, and C. M. Soukoulis, "Nonplanar chiral metamaterials with negative index", *Applied Phys. Let.*, Vol. 94, 151112(1–3), 2009.

[4] R. Ro, V. V. Varadan, and V. K. Varadan, "Electromagnetic Activity and Absorption in Microwave Composites", *Proc. Inst. Elect. Eng. H,* 139(5), 441-448, 1992.

[5] J. Margineda, G. J. Molina-Cuberos, M. J. Núñez, A. J. Garcia-Collado, and E. Martin, "Electromagnetic Characterization of Chiral Media", *Electromagnetic Waves Propagation in Complex Media*, Rijeka, Croacia: InTech, 2012, pp 3–24.

[6] E. Martín, J. Muñoz, A. J. García-Collado, J. Margineda, and G. J. Molina-Cuberos, "Chiral retrieval method based on right circularly polarized and left circularly polarized waves", *Meas. Sci. Technol.*, Vol. 25, pp 115004(1–5), 2014.